Political Economy of Voting Behavior, Rational Voter Theory and Models of Strategic Voting

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The objective of the project is extension of analysis of voting behavior and models from the standpoint of possible strategic voting (sometimes associated with manipulation). By strategic voting we mean voters' behavior promising maximization of expected individual or group utilities. By manipulation we mean such strategic behavior of one group that wants to influence some other voters to vote against their interests but for interests of manipulating group. As an innovation to existing approaches we want to introduce into the voting models categories of non-rational, semi-rational and rational voters and to study also information complexity of strategic voting. Game-theoretical and other operations research approaches (multi-criteria optimization) will be employed as a general framework of the models. There are two types of research output anticipated: theoretical models as methodological contribution to rational voters' theory, and applications, empirical studies of the new electoral history of the Czech Republic (1992-2010) and voting rules in European Union with an emphasis on elements of strategic voting.

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CONCEPTS

Voting *is the act of registering a choice between alternatives - either between candidates, parties or questions. Since in democratic societies voting occurs on everything from town meeting questions to presidential elections, it is not surprising that both economics and political sciences are focusing attention to the topic for about last 250 years*

Political economy of voting *is based on the rational voter model (FELDMAN and SERRANO, 2006, ELIN, GELMAN, KAPLAN, 2007), derived from rational choice theory. In this model, voters are short-term instrumentally rational. That is, voters have a set of sincere preferences, or utility rankings, by which to rate candidates (alternatives); voters have some knowledge of each other's preferences; and voters understand how best to use voting to their advantage (BRAHAM, STEFFEN 2008). The extent to which this model resembles real-life elections is the subject of considerable academic debate (STODDER 2005).*

Sincere voting *– individual voter's choice is based on selecting the best alternative (maximizing her utility providing it is the winner) independently on information about other voters' choice.*

In voting systems, ***strategic voting*** *(or tactical voting) occurs when voter anticipates behavior of other voters and misrepresents his or her sincere preferences in order to gain a more favorable outcome (FISHER, 2001). Any minimally useful voting system has some form of tactical voting (GIBBARD 1973, SATTERTHWAITE 1975). However, the type of tactical voting and the extent to which it affects the results of the election vary dramatically from one voting system to another (BLAIS, NADEAU, GINDEGIL and NEVITTE, 2001, DUTTA, JACKSON and Le BRETON 2001, MacINTYRE 1995).*

***Manipulation** - such strategic behavior of one group that wants to influence some other voters to vote against their interests but for interests of manipulating group.*

*Strategic voting depends on **information** the voters have about other voters' behavior and sophistication of their **analytic skills**. Innovation of existing approaches - we want to introduce into the voting models categories of non-rational, semi-rational and rational voters (sophisticated voters) and to study also information complexity of voting.*

***Non-rational voters** are not able to evaluate alternatives, they vote randomly, selecting from multiple alternatives with equal probability.*

***Semi-rational voters** are aware about their utility from different alternatives if they are selected, they vote according to their evaluation of alternatives, selecting best alternative from point of view of their individual preferences, not anticipating other voters' behavior (sincere voters).*

***Rational (sophisticated) voters** are anticipating behavior of other voters and are looking for second or third best alternatives to maximize expected benefit from final outcome.*

***Game-theoretical and operations research approaches** (multi-criteria optimization) will be employed as a general framework of the models. There are two types of research output anticipated: theoretical models as methodological contribution to rational voters' theory, and applications, empirical studies of the new electoral history of the Czech Republic (1992-2010) and voting rules in European Union with an emphasis on elements of strategic voting.*

ELEMENTS OF METHOGOLOGY

General model of voting as it is treated by social choice theory (SCHOFIELD 1996) is based on the concept of voting choice function. Voting choice function is defined as a mapping

$$C : T \times \mathrm{R}^{\mathrm{U}} \rightarrow 2^{U}, C(A,R) \subseteq A \ \textit{for all} \ (A,R) \subseteq T \times R^{U}$$

where U is the universe of alternatives, A $\subseteq$U, T = 2^{U} – ø be the set of all non-empty subsets of U, R^{U} be a non-empty finite set of all preference profiles defined over U. That is, for each element (A,R)$\in$ T x R^{U} the voting choice function C chooses precisely one element C(A,R)$\in$ 2^{U}, subject to the restriction that C(A,R) is a subset of A for all (A,R) in T x RU. So the voting choice function is not just a rule for some particular voting situation, but it is a function defined on the set of all thinkable voting choice situations from the set T x R^{U} with domain of "values" 2^{U} (the "values" are subsets of U).

Voting constraint

To formulate a model of single voter behavior we shall start with an analogy of the consumer's budget constraint, which we shall call a voting constraint. Let m be the number of candidates (i = 1, 2, ..., m) and v be the number of votes the voter can use. Let xi be the number of votes the voter gives to the i-th candidate. In general x_i may be any non-negative number. A vector $x = (x_1, x_2, \ldots, x_m)$ permissible under a particular voting system will be called a voter's feasible strategy. In a general case voting means to select a feasible strategy from the set

$$X = \left\{ \mathbf{x} \in R_m ,\ \sum_{i=1}^{m} x_i \leq v,\ x_i \geq 0, \mathbf{x} \in D \ \right\}$$

Here D is an additional requirement (e.g. only integers are allowed) on the votes reflecting properties of a specific voting system. The algebraic system $\sum_{i=1}^{m} x_i \leq v, x_i \geq 0, \mathbf{x} \in D$, defining feasible voting strategies, is termed a **voting constraint**. From the point of view of an individual voter, a voting decision (under some particular voting system) means to select and state or submit exactly one feasible voting strategy from the corresponding feasible set, given by the voting constraint. We can suppose that the voter is a rational agent that uses his resources (given by the voting constraint) in the "best" possible way: he selects one of his feasible strategies which is "most preferred" by him in some sense.

The following examples illustrate different voting constraints (feasible sets) for several well-defined voting procedures:

a) **Single-vote plurality**:

$$X_s = \left\{ \mathbf{x} \in R_m, \sum_{i=1}^{m} x_i \leq 1, x_i \in \{0,1\} \right\}$$

is the set of a voter's feasible strategies under single-vote plurality system (most frequently used standard voting system, the voter has only one vote no matter how many candidates offer themselves for his choice).

b) **Approval voting**:

$$X_A = \left\{ \mathbf{x} \in R_m, \sum_{i=1}^{m} x_i \leq m, x_i \in \{0,1\} \right\}$$

is the set of a voter's feasible strategies under approval voting (the voter casts one vote for each candidate he approves and no vote for other candidates, BRAMS and FISHBURN 1986).

c) **Cumulative voting**:

$$X_C = \left\{ x \in R_m, \sum_{i=1}^{m} x_i \leq k, x_i \geq 0, x_i - \text{integer} \right\}$$

is the set of a voter's feasible strategies under cumulative voting, where k is the size of the committee (the voter has as many votes as the number of seats to be filled, and is allowed to divide them as he pleases, perhaps giving them all to one candidate or distributing them among more candidates he trusts, MERRILL 1988).

d) **Borda voting**:

$$X_B = \left\{ x \in R_m, \sum_{i=1}^{m} x_i = m^2 - \sum_{i=1}^{m} i, \; x \in \wp \right\}$$

Here $\wp$ is the set of all permutations of (m-1, m-2, …, 1, 0). The set of a voter's feasible strategies under Borda voting (each voter is permitted to assign weights - integers m-1, m-2, ..., 0 in a one-to-one fashion to the m candidates, BORDA 1786.

e) **Interval voting**:

$$X_I = \{x \in R_m, 0 \le x_i \le M\}$$

where M is a positive constant, is the set of feasible strategies under interval voting (each voter is asked to rate candidates on a scale from 0 to M, e.g. from 0 to 100, see e.g. JOSLYN 1976, RIKER 1982).

Rational choice

Usually it is supposed that each rational voter has a well-behaved preference relation on the set of candidates, which is at least complete, reflexive and transitive (weak ordering), sometimes also anti-symmetry is required (strong ordering).
But the process of discovering this preference relation remains unclear. Clearly the candidates' qualifications may be judged by multiple criteria, such as trustworthiness and/or honesty, capabilities, general political stance (conservative, moderate, liberal), and positions on specific political issues, evaluated from the standpoint of voter's interests. These criteria are summarized in the voter's mind, to produce a value (utility) function.
Then the voter rates the candidates as to the first choice, second, third, etc., based upon the voter's utility function toward the candidates. Multiple criteria decision making theory provides an appropriate methodology to describe and analyze this rather vague process of forming an individual voter's utility function.

Stating an individual voter's optimal choice problem as a multi-criteria optimization problem we shall start with several trivial and, in a sense, simplifying assumptions:

a) *Multiple issues*. We shall suppose that there is a list of major political issues $\boldsymbol{\pi} = (\pi_1, \pi_2, ..., \pi_k)$ where $k > 1$ and that an individual voter is able to rank the issues by their importance to him. Let us denote the voter's ranking over this list by $\mathbf{b}_j = (b_{j1}, b_{j2}, ..., b_{jk})$, where b_{js} is an integer between 0 and t, expressing the position of the s-th issue in the j-th voter's ordering, or a cardinal measure (say, between 0 and 100) expressing the relative importance of the issue to the voter.

b) *Observability of candidates' position*. We suppose that each candidate or party publicly states his own ranking over the list of major political issues. Let us denote the i-th candidate's ranking by $\mathbf{a}_i = (a_{i1}, a_{i2}, ..., a_{ik})$.

c) *Trustworthiness* of the candidates. We suppose that there is no uncertainty about the future position of the candidates (publicly declared rankings express the true position of the candidates).

Portfolio voting, semi-rational voters, optimization model

Example of a possible model

Let us suppose the following proportional voting rule for electing a committee, called a *portfolio voting rule* (TURNOVEC, 1995): Each voter chooses among party lists. Let *n* be the number of voters, *m* be the number of parties, and *v* be the number of seats in a committee. Each voter has *k* votes (as many votes as the number of seats). By x_{ji} let us denote a number of seats assigned by the *j*-th voter to the *i*-th party. For simplicity, we suppose that all voters take part in the election and use all their votes, then

$$\sum_{i=1}^{m} x_{ji} = v, w_{ji} = \frac{x_{ji}}{v}$$

(for all $i = 1,2,...,m$ and $j = 1,2,...,n$) and $\mathbf{w}_j = (w_{j1}, w_{j2}, ..., w_{jm})$ is an *j*-th *individual voting portfolio*, or an *j*-th voter feasible voting strategy. Then $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, ..., \alpha_m)$ such that $\alpha_i = \frac{\sum_{j=1}^{n} w_{ji}}{n}$ is a *social voting portfolio* according to which the seats are to be distributed in a committee.

As before we denote the j-th voter ranking over a list of major political issues by $\mathbf{b}_j = (b_{i1}, b_{j2}, ..., b_{jk})$. Each party declares its own ranking over the issues $\mathbf{a}_i = (a_{i1}, a_{i2}, ..., a_{ik})$.

The individual voter's ***semi-rational behavior*** can then be defined by an optimal solution to the following goal programming problem:

minimize

$$d(\sum_{i=1}^{m} w_{ji} \mathrm{a}_i,\ \mathrm{b}_j) \qquad (1)$$

subject to

$$\sum_{i=1}^{m} w_{ji} = 1,\ \ w_{ji} \geq 0 \qquad (2)$$

where *d* is a distance between the *j*-th individual ranking and the aggregate committee ranking generated by the *j*-th voter's voting portfolio. An optimal solution $\mathbf{w}_j^0 = (w_{j1}^0, w_{j2}^0, ..., w_{jm}^0)$ to the problem (1) - (2) we can call an *j-th individual optimal voting portfolio*. Traditional model with possibility to vote for one alternative (candidate) only is a special case of (1)-(2) with 0-1 variables.

Selection of individual voting portfolio maximizing her utility is an optimization problem.

Simple example

Consider a voting situation with 3 candidates A, B, C and 3 political issues x, y, z, characterised by Table 1 (rankings of the issues by the candidates and by the voter, where 1 means top ranking and 3 bottom ranking).

Table1

	A	B	C	voter
x	3	1	2	2
y	1	2	3	1
z	2	3	1	3

We shall derive optimal voting strategies for different voting procedures on the basis of the following criterion: The voter prefers a voting strategy (distribution of votes among the candidates) S_1 to a voting strategy S_2, if the distance between his ranking of political issues and aggregated candidates' ranking generated by voting strategy S_1 is less than the distance generated by S_2.

We shall use absolute value distance function. For example, let the selected voting strategy be (1, 1, 0), i.e. voter casts one vote for candidate A, one vote for candidate B and no vote for candidate C, then the aggregated candidates' ranking of the issues x, y, z generated by this strategy is

$$\frac{1}{2}\begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ 3/2 \\ 5/2 \end{pmatrix}$$

and the distance between the voters' ranking

$$\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$$

and aggregated ranking is

$$abs(2-2) + abs(1-\frac{3}{2}) + abs(3-\frac{5}{2}) = 1$$

In Table 2 we compare aggregate ranking generated by different feasible voting strategies to individual voter's ranking.

Table2

feasible voting strategy	distance
(0, 0, 0)	6
(1, 0, 0)	2
(0, 1, 0)	2
(0, 0, 1)	4
(1, 1, 0)	1
(1, 0, 1)	3
(0, 1, 1)	3
(1, 1, 1)	2
(2, 1, 0)	3
(1, 2, 0)	4/3
(0, 2, 1)	10/3
(0, 1, 2)	8/3
(2, 0, 1)	8/3
(1, 0, 2)	10/3

The first four rows of Table 2 correspond to the feasible voting strategies under single-vote procedure. We can see that in this case the optimal voting strategies are (1,0,0) and (0,1,0), i.e. the voter gives his single vote either to the candidate A or to the candidate B. By this approach we can also get an individual voter's ordering of the candidates: $A \approx B$, $A \succ C$, $B \succ C$.

The first 8 rows of Table 2 correspond to the feasible voting strategies under approval voting procedure. The voter's optimal strategy is (1,1,0), i.e. he gives his approval votes to the both candidates A and B.

The last six rows of Table 2 correspond to the feasible voting strategies under Borda voting procedure. The voter's optimal strategy in this case is (1,2,0), i.e. for the voter it is optimal to give one vote to the candidate A and two votes to the candidate B.

Table 2 gives complete list of all discrete feasible strategies (14 strategies) under cumulative voting. The voter's optimal strategy under cumulative voting is in this case the same as under approval voting: (1,1,0), i.e. for the voter it is optimal to give one vote to the candidate A and one vote to the candidate B.

An obvious criticism of this model of semi-rational voting behavior in portfolio voting follows from the fact that the level of satisfaction of an individual voter with the results of voting depends not only on his decision, but also (and to a very great extent) on decisions of many other voters. It ignores strategic aspects of voting

Portfolio voting, sophisticated voters, game-theoretical model

A ***sophisticated voter*** is aware of the fact that his individual voting portfolio will certainly differ from the social voting portfolio and his satisfaction should be measured by a distance between his individual ranking and the aggregate committee ranking generated by the social voting portfolio, rather than by the individual voter's portfolio.

The distance between the *r*-th individual ranking and the aggregate committee ranking generated by a social voting portfolio α can be measured by the *r*-th voter's distance function

$$\delta_r(w_r, w_1, \ldots, w_{r-1}, w_{r+1}, \ldots, w_n) = d\left(\frac{1}{n}\sum_{i=1}^{m}(w_{kj} + \sum_{j \neq r} w_{ji})a_i,\ b_r\right) \tag{3}$$

where only the individual portfolio variables w_{ri} are under the control of the *r*-th voter. Therefore, we can formulate a game of the *n* voters with the pay-off functions (3) and the strategy sets

$$W^{(r)} = \left\{ w_r \in R_m, \sum_{i=1}^{m} w_{ri} = 1, w_{ri} \geq 0 \right\} \tag{4}$$

and look for optimal strategies in such a game.

A general model framework for analysis, different voting systems could be presented as special cases of the problem (1)-(2) and game (3)-(4).

Using and extending models (1)-(4) we can introduce explicitly strategic voting based on different levels of different voters' information about the other voters' preferences, use approaches of games against nature for non-rational voters, games with sophisticated and indifferent players, a coalitional cooperation etc.

Difference between strategic voting and manipulation: while strategic voting is legitimate expression of voters' rationality, manipulation consists in misrepresentation of candidates rankings of political issues in order to get a more favorable outcome

Possibilities:

Theory: development of the model and different information setups

Empirical experiments with small groups having different levels of information and using different voting procedures

Analyses of forthcoming elections (European Parliament, Czech parliamentary election)