Charles University in Prague
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RIGOROUS THESIS

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Pricing of Gas Swing Options

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Academic Year: 2009/2010

Declaration:

Hereby I claim that I elaborated this rigorous thesis on my own, and that the only literature and sources I used are those listed in references.

Prague, February 15, 2010 **Andrea Pokorná**

Acknowledgments

I would like to express my gratitude to Prof. Ing. Karel Janda, M.A., Dr., Ph. D. for supervising my thesis. My thanks is also extended to my consultants, PhDr. Jozef Baruník and Mgr. Tomáš Václavík, for their valuable comments.

Abstract

Even though contracts for the purchase and sale of natural gas providing an offtake flexibility concerning volume and time (gas sales agreements) have been commonplace in the natural gas industry for many years, the development of techniques for pricing them has not followed at the same pace. This thesis is motivated by the changing nature of the natural gas industry in the European Union, which is asking for a mark-to-market evaluation of these contracts. The flexibility provided by these contracts is then regarded as a financial option, called a “gas swing option”. Since the gas swing option is actually a set of several American puts on a spread between prices of two or more energy commodities, we devote one section of the text to the theory on spread option pricing. Due to the specific features of the energy markets the existing analytic approximations for spread option pricing are hardly applicable to our framework. That is why we employ numerical methods and model the spot price dynamics through stochastic processes capturing such features. The price of an arbitrarily chosen gas swing option is then computed in accordance with the concept of risk-neutral expectations, i.e. is considered as an expectation of discounted future cash flows for a probability structure called risk-neutral. Finally, our result is compared with the ex-post value of the option.

Abstrakt

Přestože plynárenský průmysl využívá smlouvy o prodeji a nákupu zemního plynu poskytující objemovou a časovou flexibilitu v hojné míře, metody jejich oceňování nejsou zdaleka tak vyvinuté. Diplomová práce je motivována měnící se povahou plynárenského průmyslu v Evropské unii, která si žádá, aby tyto kontrakty byly oceňovány tržně. Na flexibilitu, kterou nabízejí, je pak nahlíženo jako na finanční opci, tzv. „gas swing opci". Jelikož gas swing opce je ve skutečnosti množina amerických put opcí vypsaných na spread mezi cenami dvou či více energetických komodit, je nemalá část textu věnována teorii o oceňování spread opcí. Existující analytické aproximace pro spread opce se z důvodu specifických vlastností vykazovaných energetickými trhy zdají být problematicky aplikovatelné pro oceňování swing opcí. Proto je využito numerické metody a dynamika spotových cen je modelována pomocí stochastických procesů, které tyto specifika zohledňují. Cena libovolně zvolené swing opce je pak spočtena v souladu s konceptem rizikově neutrálních očekávání, tj. jako očekávaná hodnota diskontovaných budoucích finančních toků pro pravděpodobnostní strukturu označovanou jako rizikově neutrální. Na závěr je výsledek porovnán s ex-post hodnotou této opce.

List of Abbreviations

ABM	Arithmetic Brownian Motion
ACQ	Annual Contract Quantity
AMQ	Annual Minimum Quantity
ARA	Amsterdam-Rotterdam-Antwerp
CEGH	Central European Gas Hub
CY	Calendar Year
DCQ	Daily Contract Quantity
DCQ_{max}	Maximum Daily Contract Quantity
DQT	Downward Quantity Tolerance
ECB	European Central Bank
EFE	Endex Futures Exchange
EFET	European Federation of Energy Traders
EU	European Union
GBM	Geometric Brownian Motion
GSA	Gas Sales Agreement
GY	Gas Year
ICE	IntercontinentalExchange®
LF	Load Factor
LNG	Liquefied Natural Gas
MCM	Monte Carlo Methods
MCS	Monte Carlo Simulation
NBP	National Balancing Point
NCG	NetConnect Germany
PSV	Punto Scambio Virtuale
SDE	Stochastic Differential Equation
SY	Storage Year
TTF	Title Transfer Facility
UK	United Kingdom

Content

1. Introduction

Contracts for the purchase and sale of natural gas providing an offtake flexibility concerning volume and time, generally called gas sales agreements, have been commonplace in the natural gas industry for many years. Most of today's European natural gas contracts are long-term contracts including a so-called "Take-or-Pay" clause. This provision obliges the buyer to pay for the minimum contract quantity whether or not this is actually offtaken by him. However, the take-or-pay obligation very often refers only to a certain part of the annual contract quantity and the remaining part can, but does not have to be, offtaken. It constitutes an option, called a "swing option", for the buyer to offtake this part of the annual quantity of gas or not. Even if these contracts are widely exercised, the techniques used for pricing them have not yet been fully developed.

This thesis is motivated by the changing nature of the natural gas industry. Specifically, in accordance with EU legislation, this industry has been liberalized with a side effect of the development of gas trading within Europe. Once there is a market where gas is liquidly traded spot and forward, the gas sales agreements can be seen in a different way, or specifically, they can be valued mark-to-market. It means that the flexibility they provide can be thought of as a financial option. It is then the aim of this thesis to derive a tool for the pricing of such options, called gas swing options.

Gas swing options can be defined as spread options on energy commodities. We thus devote much time to the spread option pricing theory. There are two main groups of models that are trying to find a fair value price of a spread option: numerical methods and analytic approximations. For the analytic examples see e.g. Kirk (1995), Poitras (1998), Carmona and Durrleman (2003), and Alexander and Venkatramanan (2007). Representatives of the numerical methods are then trinomial trees, finite difference methods, or Monte Carlo simulation. Since the trees and finite difference methods become prohibitively time consuming when the dimension of the problem increases one is referred to Meinshausen and Hambly (2004) for the Monte Carlo methods.

The difficulty of our valuation problem increases due to the fact that gas swing options are written on energy commodities. Specifically, the energy markets are fundamentally different from traditional financial security markets in several ways. Firstly, the markets of energy commodities and energy derivatives lack the same level of liquidity that the majority of financial markets enjoy. Very often one cannot find a proper hedging tool or hedging strategy for mitigation of his price risk since there is no market for the derivative product he is interested in. Secondly, the energy markets are characterized by the limited ability of market players to arbitrage since the players miss a sufficient amount of non-operating inventories. Thirdly, prices of energy commodities are typically exposed to very high volatility and large shocks. Finally, the prices tend to show strong mean-reverting trends and seasonality. These facts are the reasons why we employ numerical methods rather than an analytic approximation to derive the tool for the pricing of gas swing options.

Before doing so, we firstly focus, in chapter two, on a contractual relationship between a seller and a buyer of natural gas. We introduce the main features of a general gas sales agreement, along with the specification of obligations and rights the agreement establishes for the contracting parties. A combination of the obligations and rights then constitutes the (gas swing) option for the buyer. In chapter three, we define conditions under which the option becomes a financial option. Assuming these conditions are satisfied, we continue with a description of main option features. It helps us to disclose that the option is actually a set of several American puts on a spread between the prices of two or more energy commodities. Chapter four is consequently devoted to the spread option pricing theory. Going through the theory we recognize which methods are best applicable to the pricing of gas swing options. The Monte Carlo simulation is then the one that is subsequently employed in chapter five to derive the option pricing model. In this chapter, an arbitrary gas sales agreement is chosen to test the model. After applying the model to the pricing of this agreement, we finally compare the model results with the ex-post value of the option. Concluding remarks are summarized in the conclusion.

2. Gas sales agreement

The gas industry has been traditionally characterized by long-term take-or-pay contracts between producers and wholesale traders (or shippers, or importers). The major part of gas export to the EU-region has, until now, been sold under such contracts (Asche et al., 2002b). However, take-or-pay contracts have also been commonplace at lower stages of the gas provision process. Specifically, contracts between wholesale traders and retail traders have taken a similar form when compared to contracts concluded at the production stage. Regardless of the stage of conclusion one commonly calls a contract for the purchase and sale of natural gas, including a so-called Take-or-Pay provision, a “gas sales agreement” (GSA).

To understand why this type of contract is so widely used in the gas industry it is necessary to firstly focus on the traditional contractual relationship between a producer and a wholesale trader. This relationship is influenced by the special nature of the gas industry. Specifically, the industry is highly capital intensive. Before producers can produce and sell natural gas they must make a transaction-specific investment like a gas-field development. It is the reason why they ask for some kind of guarantee that the initial cost will be recovered. Long-term take-or-pay contracts appear to be the right instruments on how to avoid such volume risk. They include a Take-or-Pay clause (as described later in the chapter) which ensures producers a minimum amount of their future sales. Regarding traders’ incentives it is necessary to make a distinction between a monopoly and a competitive organization in the market. Under a monopoly, long-term take-or-pay contracts help traders to capture a certain market share and avoid price risk in exchange for taking volume risk. In addition, this market organization enables the traders to shift a big fraction of the volume risk to their customers through take-or-pay contracts. However, the situation changes in the face of competition. Since there are competitors in the market the traders face a higher risk of a loss of their customers. In other words, the volume risk the traders bear considerably increases. This fact then raises the question what the future of long-term take-or-pay contracts will look like. For an answer and for more information about long-term take-or-pay contracts see for example Asche et al. (2002a & 2002b), Creti & Villeneuve (2003), or Neumann & Hirschhausen (2005).

In this chapter we focus on the provisions of a general take-or-pay contract, or gas sales agreement, regardless of the stage at which it is concluded. In other words, we do not investigate who the contracting parties are. We simply refer to them as the buyer and the seller. Our main interest then concerns the basic obligations and rights this type of contract establishes.

2.1. Basic definition

A gas sales agreement (GSA) is a contract for the purchase and sale of natural gas providing an offtake flexibility concerning volume and time. It is based on the commitment of a seller to deliver an agreed quantity of gas to a buyer. The buyer is, on the other hand, obliged to pay for the offtaken volume. However, by specifying a minimum and maximum quantity the contract sets limits on the buyer's offtake. The buyer always has to pay for the minimum quantity even if it was not offtaken by him (a so-called take-or-pay mechanism) and his offtake cannot exceed the maximum quantity limit. In other words, the offtake can swing between these two constraints. That is why these contracts are known as "take-or-pay" or "swing contracts".

2.2. Main features

A GSA establishes the obligations and rights of all parties involved through its provisions. Although the exact contents of existing GSAs are guarded by contracting parties, the general contract structure is common knowledge in the gas industry.[1] The following terms are then the most important ones regarding the distribution of obligations and rights. In addition, their final wording is crucial for a risk allocation between the contracting parties.

[1] See for example http://contracts.onecle.com/warren/gas.supply.2001.01.01.shtml.

- Delivery Period
  - starts often either on 1.1. (Calendar year) or 1.10. (Gas year)
  - duration of one or more contract years
- Point of Delivery
  - point of conveyance of the title
  - often either a border point or virtual point within a country
- Quantity
  - annual contract quantity (ACQ)
  - daily contract quantity (DCQ)
  - downward quantity tolerance (DQT)
  - annual minimum quantity (AMQ)
- Take-or-Pay clause

  The buyer is obliged to offtake and pay for, or to pay for at least a minimum annual quantity (AMQ) (if it is delivered by the seller) defined as a percentage of the ACQ.
- Make-Up Gas

  When the buyer has incurred the Take-or-Pay obligation he may have rights in subsequent years to receive an equivalent gas volume (Make-Up gas) for free or at reduced price after he has taken gas in excess of an agreed threshold volume. This is commonly the ACQ but may, for example, be AMQ.
- Carry Forward

  A provision under which the buyer, who lifts more than his ACQ in any year, is allowed in subsequent years to offset such overlifting against underlifting which might otherwise have incurred some form of sanction, such as Take-or-Pay.
- Quality
  - chemist, content of combustion, dew-point etc.
- Price
  - fixed price or gas price formula (discussed later in the chapter) at the delivery point

- Price Review
  - right of the parties to ask for a renegotiation of the contract provisions with reference to a substantial change of nature of the European energy markets
  - often stipulates a price review period – minimum interval between two price reviews
- Force Majeure clause

  It is a clause which essentially frees both parties from liability or obligation when an extraordinary event or circumstance beyond the control of the parties, such as fire, explosion, natural disaster etc., prevents one or both parties from fulfilling their obligations under the contract.
- Arbitration Proceedings
  - stipulation of arbiters which will resolve a dispute in case the matters at issue are not resolved within negotiations between the parties

2.2.1. Contractual quantity

Deliveries under take-or-pay contracts are basically defined by an annual contract quantity and a daily contract quantity.

Annual contract quantity (ACQ) is the amount of gas which the seller must deliver and the buyer must offtake in a given contractual year. It may be expressed as a discrete number or as a multiple of the daily contract quantity (eq. (2.1)). In practice, many contracts are written in the form allowing the buyer to offtake considerably below the stated ACQ, i.e. providing a downward quantity tolerance (defined below).

Daily contract quantity (DCQ) is the amount of gas which the buyer nominally undertakes to purchase and the seller undertakes to deliver in a defined 24 hour period. Although featured in many contracts, in practice this expression is of little meaning in itself. It may serve as a mean of expressing ACQ (eq. (2.1)) or define maximum daily quantity.

$$\mathrm{ACQ} = n\,\mathrm{DCQ}, \tag{2.1}$$

where n denotes the number of days in a contract year.

Maximum daily quantity (DCQ_{max}) is given by

$$\mathrm{DCQ}_{\max} = \frac{\mathrm{ACQ}}{nLF}, \tag{2.2}$$

where *n* denotes the number of days in a contract year and *LF* denotes a load factor, which takes a value from the interval (0,1].[1]

Downward quantity tolerance (DQT) is the amount by which the buyer may fall short of its full annual contract quantity without incurring sanctions. Basically, it defines an annual minimum quantity.

Annual minimum quantity (AMQ or MinTake) is the amount the buyer has to pay for even if he does not offtake it with respect to defaults on the seller's side.

$$\mathrm{AMQ} = \mathrm{ACQ} - \mathrm{DQT} - \mathrm{D}, \tag{2.3}$$

where D denotes defaults on the seller's side.

2.2.2. Contractual price

Contracting parties can agree either on a fixed price or on an indexed price. As the typical duration of a GSA is too long for a fixed price to capture future market conditions, price indexation has been quite common in gas sales agreements. The prices of gas delivered under the actual European gas sales agreements are usually determined by a price formula indexed to the changes of the market prices of competing fuels (Asche et al., 2002b). However, with the development of the European gas spot and forward trading the features of the price formula are supposed to change. More precisely, one can expect a replacement of the indexation to competing fuels by an indexation to the market price of natural gas itself.

The price formula, as mentioned in Asche et al. (2002b), links the current gas price to the price changes of relevant energy substitutes. The purpose of such indexation is to make the price of gas competitive when compared to the prices of other fuels.

[1] Load factor is an important parameter of the contract as in case it is lower than one it is a source of additional flexibility (time flexibility) for a buyer of the GSA (discussed hereinafter). It can also be expressed in hours, i.e. it can take value from the interval (0;8760] in case of a non-leap year and from the interval (0;8784] in case of a leap year.

Examples of alternative energy commodities are crude oil, fuel oil, gas oil, coal, electricity and wood. The formula consists of two parts, a constant basis price and an escalation component linking the gas price to prices of alternative fuels. The basis price reflects the value of gas at the time of entering into the contract. The indexation to alternatives requires a value to be given to factors such as weight, an energy conversion factor and a pass through factor. A general gas price formula is then given by

$$P_t = P_0 + \sum_{j=1}^{m} \alpha_j (AC_{jt} - AC_{j0}) ECF_j \lambda_j \ , \qquad (2.4)$$

where P_t is the price paid for a unit of gas delivered at the delivery point within month t, P_0 is the constant basis price of natural gas, α_j is the weight of the alternative commodity j, where $j=1,\dots,m$, and $m < \infty$, AC_{jt} is the price of the alternative commodity j calculated at time t for its reference period, AC_{j0} is the constant basis price of the alternative commodity j, ECF_j is the energy conversion factor of the commodity j, and λ_j is the pass through factor assigned to commodity j. The reference periods for the calculation of the prices of the alternative commodities are usually from three to nine months preceding immediately, or with a certain lack, the month of delivery. This means that the price of gas reflects changes in prices of the competing fuels with a certain delay.

As we have already mentioned, the exact contents of gas sales agreements are highly protected from public. A concrete appearance of the price formula is the most guarded part. Fortunately, there is one exception represented by the formula (2.5), which was published by the German company Ruhrgas for a gas auction in 2004.[1]

$$P = P_0 + 0.0035 \text{ x } (GO - GO_0) + 0.00175 \text{ x } (FO - FO_0)^2 \qquad (2.5)$$

It is the price of gas in Euro cents per kWh, where P_0 is determined as 95 % of 1.2232 Euro cents per kWh (BAFA value as of 01.01.2003), i.e. it is equal to 1.16204 Euro cents per kWh.

[1] In 2002, the German Federal Minister of Economics and Labor approved the acquisition of Ruhrgas by E.ON AG. The Approval was subject to a number of obligations on Ruhrgas. One of these was to establish a gas release programme to release 200 billion kWh of gas from its long-term import contracts. In accordance with the Approval, Ruhrgas started the process of six separate annual auctions. For the second one, Ruhrgas introduced a price formula with fuel oil indexation as an alternative to the BAFA-based price formula (the average import price of gas delivered to Germany) laid down in the Approval.

[2] See “Ruhrgas Gas Release Programme 2004”, Information Memorandum, 31 March 2004.

GO denotes the arithmetic average of the eight values of the product GASOIL multiplied by USD/EUR for the eight months ending one month prior to each Recalculation Date for GO which is the first day of January, April, July and October of each calendar year. GO_0 *is then stated as 223.757 Euro per ton. The product GASOIL refers to the monthly average of the daily quotations high and low for Gasoil 0.2 PCT FOB Barge Rotterdam in USD per ton as published in the "Platt`s Oilgram Price Report", New York edition.*

USD/EUR is equal to the monthly average of the reciprocal of the exchange rate between the US Dollar and the Euro as published by ECB.

FO denotes the arithmetic average of the four values of the product FUELOIL multiplied by USD/EUR for the four months ending immediately prior to each Recalculation Date for FO which is the first day of each delivery month. FO_0 *is then equal to 168.404 Euro per ton. The product FUELOIL refers to the monthly average of the daily quotations high and low for fuel oil 1 PCT FOB Barge Rotterdam in USD per ton as published in the "Platt's Oilgram Price Report", New York edition.*

Using the values of the formula parameters we can rewrite the formula with

$$P = 1.16204 + 0.0035 \text{ x } (GO - 223.557) + 0.00175 \text{ x } (FO - 168.404). \qquad (2.6)$$

2.3. Allocation of obligations, rights and risks

As some of the above mentioned provisions are crucial for our subsequent evaluation we will learn more about them. We will describe obligations and rights these provisions establish, together with their influence on a risk allocation. Generally, an efficient allocation of risk is necessary as it apportions risk to the party that is in the best position to bear the burden of that risk.

2.3.1. Take-or-Pay clause

The Take-or-Pay clause is a common provision in gas sales contracts constituting an obligation to the buyer. If the buyer fails to take the agreed quantity in a given year (assuming that the seller was able to deliver the gas) for reasons other than force majeure, he shall pay the seller for that quantity as if the gas had been offtaken.

The Take-or-Pay provision is crucial for the allocation of volume risk. It enables a seller to shift his volume risk to a buyer. In case of the traditional contractual relationship between a producer and a wholesale trader we speak about a mitigation of a “hold-up” problem. The hold-up problem occurs when two factors are present. First, the party of a future transaction must make non-contractible specific investments to be able to offer a product. Second, the exact form of the optimal transaction (e.g. how many units if any, price of product) cannot be specified with certainty ex-ante. It depends on the resolution of uncertain parameters and these parameters cannot be objectively measured and contracted upon. Under these factors two parties (the producer and the trader) may be able to work most efficiently by cooperating, but one party (the producer) refrains from doing so due to concerns that he may give the other party (the trader) an increased bargaining power, and thereby reduce his own profits. It implies that without adequate contractual protection, the producer’s anticipation of the trader’s opportunistic behavior will result in a less than socially optimal level of investment. For more information about the hold-up problem see for example Klein et. al (1978) or Masten and Crocker (1985).

While under the monopoly the trader could shift the volume risk further to his customers in return for secure supplies, however, it is not possible under the competitive market any more. In addition, the competition significantly increases the volume risk as the gas demand is satisfied by more competing subjects. This increasing risk burden of the trader can be reduced by introducing the following provision.

2.3.2. Downward quantity tolerance

The downward quantity tolerance (DQT) provision sets the amount of gas by which the buyer may fall short of its full annual contract quantity without incurring sanctions. It allows a minimum annual quantity (AMQ) to be lower than an annual contract quantity (ACQ). In other words, it constitutes the right to the buyer to not offtake a certain amount of the agreed quantity of gas without incurring sanctions.

The provision reduces the buyer's burden of volume risk by shifting a part of the risk back to the seller. Such allocation of risk seems to be more appropriate when taking into account the changing nature of the gas industry.

2.3.3. Price indexation

Agreeing on a fixed price of gas would mean the allocation of price risk to the buyer. However, since the buyer bears the burden of volume risk it would not be appropriate to impose the price risk on him as well. That is why the seller is willing to take the price risk in exchange for shifting the volume risk. The result is the floating price of gas given by the price formula indexed to market price changes of competing fuels (as described in the subchapter 2.2.2.).

2.4. Breach of contract

Contracts can of course be breached in many ways but a failure to take or supply gas is one of the most interesting and important ones.

Buyer's breach of contract

The buyer can potentially be in breach of contract if he has taken less gas than agreed in the contract by an ACQ provision and if it was not caused by force majeure. In this case, the buyer firstly claims the two following provisions:

- Downward Quantity Tolerance, which may either excuse or at least limit potential penalties, and
- Carry Forward provision.

If, taking into account these adjustments, there is still a shortfall, the buyer is in breach of contract and the seller may request the penalties as specified by the Take-or-Pay provision (see subchapter 2.3.1.). However, the buyer can call for the volumes that he has already paid for, but which have not been taken by him in subsequent years and so set up a claim given by the Make-Up gas provision.

Seller's breach of contract

The seller must be able to deliver an agreed quantity of gas (ACQ) on any occasion the buyer requires it, except for cases of force majeure. Volumes the seller fails to deliver are often called defaults and the buyer is compensated for such seller's failure by reduced prices charged by the seller for the delayed delivery. However, the buyer has the right to diminish his offtake obligation by these default volumes.

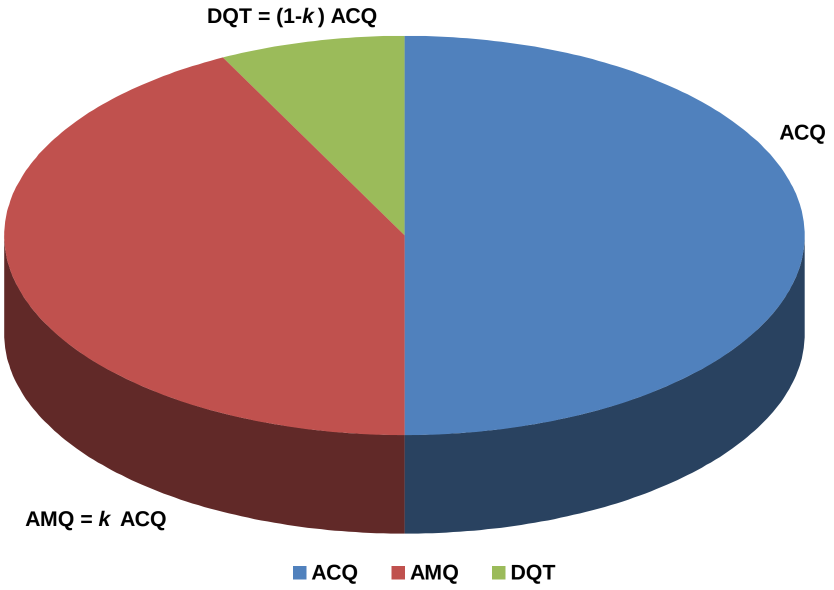
Through a brief definition of the general gas sales agreement we have found out what the main obligations and rights of the contracting parties are. Specifically, a combination of these obligations and rights constitutes an option to the buyer not to offtake a certain amount of the agreed quantity of gas from the seller without incurring any sanctions. This option is known as a "gas swing option".

3. Gas swing option

As noted previously, many take-or-pay contracts provide the downward quantity tolerance (DQT), allowing the minimum annual quantity (AMQ) to be lower than the annual contract quantity (ACQ). It means there is an amount (flexibility volume), usually expressed as a percentage of ACQ (*1-k %*, where $k \in [0;1)$ [1]), by which the buyer may fall short of its full annual quantity without incurring any sanctions (see figure 1). It constitutes an option (a gas swing option) to the buyer to offtake this amount of gas or not. So the buyer to a GSA becomes at the same time a buyer of the option (hence only the "buyer").

1 Coefficient *k* corresponds with the take-or-pay provision of GSA. It states how many percents of the ACQ have to be offtaken and paid for or paid for by the buyer.

Figure 1: Gas swing option



Source: Author

The buyer faces two incentives to purchase the option. Firstly, the option reduces the buyer's volume risk. Specifically, it helps the buyer bring his offtake from the seller into line with a consumption of his customers which is not known ex-ante. The consumption depends on the resolution of uncertain parameters such as weather, competing forms of energy, and the standard of living of the customers. Secondly, in consequence of the liberalization process in the gas industry and the subsequent development of spot and forward trading within Europe the option extends to a different dimension and becomes a financial option. It means the option changes its force. It represents the right of the buyer to choose between two purchase sources of the option volume of gas: the agreed GSA and spot gas market.

3.1. Necessary conditions for mark-to-market evaluation

To be able to value GSA mark-to-market, or in other words, to think of the option it provides as a financial option, there must be a market where gas is traded spot and forward. In addition, the market must be liquid and open to contracting parties.

3.1.1. Definition of market liquidity

"Liquidity can be a somewhat elusive concept, since it incorporates four distinct characteristics of a market namely: depth, breadth, immediacy, and resilience. Deep markets are ones in which large volumes can be bought or sold without moving the price excessively, and wide markets are ones in which a large number of different bids and offers are present in the market. Immediacy on the other hand relates to the ability to trade large volumes in a short period of time, and resilience to the ability of the market to recover towards its natural supply/demand equilibrium after having been exposed to a shock. Liquidity itself tends to develop as market players become more confident in the fairness of a market – and once liquidity increases it tends to form a virtuous circle." (Cronshaw et al., 2008)

3.1.2. Concept of a gas trading hub

Gas trading can develop in a place where a number of pipelines and cargo hauls (carrying LNG) cross and bring gas from various gas producers. Such place is then called a "gas hub". It is represented either by a single point or by an area (so-called "virtual hub"). Legally, it is the place where title transfers occur.

To approximate to the liquidity in accordance with the above mentioned definition it must be possible to easily move gas in and out of the hub, i.e. the existence of a developed gas transmission network within Europe is a precondition. More precisely, to reach the depth of the market, large volumes of gas must be traded in the hub, hence an adequate transport capacity must be available to get these volumes in and out of the market. In addition, to reach the breadth, a large number of different bids and offers must be present in the market. Hence it is necessary for the capacity to be open to all interested subjects for a price resulting from a matched capacity demand with a capacity supply. It can only be satisfied in a competitive environment. The requirement of immediacy is then limited by the fact that gas cannot be easily stored near the hub. It must be moved into the hub from various corners of Europe resulting in a time gap between a trade and its settlement (i.e. delivery). That is the reason why most short-term gas trades are day-ahead (i.e. with the delivery taking place a day after the trade) and not immediate.

Another major element of trading hubs is the legal and financial framework of the marketplace. This is because it determines conditions of entry for new players and rules of trading, thereby transaction costs, and constitutes a confidence in the market.

For essential operational and commercial requirements to create the successful development of a gas trading hub see the paper published by EFET Gas Committee in February 2003.

3.1.3. Spot and forward trading

A spot gas trade is a purchase or sale of gas for immediate delivery in the trading hub. Deliveries which shall occur on a certain day in the future are traded under forward contracts (or standardized forward contracts called futures contracts). However, gas trades with settlement occurring in less than one month are also treated as spot trades. That is the case of day-ahead gas trades which outweigh within-day gas trading. The process of a short-term trading development is accompanied with the development of a forward (or futures) market, as forwards (or futures) are essential risk management tools.

For financial markets and many commodities markets we can observe a relationship between spot and forward prices of the form

$$F(t,T) = S(t)\exp\{(r-y)(T-t)\}, \tag{3.1}$$

where $F(t,T)$ denotes a forward price at time t of a commodity with delivery at time T, $S(t)$ stands for a spot price of the commodity at time t, r denotes a risk-free interest rate and y refers to a convenience yield which is a nonzero in case of commodities. The convenience yield then captures the benefit from owning the commodity minus the cost of storing it.[1] This relationship is a product of a model for finding the fair price of a forward contract, known as a cost-of-carry model. The fair price is then that which

[1] The expansion to pricing of forward contracts in the form of introducing a convenience yield y for commodities markets where an underlying asset can be stored was established by Brennan and Schwartz (1985).

makes any riskless arbitrage impossible. For more information see Brennan and Schwartz (1985).

However, there are markets where ability to arbitrage is limited by an absence of non-operating inventories. When forward market prices are below the theoretical price $F(t,T)$, the arbitrage is possible for anyone who can sell into the current market for $S(t)$ (higher priced synthetic forward) and buy forward. It requires the ability to sell from current inventories. However, most energy sellers do not dispose of inventories which are not required for operations. Consequently, they are unable to take advantage of market prices being below the theoretical (arbitrage-free) prices. It is the case of the energy markets which holds for that the price of a forward contract cannot be derived from today's spot price of a commodity just by applying the equation (3.1).

Another presumption concerning forward prices is that they are the market's "best guess" (unbiased estimates) of future spot prices. In other words, a forward curve observed at a certain time in the market represents the expectation of the market about future spot prices of the relevant commodity based on all, at that time in the market available, information.

3.1.4. European gas markets

The gas directive issued by the European Parliament and the Council on 22 June 1998 (Directive 98/30/EC) started the process of liberalization in the European gas industry with completion date on 1 July 2007. Along with this process gas trading has been developing around specific delivery points or market "hubs". Prices established in hub trading started being used as the pricing basis for gas supply contracts (mainly with a large customer segment), leading to the development of gas-indexed prices, rather than oil- or coal-indexed ones, as has traditionally been the case. Gas-indexing is well established in the UK as the local gas hub (NBP) is the most liquid market within Europe. In addition, it is a benchmark for other developing markets, such as Zeebrugge Hub, the Dutch TTF and German BEB virtual hub.

The UK is the most competitive, liquid and transparent gas market in Europe. Gas delivered at the National Balancing Point (NBP), a virtual balancing point for the

UK's transmission network, is traded through two exchanges. Physical short-term trading is handled by APX Gas NL B.V.[1] and futures contracts for physical delivery are traded through the IntercontinentalExchange® (ICE)[2].

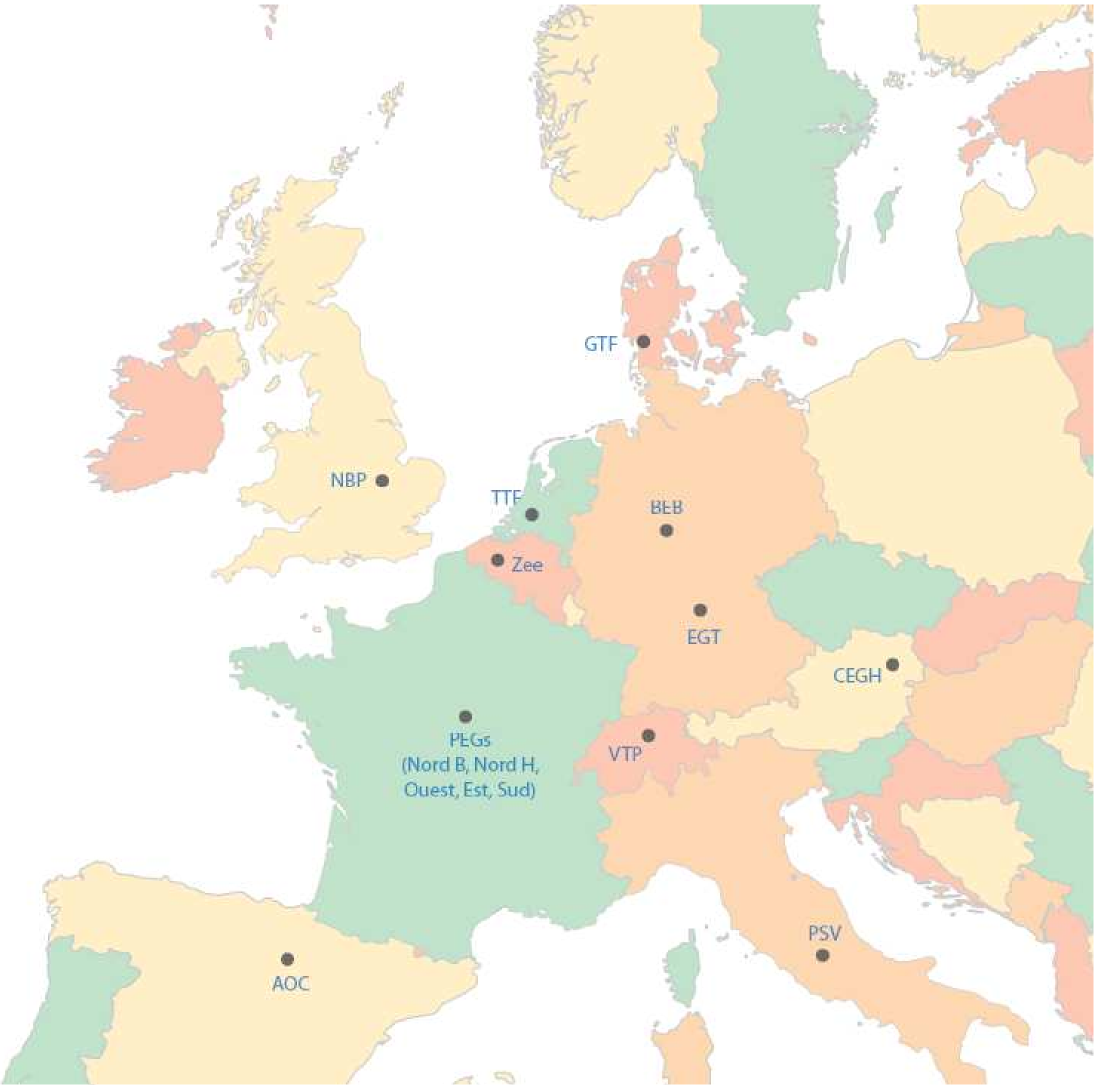
In continental Europe, the most active gas trading hub used to be Zeebrugge in Belgium. Zeebrugge is a physical hub, with gas pipelines from France, Germany, Norway, the Netherlands and the UK, and an LNG terminal. However, in recent years, trading volumes at Zeebrugge have been eclipsed by a strong growth in trading at the Dutch network's virtual balancing point – the Title Transfer Facility (TTF). TTF has become a focus for Dutch trading activities and a dominant reference point in the region. Just like in the case of NBP, both delivery points are traded short-term through APX Gas NL B.V. Futures contracts for physical delivery at TTF are then operated by Endex N.V.[3] On the Endex Futures Exchange (EFE), Endex members can trade hi-calorific gas under long-term contracts covering the gas curve from 3 months ahead, 4 quarters ahead, 4 seasons ahead till 3 calendars ahead.

Other key hubs are the German virtual hubs BEB and NCG. In addition, trading is developing in Italy (PSV), in France (PEG) and in Austria (CEGH), but without a sign of liquidity yet (see figure 2). One can find more information on the liberalization process in the European gas industry and on gas markets development in Haase (2008).

There are some ways to measure the market liquidity. Liquidity is mainly assessed via a width of spreads, existence of balancing markets, brokers, exchanges and independent price quotes. Based on such parameters, the Heren hub liquidity index (graph 1) shows the liquidities of the European gas markets.

[1] http://www.apxgroup.com/
[2] https://www.theice.com/homepage.jhtml
[3] http://www.endex.nl/index.php

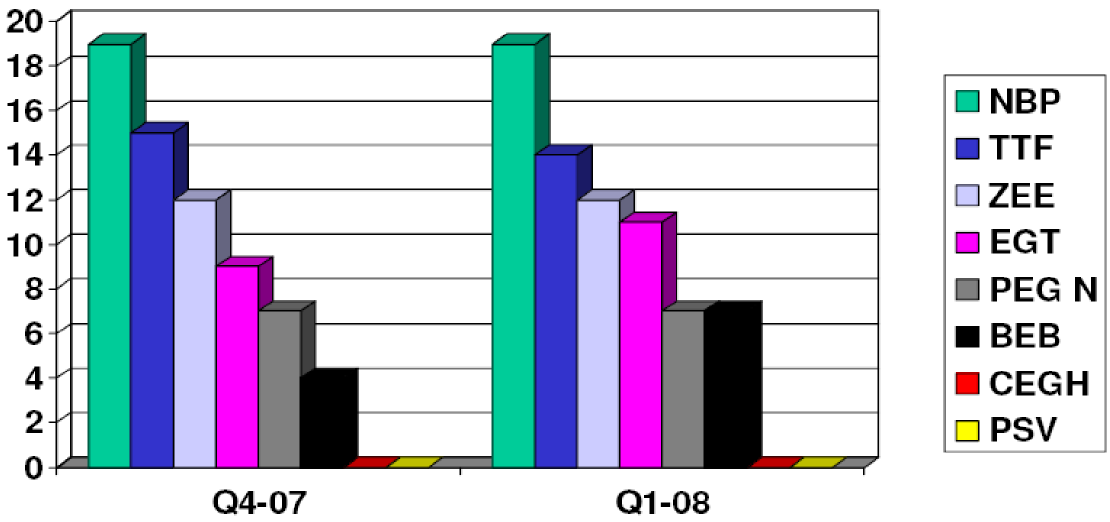
Figure 2: Map of European gas trading hubs



Source: European Gas Hub Report published by ICIS Heren[1]

[1] ICIS Heren is a world-leading publisher of gas, power and carbon market information (www.heren.com).

Graph 1: Heren hub liquidity index



Source: ICIS Heren

Even if the liquidity of some European gas markets has been growing gradually there is no guarantee of an existence of a liquid gas market to all intents and purposes at the end of the process. A liquid market requires a large number of participants, both on a demand side and on a supply side. However, the gas industry is characterized by a limited number of producers.[1] The lack of competition on the supply side raises the risk of price management by the gas producers through governing the gas supply. Looking for remedies, an expansion of LNG imports may represent the start of a more global gas market, where Europe competes with the USA for Atlantic basin LNG supplies.

The next restraint is represented by a territorial restriction clause present in existing contracts for Algerian and Russian exports. This clause forbids Russian and Algerian customers from reselling gas outside their respective territories and thereby limits gas trading within Europe. However, this problem seems to be negotiable and a breakthrough in this area has already occured.[2]

[1] There are few countries managing the world natural gas resources. Regarding the Europe it is supplied partly by its own resources (Norway, The United Kingdom and The Netherlands) and the rest of European gas consumption is satisfied by imports mainly from Russia and Algeria.

2 The Italian oil and gas company ENI managed to renegotiate a number of restrictive clauses in its existing contracts with the Russian gas producer Gazprom in 2003. Under the new settlement, ENI has no longer been prevented from reselling the purchased gas from Gazprom outside Italy.

For the remaining part of this thesis we will assume that a liquid spot and forward market exist and that they are open to the contracting parties.

3.2. Option features

As noted previously, the DQT provision constitutes the right for the buyer to not offtake a certain amount of the agreed quantity of gas, without incurring any sanctions. One can see it as the right of the buyer to sell a part of the ACQ back to the seller for the contractual price. Such view of this problem is crucial for the following description of swing option features.

3.2.1. Definition of an option

An option is a derivative instrument, which means that its price is derived from the price of another asset (an underlying asset). Every option is represented by a contract between a buyer and a seller of the option. The seller (writer) has the obligation to either buy (put option) or sell (call option) the underlying asset from/to the buyer by a certain day (expiration date or maturity) for a certain price (exercise price or strike price), whereas the buyer has the right, but not the obligation, to complete the transaction. When the option expires, the buyer decides whether to exercise it or not based on the money-position generated by this transaction. The buyer is supposed to exercise the option only when it is in-the-money. However, the right is not for free and the buyer has to pay a so-called option premium to get it. It is the fair value price that the option writer charges for that option at a particular point in time. A review of the option theory can be found in Hull (2006).

3.2.2. Buyer's decision making

Since the buyer is supposed to be a rational subject driven by profit maximization, we can assume that he buys gas as cheap as possible. This means that if the contractual price is lower than the market price of gas, the buyer will want to buy gas under the GSA. In case of the opposite situation, the buyer wants to purchase in the market. However, it is important to take into account the distance between the delivery point under the GSA and the market location. If the distance is equal to zero, the problem is described as above. If it is nonzero, the buyer adds a location differential to the market price and again follows the above mentioned criteria.

The buyer's decision making problem can thus be summarized by the exercise condition:

$$\textit{Contractual price} > \textit{Market price} + \textit{Location differential}, \qquad (3.2)$$

where all elements are expressed as unit costs and *Location differential* can take both a positive and negative value. When this inequality is satisfied the buyer is supposed to exercise the right to not offtake gas from the counterparty under the GSA, or in other words, to sell gas back to the seller for the contractual price.

However, the exercise condition represents only a basic incentive of the buyer. It does not take into account other decision-shaping factors like provisions about Take-or-Pay, Make-Up or Carry Forward gas. This will be handled later in the thesis. Any reader interested in modeling take-or-pay decisions is referred to Schultz (1997).

3.2.3. Option volume

The buyer follows the above mentioned criteria on a daily basis. It means he decides every day on the source of his purchase. The GSA defines the value of the maximum daily quantity (DCQ_{max}) which is, in most cases, less than the amount of the flexibility volume. It keeps the buyer off the possibility to exercise the whole swing option volume (flexibility volume) given as $ACQ - AMQ$ in one day. As the DCQ_{max} depends on an agreed load factor (see eq. (2.2)), it is time to distinguish between two types of flexibility provided by typical gas sales agreements, or take-or-pay contracts.

When the load factor is less than one, or less than 8,760 hours (8,784 hours in case of a leap year), we speak about time flexibility. It allows the buyer to offtake on some days of the contract year more and on some days less than the average daily contract quantity (DCQ). As a result, the buyer can shift a part of his summer offtake to winter months and so earn some extra money from reselling the shifted volumes in the market for higher winter prices.

The provision about a downward quantity tolerance is a source of another type of flexibility, volume flexibility. It allows the buyer to offtake less than the ACQ, but not less than the AMQ, without incurring any sanctions.

Although many take-or-pay contracts provide both time and volume flexibility this thesis is interested solely in the pricing of the volume flexibility. It means a load factor of 1 or 8,760 hours (8,784 hours in case of a leap year) is assumed. The maximum daily quantity is then equal to the average daily quantity, i.e. $\text{DCQ}_{\max} = \text{DCQ}$. Regarding a minimum daily quantity we assume that there is no limit, i.e. it is equal to zero. These daily constraints then determine how big the fraction of the option volume can be exercised within one day. It is less than or equal to the DCQ.

However, we do not assume that there is a day when the buyer offtakes less than the DCQ but more than zero since such offtake does not represent optimal behavior under assumptions of the mark-to-market evaluation. It means the daily option volume is equal to the DCQ.

3.2.4. Number of options

As the swing option volume (flexibility volume) cannot be exercised within one day but only gradually on some days of the delivery period, it seems appropriate to think of the swing option as if it was a set of many options on a volume equal to the DCQ. The number of options *N* the buyer owns is then given by the number of days necessary to exercise the entire flexibility volume, i.e.

$$N = \frac{\text{ACQ} - \text{AMQ}}{\text{DCQ}} \in (0; n], \tag{3.3}$$

for ACQ = *n*DCQ > AMQ and DCQ > 0, where *n* denotes the number of days in a contract year.

3.2.5. Type of the options

There are two basic types of options, call options and put options. A call option gives its owner the right to buy an underlying asset by an expiration date for a strike price. A put option gives its owner the right to sell an underlying asset by an expiration date for a strike price. As the buyer has the right to sell some of the contractual volumes back to the seller and the right to sell is characteristic for put options we can conclude that all of the options the buyer owns are put options.

Another important classification of options results from the manner of option exercise. There are two families of financial options, European options and American options. The European options can be exercised only on the expiration date, whereas the American options can be exercised at any time up to the expiration date. The buyer's right to sell back (or not to offtake) some of the contractual volumes is not restricted to certain days of the contract year. This means that the buyer is allowed to exercise the options he owns at any time from the first day till the last day of the contract year. In other words, the buyer owns N number of American put options.

3.2.6. Expiration dates

Since on each day of the contract year no more than one option can expire, there must be N number of different expiration dates. One of the options expires on the last day of the contract year, i.e. at time T, and the remaining ones on the days before, i.e. at time $T-1$, at time $T-2$, up to the one that expires at time $T-N+1$. Each of the options can then be exercised at any time from the first day of the contract year till its expiration date.

3.2.7. Option payoffs

Following the exercise condition (3.2), a buyer's payoff from an exercise of one of the N options at time T is equal to

$$(P_T - S_T - LD)^+ DCQ, \tag{3.4}$$

where $(\ldots)^+$ stands for max(…;0), S_T denotes a spot (day-ahead) price of gas present at the time of the exercise T, P_T refers to the value of the contractual price at time T and LD denotes the location differential.

The contractual price P_T can be replaced by the formula defined in the subchapter 2.2.2. The general formula $P_T = P_0 + \sum_{j=1}^{m} \alpha_j (AC_{jT} - AC_{j0}) CF_j \lambda_j$ can be reorganized to

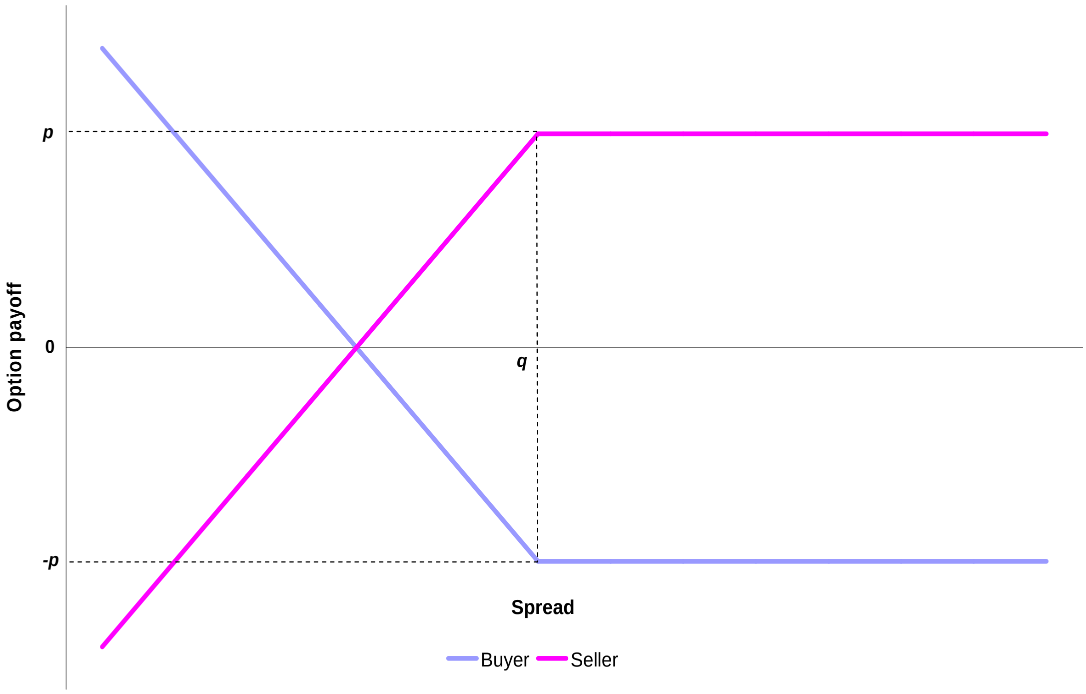
$$P_T = P_0 - \sum_{j=1}^{m} \alpha_j CF_j \lambda_j AC_{j0} + \sum_{j=1}^{m} \alpha_j CF_j \lambda_j AC_{jT} ,$$

where $P_0 - \sum_{j=1}^{m} \alpha_j CF_j \lambda_j AC_{j0}$ is a fixed term and only term $\sum_{j=1}^{m} \alpha_j CF_j \lambda_j AC_{jT}$ floats in time. Introducing coefficients k_j equal to $\alpha_j CF_j \lambda_j$ we can rewrite the equation (3.4) with

$$\left[q - \left(S_T - \sum_{j=1}^{m} k_j AC_{jT} \right) \right]^+ DCQ , \qquad (3.5)$$

where q is equal to $P_0 - \sum_j k_j AC_{j0} - LD$ and is constant for the whole contract year.

Graph 2: Option payoff



Source: Author

The option payoff given by the equation (3.5) depends on a spread between the market prices of at least two commodities, namely the price of natural gas (*S*), and the prices of one or more alternative commodities (*ACj*, for $j=1,\ldots,m$). For clearness, one can see the relationship between values of the option payoff and the spread on graph 2, where *p* denotes an option premium (discussed later in the chapter).

Generally, the buyer is not interested in the price movements of a single asset, but he focuses on movements of a spread between the prices of two or more assets. This characteristic is typical for so-called "spread options" falling into a group of "correlation options". The payoff from a general put option on a spread $bS_{2T} - aS_{1T}$ at time of an exercise *T* is then given as

$$[q - (bS_{2T} - aS_{1T})]^{+}, \tag{3.6}$$

where S_{1T} and S_{2T} are spot prices of two various assets present at time *T*, *a* and *b* are constants and *q* is a strike price.

This is an important finding for our valuation problem and the reason why we devote the whole chapter 4 to the theory on spread option pricing.

3.2.8. Underlying assets

Focusing on (3.5), one can see that the options are written on a spread between the market price of gas and the prices of competing fuels. It means that natural gas and competing fuels are the underlying assets. However, unlike the market price of gas the prices of competing fuels are not taken into account by their full amounts. They are multiplied by parameters k_j. Using the notation of (3.6) we can say that $a = k$ and $b = 1$.

3.2.9. Strike prices

One compares the value of the spread with a constant equal to $q = P_0 - \sum_j k_j AC_{j0} - LD$. It means this constant represents a strike price which is the same for all *N* options the buyer owns.

3.3. Option premium

The fair value price of an option, i.e. option premium *p*, is comprised of an intrinsic value and a time value (extrinsic value). The intrinsic value is calculated as a difference between the market value of an underlying asset and the strike value, but only if the option is in-the-money. For out-of-the-money options, it is equal to zero. In other words, the intrinsic value is a measure of guaranteeable “exercise profit” the option offers. The “exercise profit” is a result of a forward sale (for a call) or a forward purchase (for a put) and the option execution. In addition, for some price movements the option owner can earn more than the intrinsic value. This ability is charged by the option writer in the form of a time or extrinsic value.

Concerning our problem, we can see the intrinsic value as an additional profit/loss of the buyer/seller stemming from the DQT provision in the case that the option was immediately exercised. The exercise would mean a buyer’s forward purchase of the option volume in the market instead of using the GSA as a source. One can understand it as though the buyer decided to sell the option volume to the seller for the contractual price and to buy the same volume in the forward market for the market price. However, the forward purchase in the market itself does not guarantee the buyer the acquisition of the profit. Since the contractual price is indexed to the market prices of competing fuels which can float in time it is necessary to hedge against such a price risk. The option execution, together with the forward purchase in the market and the hedging, would finally bring the buyer the extra profit.

Moreover, until the delivery occurs prices can move in (for the buyer) favorable directions ensuring the buyer some additional gains above the ex-ante profit. As such gains simultaneously represent extra expenditures to the seller, he charges more than the intrinsic value (by the extrinsic value) for the option.

Describing the main features of the gas swing option we have disclosed that the option is actually a set of many American put options on a spread between prices of two or more energy commodities. It means, instead of concerning in price movements of gas, the buyer is interested in changes of a spread between prices of two or more energy commodities (gas and its competing fuels). As noted previously, such characteristic is typical for so-called “spread options” whose pricing represents a multidimensional valuation problem. To be able to derive a tool for pricing of gas swing options it is firstly necessary to go through the theory on spread option pricing.

4. Spread option pricing theory

A spread option is a type of option deriving its value from a difference between the prices of two or more assets. In other words, spread options allow investors to simultaneously take positions in two or more assets and profit from an increase (call option) or decrease (put option) of a spread between the assets prices. Because of their generic nature, spread options are used across various markets like currency and foreign exchange markets, fixed income markets, commodity futures markets, and energy markets.

Focusing on energy markets, spread options are frequently used as risk mitigation tools. Subjects that compete against other commodity prices or earn margins given by a spread between two or more commodities (power plants etc.) want to hedge against their price risk by a spread option trade. Examples of liquidly traded energy spread option products are: crack spreads, spark spreads, or dark spreads. One can trade them through an exchange (e.g. NYMEX[1]), but the bulk of their volumes is traded OTC (e.g. NYMEX, Spectron[2]).

There is a couple of models that have been developed by mathematicians while trying to find a fair value price of a spread option. One can distinguish between two main groups: numerical methods and analytical approximations. The analytical methods generally seek for a closed-form formula for calculation of the option premium. They are very popular because of their user-friendly application. However, a derivation of closed-form formulas for multidimensional valuation problems, such as pricing of spread options, stays a real challenge. One is referred to Kirk (1995), Poitras (1998), Carmona and Durrleman (2003), and Alexander and Venkatramanan (2007). In case

[1] http://www.nymex.com/index.aspx
[2] http://www.spectrongroup.com/

there is a lack of closed-form formulae the numerical methods can always be used to produce values approximating the price of a spread option. Representatives of this group are trinomial trees, finite difference methods or Monte Carlo simulation. However, methods like trees or finite difference methods become prohibitively time consuming when the dimension of the problem increases. It means most papers focused on the application of numerical methods to the spread option valuation problem engage the Monte Carlo methods. Meinshausen and Hambly (2004) can be an example.

The next important classification of option pricing models is based on a way of forming expectations about future spot prices. Some models use stochastic processes for modeling the spot price dynamics ("spot price models"). Another group of models relies instead on expectations shaped by market players ("forward curve models"). The spot price models are extremely popular because of their intuitive appeal and because of their mathematical tractability. See e.g. Schwartz (1997), Alaton et al. (2002), or Svec and Stevenson (2007). However, these models are not always satisfactory as they produce expectations which are very rarely consistent with the actual forward curves observed in practice. This is mainly the case of energy markets where the ability to arbitrage is limited and where forward prices can settle on values different from the theoretical arbitrage-free prices. This major shortcoming is at the core of the search for more sophisticated models that can account for the observable features of forward curves. One is referred to Miltersen (2003), or Santa-Clara and Sornette (2001).

4.1. Price dynamics of underlying assets

As today's value of the spread option price depends on future spot prices of its underlying assets and these are not known ex-ante, it is necessary to model their dynamics. Or one can focus on the dynamics of an entire forward curve instead of modeling only the dynamics of its leftmost point. The models reviewed thereinafter assume perfect markets and continuous trading. This means the prices of underlying assets are supposed to be continuous in time. In addition, they can take any value within a certain range, i.e. they are continuous in their values.

The price dynamics are mostly represented by stochastic differential equations (SDEs) whose coefficients are usually assumed to be Markovian.[1] More precisely, the option pricing theory uses mainly a generalized Wiener process[2] for modeling the price dynamics. We thus review some of their most frequently used specific types. For deeper information on stochastic differential equations see the Sobczyk's book (1991).

4.1.1. Geometric Brownian motion

A geometric Brownian motion (GBM) (occasionally, exponential Brownian motion) is the most used stochastic process in financial economics theory and in practice. It is a continuous-time stochastic process coming under the Markov processes. Stochastic process S_t is said to follow the GBM if it satisfies a stochastic differential equation of the form

$$dS_t = \alpha S_t dt + \beta S_t dW_t, \tag{4.1}$$

where W_t is a Wiener process or Brownian motion and α and β are constants.

A natural logarithm of the random variable S with the dynamics of GBM then follows an arithmetic Brownian motion (reviewed in subchapter 4.1.2.) with SDE of the form

$$d \ln S_t = \left(\alpha - \frac{1}{2}\beta^2 \right) dt + \beta dW_t. \tag{4.2}$$

These dynamics imply that the value of lnS at time t is normally distributed.[3] Since the natural logarithm of the variable is normally distributed the variable itself has a lognormal distribution.

For the arbitrary initial value S_0 the equation (4.1) has an analytic solution

$$S_t = S_0 e^{\left(\alpha - \frac{\beta^2}{2} \right) t + \beta W_t}.$$

[1] *"A Markov process is a particular type of stochastic process where only the present value of a variable is relevant for predicting the future. The past history of the variable and the way that the present has emerged from the past are irrelevant."* Hull J. (2006)

[2] Definition of the Wiener process can be found in the Appendix A.

[3] The process followed by the natural logarithm is derived by using the Itô's lemma (see Appendix A for its definition).

It is a log-normally distributed random variable with an expected value $E(S_t) = S_0 e^{\alpha t}$ and a variance $Var(S_t) = S_0^2 e^{2\alpha t}(e^{-\beta^2 t} - 1)$. One can see that the expected value grows exponentially in time and the variance increases proportionally to the time interval. It means the further we move in time, the greater our uncertainty is regarding the value of the variable.

4.1.2. Arithmetic Brownian motion

Sometimes it can be useful to work with an arithmetic Brownian motion (ABM) instead of a geometric one. An example is the case when one prefers working with the natural logarithm of a log-normally distributed variable to deal with the variable itself.

A stochastic process S_t is said to follow the ABM if it satisfies a stochastic differential equation of the form

$$dS_t = \alpha dt + \beta dW_t, \tag{4.3}$$

where again W_t is a Wiener process or Brownian motion and α and β are constants.

For the arbitrary initial value S_0 the equation (4.3) has an analytic solution

$$S_t = S_0 + \alpha t + \beta \int_0^t e^{\alpha(t-u)} dW_u .$$

It is a normally distributed random variable with an expected value $E(S_t) = S_0 + \alpha t$ and a variance $Var(S_t) = \beta^2 \frac{e^{2\alpha t} - 1}{2\alpha}$. Now the expected value grows arithmetically in time and the variance increases proportionally to the time interval.

4.1.3. Mean reverting processes

The geometric Brownian motion models are appropriate for modeling the price dynamics of various assets, however, they fail to capture one of the main features of interest rates and commodity prices: mean reversion.

Suppose that the random variable S follows a geometric mean-reverting process given by SDE of the form

$$dS_t = \alpha S_t (M - S_t)dt + \beta S_t dW_t, \quad (4.4)$$

where *M* denotes a long-run equilibrium level (or a long-run mean value which the variable tends to revert to), α is the speed of reversion and the other terms have the same meaning as in the case of the geometric Brownian motion presented before. We then assume the variable is lognormally distributed and we say it follows a geometric Ornstein-Uhlenbeck process (the name explained below).

The mean-reverting process and the GBM differ from each other through a drift term. In case of the mean-reverting process, the drift is positive when the current price level *S* is less than the equilibrium level *M,* and is negative when *S* is greater than *M*. In others words, the equilibrium level attracts prices in its direction. The size of a price change is then given by the distance between the current price and the equilibrium level: the wider the spread is the larger the change expected. This kind of dynamics influences the variance of the random variable. The variance does not grow proportionally to the time interval as in case of the Brownian motion. It grows at the beginning and after some time it stabilizes on a certain value.

Sometimes work with an arithmetic process for the natural logarithm of the stochastic variable is favored, mainly due to its simplicity. Consider that a random variable *S* follows an arithmetic Ornstein-Uhlenbeck process[1] towards an equilibrium level *M* described by SDE of the form

$$dS_t = \alpha(M - S_t)dt + \beta dW_t. \quad (4.5)$$

For the arbitrary initial value S_0 the equation (4.5) has then an analytic solution

$$S_t = S_0 e^{-\alpha t} + M(1 - e^{-\alpha t}) + \beta e^{-\alpha t}\int_0^t e^{\alpha u} dW_u .$$

In this case, *S* is a normally distributed variable with an expected value $E(S_t) = S_0 e^{-\alpha t} + M(1 - e^{-\alpha t})$ and a variance $Var(S_t) = (1 - e^{-2\alpha t})\frac{\beta^2}{2\alpha}$. Unlike the

[1] This name is due to the authors of the paper that first discussed the model using such process: see Uhlenbeck and Ornstein (1930).

Brownian motion, the variance is bounded, i.e. it does not take the infinite value for any instant of time. With growing t it tends to a value $\frac{\beta^2}{2\alpha}$.

There can be some adjustments to these processes similar to the one used by Schwartz (1997), which is represented by SDE of the form

$$dS_t = \alpha(M - \ln S_t)S_t dt + \beta S_t dW_t . \tag{4.6}$$

This process enables its parameters to be estimated easier from historical data.

4.1.4. Seasonality incorporating processes

Seasonality occurs with some commodities like natural gas, heating oil, gasoline and electricity. It can be incorporated into the traditional stochastic processes by changing the drift term. Taking into account the mean-reverting process with the SDE (4.4) the seasonality can be incorporated by replacing the fixed long-run mean value M with a periodic function M_t capturing a cyclical nature of commodities prices (see e.g. equations (4.7) and (4.8)).

$$dS_t = \alpha S_t (M_t - S_t)dt + \beta S_t dW_t \tag{4.7}$$

$$dS_t = \alpha(M_t - S_t)dt + \beta dW_t \tag{4.8}$$

Suitable representatives of the function M_t are sine/wave functions with their maximum at the demand peak season. An example could be the deterministic function

$$M_t = a + b\sin(\varpi t) , \tag{4.9}$$

where a and b are constants, and the frequency ω is equal to $2\pi/365$.[1]

4.2. Spread option pricing

Although thirty five years have already elapsed since Fischer Black, in cooperation with Myron Scholes (1973) and Robert C. Merton (1973), published their pioneering papers no better framework for option pricing has yet been developed.

[1] This type of periodic function has been used by Dornier and Queruel (2000) for modeling Chicago's temperature.

Following their work, pricing and hedging schemes for various sorts of financial derivatives have been shaped. The authors provided practitioners with a user friendly closed-form formula applicable to pricing of one-factor options.[1]

However, under certain cases one cannot restrict the problem to evaluation of the one-factor option. This is very often the case of real options pricing when one is interested in the dynamics of a spread between two or sometimes more variables instead of the dynamics of the variables themselves. We then speak about a multifactor valuation problem that cannot be solved simply by applying the Black-Scholes formula. A result of the considerable effort of mathematicians to develop a closed-form formula for pricing of two-factor options is a couple of closed-form approximations, but no analytic formula (Alexander and Venkatramanan, 2007). That is why numerical methods are so frequently used for the calculation of a multi-factor option premium. This section reviews the derived closed-form approximations and the numerical method that is most commonly employed in case of multifactor valuation problems.

Before going into details, it is important to note that all undermentioned methods and models are based on the same concept of risk-neutral expectations. More precisely, an option price is considered as an expectation of discounted future cash flows for a probability structure called risk-neutral. Since our valuation problem concerns put options we demonstrate the concept on a put. Specifically, the price of a put spread option is given by the risk-neutral expectation

$$p = e^{-r(T-t)} E\left\{\left[q - \left(bS_{2T} - aS_{1T}\right)\right]^{+}\right\}, \quad (4.10)$$

where r is a risk-free interest rate. This finding is extremely useful in practice as it provides the natural generalization to more complex situations involving more general underlying assets with more sophisticated stochastic dynamics.

4.2.1. Analytical models

This section reviews some of the models providing a closed-form formula for pricing of European put options on a spread between two underlying variables.

[1] For the formula see the Appendix A.

4.2.1.1. Two-factor models

Even in the simple case of two price processes following correlated geometric Brownian motions with a constant volatility, no analytic formula for the price of a standard European spread option has yet been derived. The only accurate formula one can find is the formula for pricing of a special case spread option called an exchange option. It is a spread option with a zero strike price. The remaining attempts to derive an analytical tool for spread option pricing can be regarded as analytic approximations.

For all two-factor models we assume that the risk-neutral dynamics of the two underlying variables are given by geometric Brownian motions. It means they are represented by stochastic differential equations of the form (4.1) with the parameter α equaling to $r - \delta_i$, where r is a risk-free interest rate and δ_i denotes some form of cost of carry or convenience yield of commodity i ($i = 1, 2$). The parameter β then stands for constant volatilities of the underlying variables σ_i ($i = 1, 2$). In addition, we assume that the two variables are correlated through the driving Brownian motions, specifically, that $E\{dW_1(t)dW_2(t)\} = \rho dt$.

Margrabe's formula

For the special case of a zero strike price a pricing formula in a closed form was derived by Margrabe (1978). When the strike price of a spread option is zero the option is called an exchange option, since the buyer has the option to exchange one underlying asset for the other. The fact that the strike price is zero allows one to reduce the pricing problem to a single dimension, using one of the assets as a numeraire.

The price of a put spread option with a strike price $q=0$ and time to expiration $T-t$ is then given by

$$p_M = x_2 e^{-\delta_2 (T-t)} \Phi(-d_2) - x_1 e^{-\delta_1 (T-t)} \Phi(-d_1), \tag{4.11}$$

where

$$d_1 = \frac{\ln(x_1 / x_2) + \left(\delta_2 - \delta_1 + \frac{1}{2}\sigma_M^2 \right)(T-t)}{\sigma_M \sqrt{T-t}}, \quad d_2 = d_1 - \sigma_M \sqrt{T-t},$$

$x_1 = S_{1t}$, $x_2 = S_{2t}$ (i.e. spot prices of the underlying assets at time t), and $\sigma_M = \sqrt{\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2}$ is a variance of $(x_1/x_2)^{-1} d(x_1/x_2)$. Here and throughout the thesis, we use the notation φ(.) and Φ(.) for the density and the cumulative distribution function of the standard normal $N(0,1)$ distribution, i.e.

$$\varphi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \qquad \text{and} \qquad \Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-u^2/2} du\,.$$

For proof see Margrabe (1978).

Kirk's formula

Kirk (1995) presented an approximate formula for pricing European spread options on futures contracts. The method extends that of Margrabe's to non-zero but very small strike values and results in the following formula for pricing of put spread options:

$$\hat{p}_K = e^{-\left(r - \bar{r} + \bar{\delta}_2\right)(T-t)} \Phi\left(-d_{2K}\right) - Z_t e^{-\delta_1 (T-t)} \Phi\left(-d_{1K}\right), \tag{4.12}$$

where

$$d_{1K} = \frac{\ln(Z_t) + \left(r - \bar{r} + \bar{\delta}_2 - \delta_1 + \frac{1}{2}\sigma_{tK}^2\right)(T-t)}{\sigma_{tK}\sqrt{T-t}}, \qquad d_{2K} = d_{1K} - \sigma_{tK}\sqrt{T-t}\,,$$

and

$$\sigma_{tK} = \sqrt{\sigma_1^2 - 2\rho\sigma_1\sigma_2 \frac{x_2}{x_2 + qe^{-r(T-t)}} + \sigma_2^2 \left(\frac{x_2}{x_2 + qe^{-r(T-t)}}\right)^2}\,,$$

$$Z_t = \frac{x_1}{x_2 + qe^{-r(T-t)}}\,, \quad \bar{r} = \left(\frac{x_2}{x_2 + qe^{-r(T-t)}}\right) r\,, \quad \bar{\delta}_2 = \left(\frac{x_2}{x_2 + qe^{-r(T-t)}}\right)\delta_2\,.$$

Alexander and Venkatramanan (2007) presented an extension of the Kirk's formula to pricing of American spread options. They introduced an early exercise premium above the price of a European style spread option as suggested by Kirk.

A derivation of the Kirk's approximation together with its extension to American spread options can be found in Alexander and Venkatramanan (2007).

A reader interested in next analytic approximation is referred to Carmona and Durrleman (2003).

4.2.1.2. One-factor models

For the above mentioned approximations we assume that the underlying variables are modeled by means of log-normal distributions as prescribed by the geometric Brownian motion model. An important feature of this model is that it produces underlying prices that are inherently positive. But the positivity restriction does not refer to spreads between these prices. This simple remark has become fundamental for many papers proposing the use of an arithmetic Brownian motion (as opposed to the geometric Brownian motion) for the dynamics of spreads. Doing so, a simple closed-form formula can be derived for the pricing of spread options by computing Gaussian integrals.

The Bachelier model

A representative of one-factor models derived for the spread option valuation is a so-called Bachelier model.[1] This model was advocated by Wilcox (1990), Shimko (1994) and Poitras (1998). It assumes that the spread S_t between the two underlying variables follows an appropriately defined arithmetic Brownian motion given by a stochastic differential equation of the form

$$dS_t = \mu_S S_t dt + \sigma_S dW_{St}, \qquad (4.13)$$

where μ_S , σ_S and W_{St} are derived from the dynamics of the individual component variables S_{1t} and S_{2t}. We assume that these follow unrestricted arithmetic Brownian motions of the forms

$$dS_{1t} = \mu_1 S_{1t} dt + \sigma_1 dW_{1t}$$

$$dS_{2t} = \mu_2 S_{2t} dt + \sigma_2 dW_{2t},$$

with μ_i standing for $r - \delta_i$ (for $i=1,2$). In any case, μ_i is, as well as σ_i, assumed to be a deterministic constant. The Brownian motions W_{1t} and W_{2t} are correlated by a constant

[1] This terminology follows Smith (1976).

parameter ρ. As before, the initial values (present at time t) of the variables are denoted as $x_1 = S_{1t}$ and $x_2 = S_{2t}$.

The price of a put spread option is then given by

$$p = s(t)\left[\Phi\left(\frac{s(t)}{\Lambda}\right) - 1\right] + \Lambda\varphi\left(\frac{s(t)}{\Lambda}\right), \tag{4.14}$$

where

$$s(t) = x_1 e^{-\delta_1(T-t)} - x_2 e^{-\delta_2(T-t)} - q e^{-r(T-t)} \quad \text{and} \quad \Lambda = \sqrt{\nu_{11} + \nu_{22} - 2\nu_{12}},$$

where

$$\nu_{11} = \sigma_1^2\left(\frac{e^{-2\delta_1(T-t)} - e^{-2r(T-t)}}{2(r-\delta_1)}\right), \quad \nu_{22} = \sigma_2^2\left(\frac{e^{-2\delta_2(T-t)} - e^{-2r(T-t)}}{2(r-\delta_2)}\right), \quad \text{and}$$

$$\nu_{12} = \sigma_{12}\left(\frac{e^{-(\delta_1+\delta_2)(T-t)} - e^{-2r(T-t)}}{2(r-\delta_1-\delta_2)}\right).$$

For proof and deeper information see Poitras (1998) and Schaefer (2002).

4.2.2. Numerical methods

The numerical methods for option pricing are based on the computing of discounted future cash flows generated by an option. As the option payoff depends on the future spot prices of underlying assets, and these are not known ex-ante, we need to model their dynamics. This can be done by using various numerical methods. However, we limit ourselves to the methods that are useful for the pricing of gas swing options, or more generally, for multidimensional valuation problems. Specifically, methods like trees or finite difference methods become prohibitively time consuming when the dimension of the problem increases. An alternative approach, via the Monte Carlo methods, works more effectively in this framework.

Having the dynamics, the only thing left to compute the risk-neutral expectation of the equation (4.10) is to find a joint density of the couple (S_{1T}, S_{2T}) under that particular risk-neutral measure. We denote this density by $f(S_{1T}, S_{2T})$. The pricing of the spread option is then restricted to the computation of a double integral

$$p = e^{-r(T-t)} E\left\{\left[q - \left(bS_{2T} - aS_{1T}\right)\right]^{+}\right\} \tag{4.15}$$
$$= e^{-r(T-t)} \int\int \left[q - \left(bS_{2T} - aS_{1T}\right)\right]^{+} f(S_{1T}, S_{2T}) dS_{1T} dS_{2T} \ .$$

Computing the expectation by conditioning the first integral by the knowledge of S_{1T}, we get

$$p = e^{-r(T-t)} \int \left(\int \left[q - \left(bS_{2T} - aS_{1T}\right)\right]^{+} f_{2|S_{1T}}(S_{2T}) dS_{2T} \right) f_1(S_{1T}) dS_{1T} \ , \tag{4.16}$$

where $f_1(S_{1T})$ represents a density of the variable S_1 at maturity and $f_{2|S_{1T}}(S_{2T})$ denotes a conditional density of the variable S_2 at maturity, given that the variable S_1 is equal to S_{1T} at that time. One can see that the price of the spread option is an integral over S_{1T} of the prices of European puts on the second variable with strike prices $q + aS_{1T}$.

4.2.2.1. Monte Carlo methods

Most often, a good way to compute the risk-neutral expectation is using the Monte Carlo methods (MCM, or MCS for Monte Carlo simulation). Boyle (1977) was among the first ones who proposed applying the MCS to the option pricing problem. Since then, many researchers, e.g. Hull and White (1987), Johnson and Shanno (1987), or Boyle et al. (1997), have engaged the MCS for analyzing options markets.

> *"MCM are a class of computational algorithms that rely on repeated random sampling to compute their results. They are often used when simulating physical and mathematical systems. Because of their reliance on repeated computation and random or pseudo-random numbers, MCM are most suited to calculation by a computer. They tend to be used when it is infeasible or impossible to compute an exact result with a deterministic algorithm."* (Hubbard, 2007)

Since they use random number generators, i.e. deterministic algorithms that attempt to produce values that appear truly random by various measures, it seems to be a useful tool for modeling the price dynamics. The advantage of this method is its generality in being able to model "imperfect" market conditions, not easily captured in analytically tractable models. As Boyle (1977) has stated:

> *"The Monte Carlo method should prove most valuable in situations where it is difficult if not impossible to proceed using a more accurate approach."*

Disadvantages are its computational inefficiency when compared to most other numerical methods and its tendency to result in an uninsightful black box treatment of the model. However, the situation changes with increasing dimension of the problem. In multifactor option pricing framework, it is more efficient to employ the MCM, as e.g. Broadie and Glasserman (1997) or Meinshausen and Hambly (2004) did.

Specifically, considering the spread option pricing, the idea is to generate a large number of random paths of the underlying process S_1, and the same number of paths of the underlying process S_2, over the interval [0, T]. Using these paths, one can get estimates $\hat{E}_t\left\{\left[q-\left(bS_{2t}-aS_{1t}\right)\right]^+\right\}$of the risk-neutral expectations $E_t\left\{\left[q-\left(bS_{2t}-aS_{1t}\right)\right]^+\right\}$ given by the double-integral in (4.15) by computation of the mean values:

$$\hat{E}_t\left\{\left[q-\left(bS_{2t}-aS_{1t}\right)\right]^+\right\}=\frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left[q-\left(bS_{2tj}-aS_{1ti}\right)\right]^+ . \qquad (4.17)$$

The final step is to discount the estimates at the risk-free interest rate.

Since a gas swing option can be seen as a set of many spread options on energy commodities we can use some of the presented theory to derive a model for pricing of gas swing options.

5. Application to pricing of gas swing options

In this chapter, we use the presented theory for pricing of an option embedded in the gas sales agreement, i.e. a gas swing option. Since the gas swing option is actually a set of many American put options on a spread between prices of two or more energy commodities, its evaluation seems to be the case which Boyle (1977) referred to (see subchapter 4.2.2.1.). Specifically, the reviewed analytic approximations are hardly applicable to our valuation problem. It is given by a specific nature of the energy markets. They are fundamentally different from the traditional financial security markets in several ways, which increases the difficulty of the problem.

Firstly, the markets of energy commodities and derivatives lack the same level of liquidity that the majority of financial markets enjoy. Very often one cannot find a proper hedging tool or hedging strategy for mitigation of his price risk since there is no market for the derivative product he is interested to. Secondly, the energy markets are characterized by a limited ability of market players to arbitrage because the players miss a sufficient amount of non-operating inventories. Thirdly, prices of energy commodities are typically exposed to very high volatility and large shocks. And finally, the prices tend to show strong mean-reverting trends and seasonality.

All these facts support the necessity of using a numerical method to derive the pricing model for gas swing options. Specifically, we choose the Monte Carlo methods. The derived model is presented in the following text through its application to one arbitrarily chosen GSA. It means we start with a definition of such GSA.

5.1. Parameters of GSA

We choose one arbitrary GSA so that we specify values or forms of the fundamental parameters of the general GSA as described in the chapter 2. Specifically, we are interested in values or forms of the contractual quantities, take-or-pay level, price formula, delivery period, or more precisely the contract year, delivery point and the signing date.

Delivery period

Rather than the delivery period itself, its length, i.e. a contract duration, influences assumptions of the evaluation. More precisely, long-term contracts very often include provisions about make-up and carry forward gas. The rights established by these provisions then shape the buyer’s decision making. Specifically, making his decision about the source of his purchase, the buyer takes into account a possibility to shift a part of his today’s/future offtake to the next/present contract year(s) (make-up/carry forward gas). He forms his decision based on the values of forward and spot spreads between the contractual and market prices (more on it in subchapter 5.4.2.). Since it does not mean anything other than an extension of his decision horizon, we simplify the problem, without loss of generality, by assuming the contract duration of one year. Or in other words, we are going to evaluate a contract with no make-up and carry-forward gas provisions.

The practice offers more forms of a contract year definition. It can be defined as a calendar year (*CY*), gas year (*GY*), or less commonly as a storage year (*SY*).

Table 1: Types of contract year

	from				to			
General	DD	MM	YYYY	T *	DD	MM	YYYY+1	T
CY	01	01	YYYY	T	01	01	YYYY+1	T
GY	01	10	YYYY	T	01	10	YYYY+1	T
SY	01	04	YYYY	T	01	04	YYYY+1	T

** T denotes time and is usually defined as 6:00 a.m.*

Source: Author

Since the type of contract year has no influence on the model efficiency we choose it arbitrarily. Let it be the calendar year.

The only thing left is to replace *YYYY* with a concrete calendar year. The calendar year 2008 seems to be the best candidate as it satisfies the two following criteria.

1) It has already gone, which enables us to compare our valuation results with the real values. Or in other words, it provides us out-of-sample data that can be used for testing of our model.
2) At the same time we need to have enough in-sample data. Since the European spot and forward gas markets do not have a long history it means the later calendar year one chooses the better it is.

The resulting delivery period is thus defined as follows.

Table 2: Delivery period of GSA under evaluation

from:	**01.01.2008 6:00 AM**	to:	**01.01.2009 6:00 AM**
no. of days (n):			366

Source: Author

Signing date

The signing date of the contract is another important fundamental of our valuation problem. It determines time to maturities of the options the buyer owns, thereby the extrinsic value of the swing option. It is logical to choose a date that precedes the start date of the delivery period. In addition, one can assume that the parties are not willing to sign the contract too much in advance, as the further in time the delivery is, the greater their uncertainty regarding the values of the underlying variables. That is why we choose a date that precedes the start date by one quarter, i.e. 1 October 2007.

Contractual quantity and take-or-pay level

We only need to define the amount of the DCQ since the amount of the ACQ is derived by applying the equation (2.1). Under assumption that load factor *LF* is equal to 1 the DCQ_{max} equals the DCQ.

The DCQ can be defined either in calorific units or in volume units. It used to be more common to define the quantity of gas in m^3 together with a gross calorific value[1] based on the quality of supplied gas. However, it has gradually changed. Now it is more common to list the prices of gas in terms of payment per a calorific unit (MWh or Thm). The reason is that the quality of gas differs across various supply contracts and such definition makes the prices easily comparable among each other.

Since we focus on continental Europe trading we prefer using kWh (or its multiples: MWh, GWh or TWh).[2] As the amounts of the contractual volumes have no influence on the model efficiency we choose them arbitrarily.

Although the exact contents of take-or-pay contracts are protected from the public, one can find some notations regarding amounts of the take-or-pay level. We can refer to Gaylord (1989) mentioning the typical take-or-pay level of an amount between 70% and 90%. Again, we choose the level arbitrarily, only with respect to Gaylord's

[1] The gross calorific value is the heat generated by the complete combustion of a unit volume of gas in oxygen, including the heat which would be recovered by condensing the water vapour formed.
[2] In the UK gas is traded in terms (Thm).

notation. See table 3 for a summary of the amounts of volumes and take-or-pay level stipulated by the GSA under evaluation.

Table 3: Contractual quantities and ToP level

DCQ =	**240**	MWh
DCQ_{max} =	**240**	MWh
ACQ =	**87,840**	MWh
ToP =	**85**	%
AMQ =	**74,664**	MWh

Source: Author

Contractual price

The provision about a price setting mechanism is the most guarded by contracting parties. Fortunately, there is one example represented by the price formula introduced by Ruhrgas in 2004 (see subchapter 2.2.2.). It is an oil-indexed formula presented as an alternative to the BAFA-based price formula (the average import price of gas delivered to Germany). An important finding is that German import prices are linked to the price changes of such competing fuels as gas oil or fuel oil.

For the sake of simplicity, we do not use the exact form of the Ruhrgas formula. The reason is that the formula is linked to two different fuels and using it in such a form would form a valuation problem with more than two dimensions. Since a reduction of the problem to two dimensions does not mean a loss of generality we choose only one of the fuels as a variable entering into our price formula. Let it be the product *LSFO* with its reference period (see subchapter 2.2.2.).

However, some other changes of the formula parameters are necessary since their values reflected market conditions present at time of the Ruhrgas auction. It is the case of the basis prices, $LSFO_0$ and P_0. The values of $LSFO_0$ and P_0 can be derived as market values of the product *LSFO* and natural gas, respectively, present at time of signing the contract. Specifically, $LSFO_0$ is equal to an arithmetic average of the last four on that date available market prices of the product *FUELOIL*. Regarding P_0, its value equals the last on that date available market price of natural gas. Since the only relatively liquid gas market in continental Europe is the Dutch network's virtual balancing point – the Title Transfer Facility (TTF) – we use its prices as a benchmark.

Regarding coefficients like weight, conversion factor and pass through factor we must only decide on the pass through factor as the weight is equal to one and the energy conversion factor of the product *FUELOIL* is 0.08013 (for a conversion from Eur/t to Eur/MWh). As noted in Asche et al. (2002b), pass through factors are typically high, e.g. 0.85 or 0.9, meaning that natural gas prices are highly responsive to price changes of energy substitutes. Following this information we let the pass through factor be equal to 0.9. The resulting coefficient *k* is then given as a product of the weight, energy conversion factor and the pass through factor. See table 4 for a summary of the values of the formula coefficients.

Table 4: Coefficients of price formula

natural gas basis price (P_0)	**17.0125**	Eur/MWh
LSFO basis price ($LSFO_0$)	**253.4651**	Eur/t
weight (α)	**1**	
energy conversion factor (ECF)	**0.08013**	
pass through factor (λ)	**0.9**	
coefficient of LSFO (*k*)	**0.072117**	

Source: Author

We thus assume that the contracting parties agree upon the following price, expressed in Euro per MWh and recalculated monthly on the first day of each delivery month:

$$P = 17.0125 + 0.072117(LSFO - 253.4651), \tag{5.1}$$

where *LSFO* states for the product as described in 2.2.2. In addition, *EUR/USD* is supposed to be equal to 1.36. We use a prediction of EUR/USD exchange rate as made and published by the ECB in "Eurosystem staff macroeconomic projections for the Euro area" in June 2007. This assumption enables us to avoid modeling future spot exchange rates, or solving a three-dimensional problem. We take the liberty to simplify the problem because estimates of the future spot EUR/USD exchange rates would be obtained in a similar way as the estimates of the underlying commodities prices are.

Delivery point

For the sake of simplicity and without loss of generality, we assume that the delivery point is identical to the Dutch network's virtual balancing point (TTF), i.e. the location differential *LD* is equal to zero.

Specifying the GSA parameters we at the same time decided on historical periods of in-sample and out-of-sample data gathering. Specifically, we use data of the gas years 2005/06 and 2006/07 as in-sample data and data of the calendar year 2008 as out-of-sample data. Or in other words, data of gas years 2005/06 and 2006/07 are used for an estimation of model parameters and the derived model is subsequently tested through data of the calendar year 2008.

In the remaining part of the chapter we look for the fair value price (option premium) of the swing option embedded in the presented GSA on the date of contract signing, i.e. on October 1, 2007 (hence "evaluation date").

5.2. Price dynamics of underlying commodities

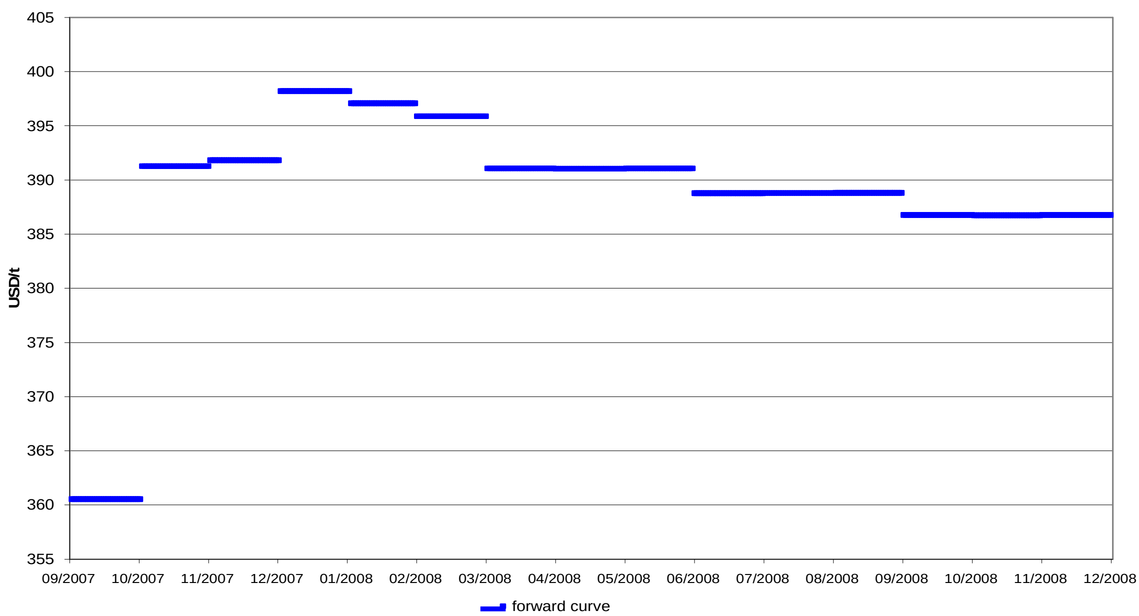
As noted previously, the options the buyer owns are written on a spread between the market price of gas and prices of competing fuels. Since we have decided on fuel oil, or more precisely on the product *LSFO*, to be the competing fuel entering into the contractual price formula the underlying assets or commodities of the options are natural gas and *LSFO*.

Since the spread option payoff depends on the future spot prices of the underlying commodities, and these are not known ex-ante, it is necessary to model their dynamics. Or one can rely on the forward prices of the underlying commodities thinking of them as they are the market's expectations of the future spot prices. As noted previously, the spot price models, i.e. models focused on the spot price dynamics, produce expectations which are mostly inconsistent with the actual forward prices formed by the market. This is an important finding since the market value of the option should be in line with the observed forward prices of the underlying commodities. It means that the price of the option implicit in expectations other than equal to the forward prices represents an arbitrage potential for the option owner or makes the option unmarketable. That is why various correction methods are used to make the spot price models consistent with the observed forward prices.

The inconsistency with observed forward prices is very typical for application of the spot price models to the energy markets. The reason is that these markets are characterized by the limited ability of market players to arbitrage. It very often results in forward prices which considerably differ from the theoretical arbitrage-free prices. Based on these facts we prefer focusing on forward price curves to the application of the spot price models.

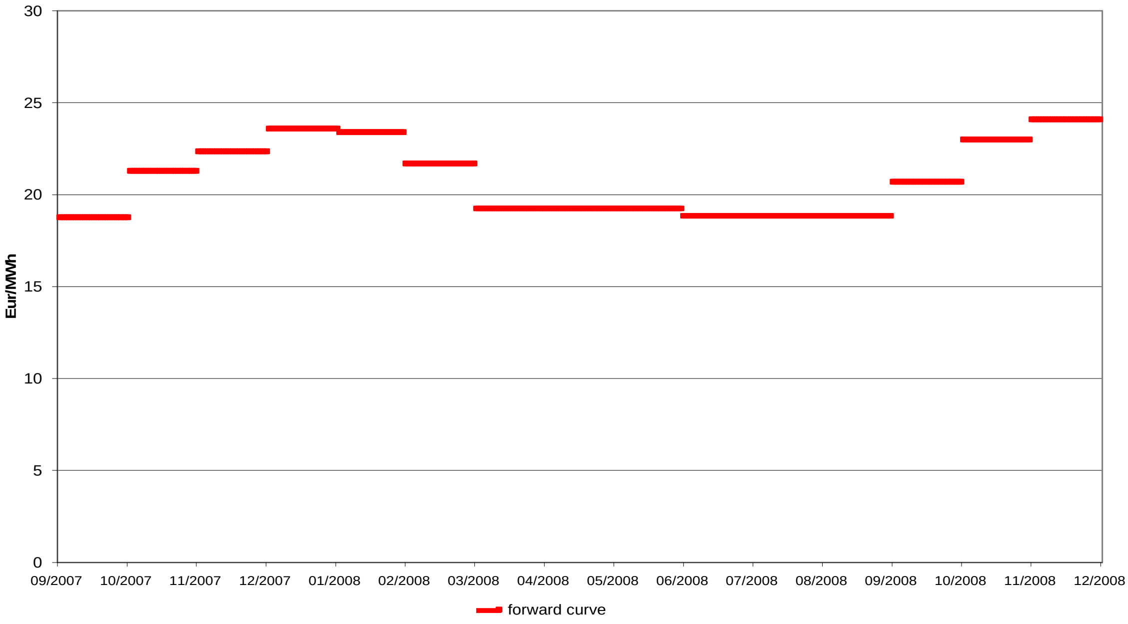
Since the evaluation date is 1 October 2007 we present the forward curves as of this date (see graphs 3 and 4).

Graph 3: Oct 1, 2007 forward prices of 1% fuel oil with delivery in ARA



Source: Author

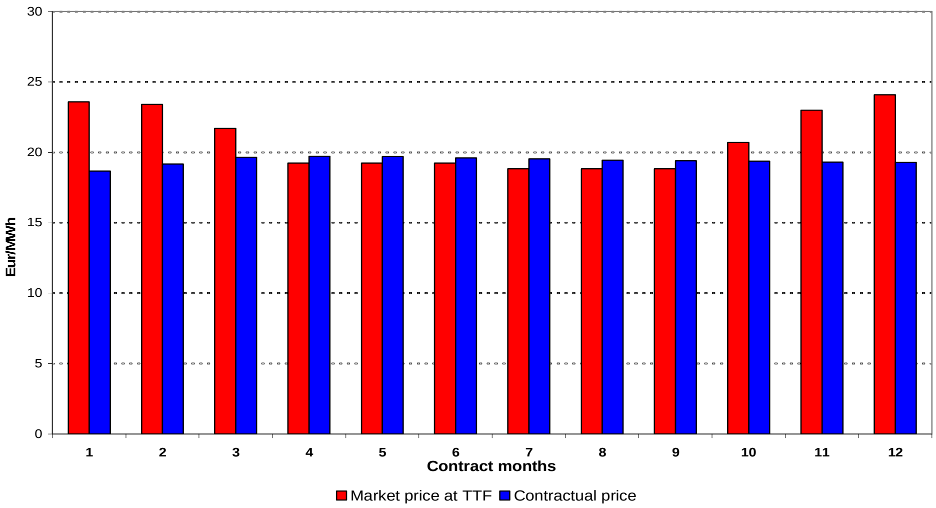
Graph 4: Oct 1, 2007 forward prices of natural gas with delivery at TTF



Source: Author

Using the forward curve shown on graph 3 and the contractual price formula, we can calculate the values of forward contractual prices of gas. See graph 5 for a comparison of these values with the forward market prices of gas. One can see that on 1 October 2007 the forward market price was lower than the forward contractual price for all summer months (April – September).

Graph 5: Market vs. Contractual forward prices (Oct 1, 2007)



Source: Author

5.2.1. Stochastic processes

Graphs 3 and 4 show the forward prices which are not continuous either in their values or in time. Their values change only at certain fixed points in time, specifically, on the first day of each calendar month, and by an amount which is a multiple of a cent. In other words, the forward prices are rounded up to two decimal points and show monthly granularity which is in virtue of the fact that delivery periods under traded futures contracts are not shorter than one month and are multiples of a calendar month. The rounding of their values to two decimal points can be easily managed, but the discontinuity in time makes the problem more difficult.

As noted previously, the buyer locks in the profit from the option purchase, without running any risks, by taking the three actions (forward purchase + hedging + execution). Moreover, until the delivery occurs prices can move in (for the buyer) favorable directions ensuring the buyer additional gains above this ex-ante profit. From the first day of the delivery period the buyer has the right to continuously choose one of his two purchase sources. Every day during the period he decides whether to buy gas under the GSA or in the market preferring the lower price and taking into account his take-or-pay limitation (or AMQ). So, to be able to price the option we need to form expectations about the values of the underlying variables for each day within the delivery period. This means that the forward curves on graphs 3 and 4 need to be transformed into forward curves with the daily granularity.

We assume that the forward prices are the best estimates of monthly averages of the future daily spot price quotations for 1% fuel oil and natural gas. As the real values of the future monthly averages depend on a resolution of uncertain parameters we must add randomness to underlying price processes given by the forward curves as shown on graphs 3 and 4. In addition, we assume that the daily quotations can randomly deviate from the monthly averages. Next, since the underlying commodities are energy commodities showing seasonality features, it seems to be appropriate to assume that price volatilities of these commodities differ across various calendar months. And finally, as we focus on modeling commodities prices that cannot take negative values, we assume they are log-normally distributed. Let 1% fuel oil with delivery in ARA be commodity 1 and natural gas with delivery at TTF be commodity 2, then the following stochastic processes summarize our assumptions about the price dynamics of these commodities.

$$\ln S_{1dm} = \ln F_{1m} + \sigma_{1m}^{M}\left(\rho^{M}\varepsilon_{2m} + \sqrt{1-\left(\rho^{M}\right)^{2}}\varepsilon_{1m}\right) - \frac{\left(\sigma_{1m}^{M}\right)^{2}}{2} + \sigma_{1m}^{D}\left(\rho^{D}\varepsilon_{2dm} + \sqrt{1-\left(\rho^{D}\right)^{2}}\varepsilon_{1dm}\right) - \frac{\left(\sigma_{1m}^{D}\right)^{2}}{2}, \quad (5.2)$$

$$\ln S_{2dm} = \ln F_{2m} + \sigma_{2m}^{M}\varepsilon_{2m} - \frac{\left(\sigma_{2m}^{M}\right)^{2}}{2} + \sigma_{2m}^{D}\varepsilon_{2dm} - \frac{\left(\sigma_{2m}^{D}\right)^{2}}{2},$$

where S_{1dm} and S_{2dm} denote the spot prices of commodities 1 and 2, respectively, on day d of a month m, F_{1m} and F_{2m} are the forward prices of these commodities for the month m, σ_{1m}^{M} and σ_{2m}^{M} stand for standard deviations of monthly averages of daily quotations from the forward prices for the month m, σ_{1m}^{D} and σ_{2m}^{D} refer to standard deviations of the daily quotations from their monthly averages in the month m, ρ^{M} is a monthly correlation coefficient, ρ^{D} is a daily correlation coefficient and finally, $\varepsilon_{1m}, \varepsilon_{2m}, \varepsilon_{1dm}$ and ε_{2dm} are random components drawing from the standard normal $N(0,1)$ distribution.

The equations (5.2) can be rewritten so one gets the following explicit solutions for the log-normally distributed random variables S_{1dm} and S_{2dm}:

$$S_{1dm} = F_{1m}\exp\left\{\sigma_{1m}^{M}\left(\rho^{M}\varepsilon_{2m} + \sqrt{1-\left(\rho^{M}\right)^{2}}\varepsilon_{1m}\right) - \frac{\left(\sigma_{1m}^{M}\right)^{2}}{2} + \sigma_{1m}^{D}\left(\rho^{D}\varepsilon_{2dm} + \sqrt{1-\left(\rho^{D}\right)^{2}}\varepsilon_{1dm}\right) - \frac{\left(\sigma_{1m}^{D}\right)^{2}}{2}\right\}, \quad (5.3)$$

$$S_{2dm} = F_{2m}\exp\left\{\sigma_{2m}^{M}\varepsilon_{2m} - \frac{\left(\sigma_{2m}^{M}\right)^{2}}{2} + \sigma_{2m}^{D}\varepsilon_{2dm} - \frac{\left(\sigma_{2m}^{D}\right)^{2}}{2}\right\}.$$

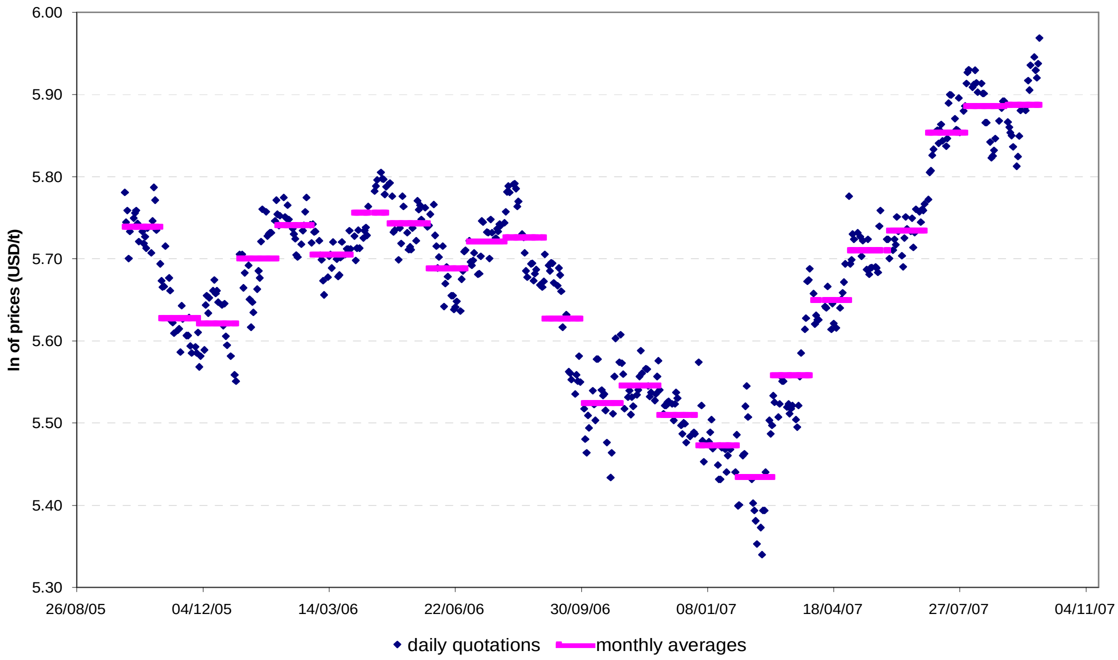
5.2.2. Parameters estimation

For an estimation of the parameters of the equations (5.2) we use the in-sample data, specifically, spot and forward prices of commodities 1% fuel oil and natural gas observed in gas years 2005/06 and 2006/07.

5.2.2.1. Estimation of σ_{1m}^{D} and σ_{2m}^{D}

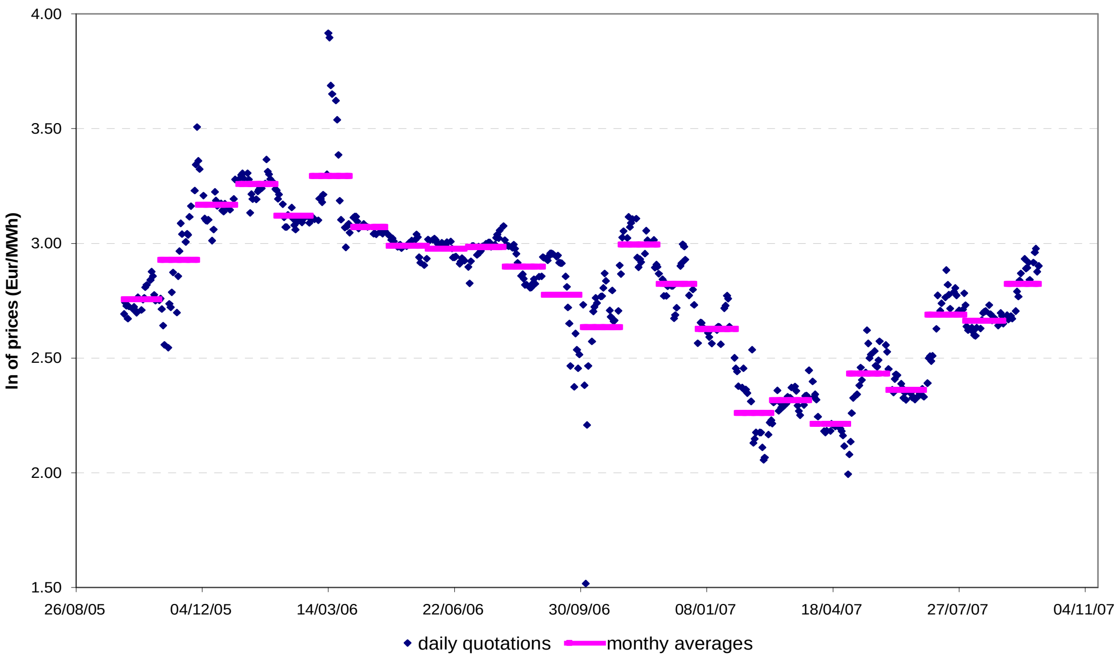
Parameters σ_{1m}^{D} and σ_{2m}^{D} are estimated using the deviations of natural logarithms of daily spot price quotations from their monthly averages as shown on graphs 6 and 7.

Graph 6: Historical spot prices of 1% fuel oil with delivery in ARA (3/10/2005-28/09/2007)



Source: Author

Graph 7: Historical day-ahead prices of natural gas with delivery at TTF (3/10/2005-28/09/2007)



Source: Author

More precisely, for each month of the in-sample period and for both commodities, we calculate a standard deviation using the formula

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}\left(\ln S_i - \ln \overline{S}\right)^2}{n-1}}\,, \tag{5.4}$$

where $\ln S_i$ denotes a natural logarithm of the daily spot price quotation for day *i*, $\ln \overline{S}$ stands for an arithmetic average of the values $\ln S_i$, for *i*=1,2,…,*n*, and *n* refers to the number of days within the relevant month.

Table 5: Standard deviations of daily spot price quotations from monthly averages

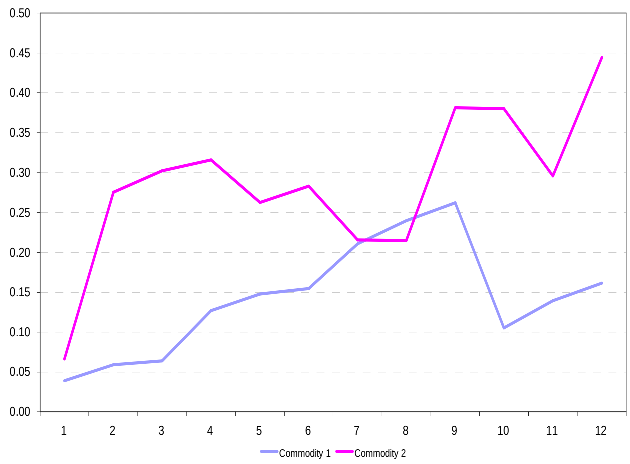
m	Commodity 1		Commodity 2	
	GY 2005/06	GY 2006/07	GY 2005/06	GY 2006/07
10	0.025	0.186	0.056	0.704
11	0.119	0.159	0.310	0.282
12	0.127	0.197	0.431	0.458
1	0.044	0.034	0.052	0.081
2	0.047	0.072	0.150	0.401
3	0.022	0.106	0.283	0.321
4	0.068	0.186	0.195	0.436
5	0.051	0.245	0.279	0.247
6	0.040	0.269	0.291	0.275
7	0.031	0.391	0.287	0.144
8	0.055	0.424	0.376	0.053
9	0.097	0.428	0.535	0.228

Source: Author

As we have two sets of results (for GY 2005/06 and GY 2006/07) we calculate their average values for each calendar month *m* = 1,2,…,12 to get the estimates of σ_{1m}^{D} and σ_{2m}^{D}.

Table 6: Estimates of σ_{1m}^{D} and σ_{2m}^{D}

m	$\hat{\sigma}_{1m}^{D}$	$\hat{\sigma}_{2m}^{D}$
1	0.039	0.066
2	0.059	0.275
3	0.064	0.302
4	0.127	0.316
5	0.148	0.263
6	0.155	0.283
7	0.211	0.216
8	0.240	0.215
9	0.262	0.381
10	0.105	0.380
11	0.139	0.296
12	0.162	0.444



Source: Author

One can see that prices of commodity 2 were more volatile than prices of commodity 1. Next, the prices of commodity 1 were more volatile in the summer months (m = 4,5,..,9) than in the winter months (m = 1,2,3,10,11,12). In the case of commodity 2, there is no such straightforward feature. We can observe only larger deviations in months representing the start and end of the summer and winter seasons (m = 3,4,9,10). However, it is necessary to emphasize that our estimates result just from two sets of observations (GY 2005/06 and GY 2006/07) which makes a deeper analysis impossible and casts a cloud over the quality of our results. More data is not available as the history of the spot (day-ahead) gas market is too young.

5.2.2.2. Estimation of σ_{1m}^{M} and σ_{2m}^{M}

Not only deviations of daily spot price quotations from their monthly averages are subjects to uncertainty. Also the forward prices represent only expectations of the future monthly averages. In other words, the real average monthly spot prices of commodities 1 and 2 can finally differ from current market's expectations given by the forward curves, namely due to random events. That is why we add noise or variability to the paths represented by the forward curves. The amount of this noise or variability is equal to σ_{im}^{M} times a random ε_{im}, for $i = 1, 2$, where ε_{im} has a standard deviation of 1.0.

We assume that σ_{im}^{M} changes every month. One can use historical data to discover to what extent market players are able to predict the future average monthly prices of commodities 1 and 2. Specifically, we examine how much twelve consecutive real average monthly prices of commodities 1 and 2 differ from forward prices for the respective months observed three months ahead of the beginning of the first of the twelve consecutive months.

For both commodities we take thirteen forward curves, including the curve as of 1 July 2005 and the curves as of the first days of the twelve consecutive months[1], and compare them with real prices. Or more precisely, we are interested in the differences of their natural logarithms. The differences are then grouped together based on how many months forward a month, for which the difference is calculated, is. For all twelve groups (from 4-month forward till 15-month forward group) we compute a standard deviation. Doing so, we get the estimates of σ_{1m}^{M} and σ_{2m}^{M} (see table 7).

Table 7: Estimates of σ_{1m}^{M} and σ_{2m}^{M}

m	$\hat{\sigma}_{1m}^{M}$	$\hat{\sigma}_{2m}^{M}$
1 (4-month fwd)	**0.141**	**0.065**
2 (5-month fwd)	**0.161**	**0.140**
3 (6-month fwd)	**0.195**	**0.218**
4 (7-month fwd)	**0.230**	**0.273**
5 (8-month fwd)	**0.263**	**0.401**
6 (9-month fwd)	**0.281**	**0.575**
7 (10-month fwd)	**0.290**	**0.652**
8 (11-month fwd)	**0.282**	**0.670**
9 (12-month fwd)	**0.280**	**0.679**
10 (13-month fwd)	**0.273**	**0.731**
11 (14-month fwd)	**0.263**	**0.671**
12 (15-month fwd)	**0.243**	**0.642**

Source: Author

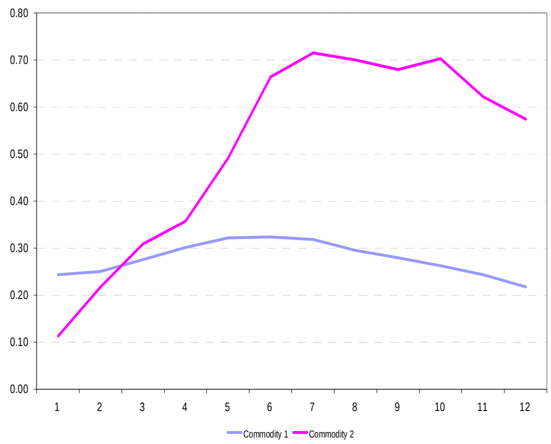
[1] The reason why we do not use older data for our estimation than those from the 1st of July 2005 is a short history of the gas forward market.

The annualized standard deviations are then given as

$$\hat{\sigma}_{im}^{AM} = \frac{\hat{\sigma}_{im}^{M}}{\sqrt{\frac{m+3}{12}}}, \text{ for } i = 1, 2. \tag{5.5}$$

Table 8: Estimates of σ_{1m}^{AM} and σ_{2m}^{AM}

m	$\hat{\sigma}_{1m}^{AM}$	$\hat{\sigma}_{2m}^{AM}$
1 (4-month fwd)	**0.243**	**0.113**
2 (5-month fwd)	**0.250**	**0.217**
3 (6-month fwd)	**0.275**	**0.309**
4 (7-month fwd)	**0.302**	**0.357**
5 (8-month fwd)	**0.322**	**0.491**
6 (9-month fwd)	**0.324**	**0.664**
7 (10-month fwd)	**0.318**	**0.715**
8 (11-month fwd)	**0.295**	**0.700**
9 (12-month fwd)	**0.280**	**0.679**
10 (13-month fwd)	**0.263**	**0.702**
11 (14-month fwd)	**0.244**	**0.621**
12 (15-month fwd)	**0.218**	**0.574**



Source: Author

One can see that the annualized deviation increases with forwardness until month 7 in the case of commodity 2 and until month 6 in the case of commodity 1. It can be explained by mean-reversion forces observable in the commodities markets. As noted in the subchapter 4.1.3., the variance (i.e. the square power of a standard deviation) of a variable following a mean-reverting process does not grow proportionally to the time interval as in case of a Brownian motion. It grows at the beginning and after some time it stabilizes on a certain value.

5.2.2.3. Estimation of ρ^D and ρ^M

We still need to estimate the cross-commodities correlations ρ^M and ρ^D. We assume that the random variables $\ln S_{1dm}$ and $\ln S_{2dm}$ are correlated through the random components of the equations (5.2). Specifically, ρ^D stands for the correlation between deviations of natural logarithms of the daily spot price quotations of commodity 1 from their averages and the deviations of natural logarithms of the daily spot price quotations of commodity 2 from their averages. Applying the formula (5.6) to the observed data we get the estimate of the correlation ρ^D (see table 9).

$$\hat{\rho}^D = \frac{1}{m-1}\sum_{i=1}^{m}\left(\frac{\ln S_{1i} - \ln \bar{S}_1}{\hat{\sigma}_1^D}\right)\left(\frac{\ln S_{2i} - \ln \bar{S}_2}{\hat{\sigma}_2^D}\right), \tag{5.6}$$

where $\ln S_{1i}$ and $\ln S_{2i}$ denote natural logarithms of the daily spot price quotations of commodities 1 and 2 for day *i*, respectively, $\ln \bar{S}_1$ and $\ln \bar{S}_2$ stand for arithmetic averages of the values $\ln S_{1i}$ and $\ln S_{2i}$, for *i*=1,2,…,*m*, respectively, and *m* denotes the number of working days within the in-sample period.

The parameter ρ^M then stands for a correlation between deviations of natural logarithms of average monthly spot prices from natural logarithms of forward prices (as specified in 5.2.2.2.) of commodity 1 and in the same way derived deviations of commodity 2. See table 9 for the estimate of the correlation ρ^M .

Table 9: Estimates of ρ^D and ρ^M

$\hat{\rho}^D$	$\hat{\rho}^M$
0.136262	**0.615764**

Source: Author

The value of $\hat{\rho}^D$ is low compared to the value of $\hat{\rho}^M$. It supports the logic that it is more likely that new market information is captured simultaneously by both commodities markets in case of forward price adjustments done once a month, than in the case of spot price adjustments done once a day.

5.3. Intrinsic value

Using the forward prices shown on graphs 3 and 4 we can calculate the intrinsic value of the swing option as an additional value generated by the downward quantity tolerance provision and by the three actions described hereinbefore (forward purchase +

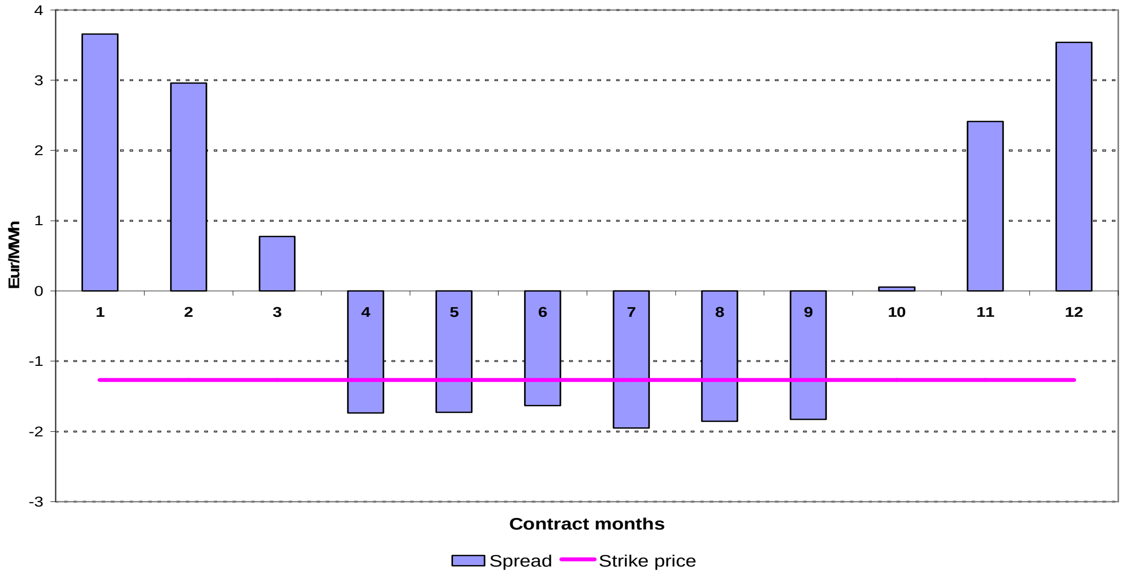
hedging + execution). As noted previously, one can see the intrinsic value as a discounted profit generated by a forward sale of the option volume back to the seller for the hedged forward contractual price, together with a forward purchase of this volume in the market for the lower forward market price. In case such actions are not profitable, i.e. if the market price is higher than the contractual price, there is no ex-ante value (or zero intrinsic value) of the DQT provision. On the date of contract signing, the measured unit intrinsic value of an option exercised in the month *m* of the contract year is then given by

$$IV_m = e^{-r(m+3)/12}\left[q - \left(F_{2m} - kF_{1m}\right)\right]^+, \quad (5.7)$$

where F_{1m} and F_{2m} denote the forward prices of commodities 1 and 2 for the delivery month *m* observed on the signing date, respectively, and the constants take values: $q = P_0 - kLSFO_0 = -1.2666$ and $k = 0.072117$.

To get the values of IV_m for $m = 1,2,\ldots,12$, we need to compare the spreads $F_{2m} - kF_{1m}$ (let us call them „forward spreads") with the strike price *q* (graph 8). IV_m is then positive when the spread of the month *m* is less than the strike price and equal to zero when the spread is greater than the strike price. As graph 8 shows, IV_m is positive for months 4, 5, 6, 7, 8 and 9, and equal to zero for months 1, 2, 3, 10, 11 and 12.

Graph 8: Forward spreads vs. strike price (Oct 1, 2007)



Source: Author

However, the buyer is not allowed to extract the whole sum of IV_m, where $m = 1,2,\ldots,12$. He is limited by the Take-or-Pay provision. Specifically, the whole amount of "exercised" gas cannot exceed the quantity specified by the DQT provision. This quantity determines how much gas the buyer is allowed to sell back to the seller. Since the buyer is supposed to be a rational subject maximizing his profits, we assume that he wants to sell back contractual gas in months with the least spreads. This means he does an optimization with respect to his limitation to maximize his ex-ante profit. The results of such optimization are summarized in table 10. Based on the forward prices as of October 1, 2007, it shows an optimized offtake under our GSA and an offtake under a contract without a DQT provision and with other parameters the same. Comparing mark-to-market values of these two contracts, we get the future ex-ante value of the DQT provision. It is equal to 8,464 Euro.

Table 10: Optimization results based on forward prices as of Oct 1, 2007

Month	Month index	MtM spreads €/MWh	Offtake (w/o DQT) MWh	MtM value (w/o DQT) €	Offtake (with DQT) MWh	MtM value (with DQT) €	Difference of MtM values
I.08	1	4.924	7,440	36,631	7,440	36,631	0
II.08	2	4.227	6,960	29,421	6,960	29,421	0
III.08	3	2.043	7,440	15,198	7,440	15,198	0
IV.08	4	-0.468	7,200	-3,370	7,200	-3,370	0
V.08	5	-0.458	7,440	-3,409	7,440	-3,409	0
VI.08	6	-0.363	7,200	-2,615	7,200	-2,615	0
VII.08	7	-0.683	7,440	-5,084	0	0	5,084
VIII.08	8	-0.589	7,440	-4,384	1,704	-1,004	3,380
IX.08	9	-0.559	7,200	-4,026	7,200	-4,026	0
X.08	10	1.320	7,440	9,823	7,440	9,823	0
XI.08	11	3.677	7,200	26,477	7,200	26,477	0
XII.08	12	4.804	7,440	35,744	7,440	35,744	0
Sum			87,840	130,404	74,664	138,869	8,464

Source: Author

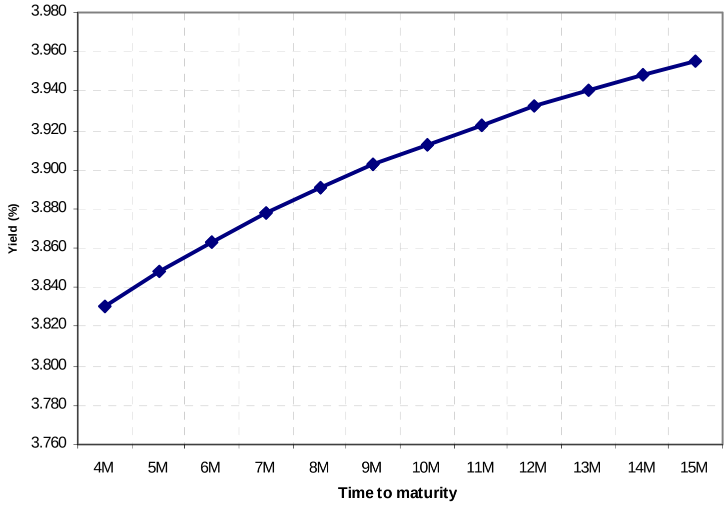
The only thing left to obtain the intrinsic value IV of the swing option is to discount the ex-ante cash flows generated by the DQT provision (as given in the column "Difference of MtM values" in table 10) at risk-free interest rates.

$$IV = \sum_{m=1}^{12} e^{-r_{m+3}(m+3)/12} CF_m , \tag{5.8}$$

where r_{m+3} denotes a risk-free interest rate for time to maturity of $m+3$ months and CF_m represents the ex-ante cash flow generated in the month m. Risk-free interest rates for maturities from four to fifteen months can be approximated by a yield curve for the Euro area as published by the European Central Bank.[1] It is constructed using AAA-rated Euro area central government bonds and its values for 1 October 2007 are summarized in table 11.

Table 11: Euro area yield curve as of Oct 1, 2007

Maturity	Yield (%)
4M	3.830
5M	3.848
6M	3.863
7M	3.878
8M	3.891
9M	3.903
10M	3.913
11M	3.923
12M	3.932
13M	3.940
14M	3.948
15M	3.955



Source: European central bank (ECB)

We conclude this subchapter with information on the resulting intrinsic value of the swing option embedded in the GSA under evaluation. It is equal to 8,182 Euro.

5.4. Extrinsic value

As until the delivery prices can move so they ensure the buyer some additional gains above the intrinsic value, or simultaneously, extra-expenditures to the seller, the seller charges more than the ex-ante value. Favorable (from the buyer's perspective) price movements are then those that lead to smaller spreads. To be able to evaluate such time value of the DQT provision the seller needs to form his expectations about future spot prices. He can do this by simulating the price dynamics of the underlying

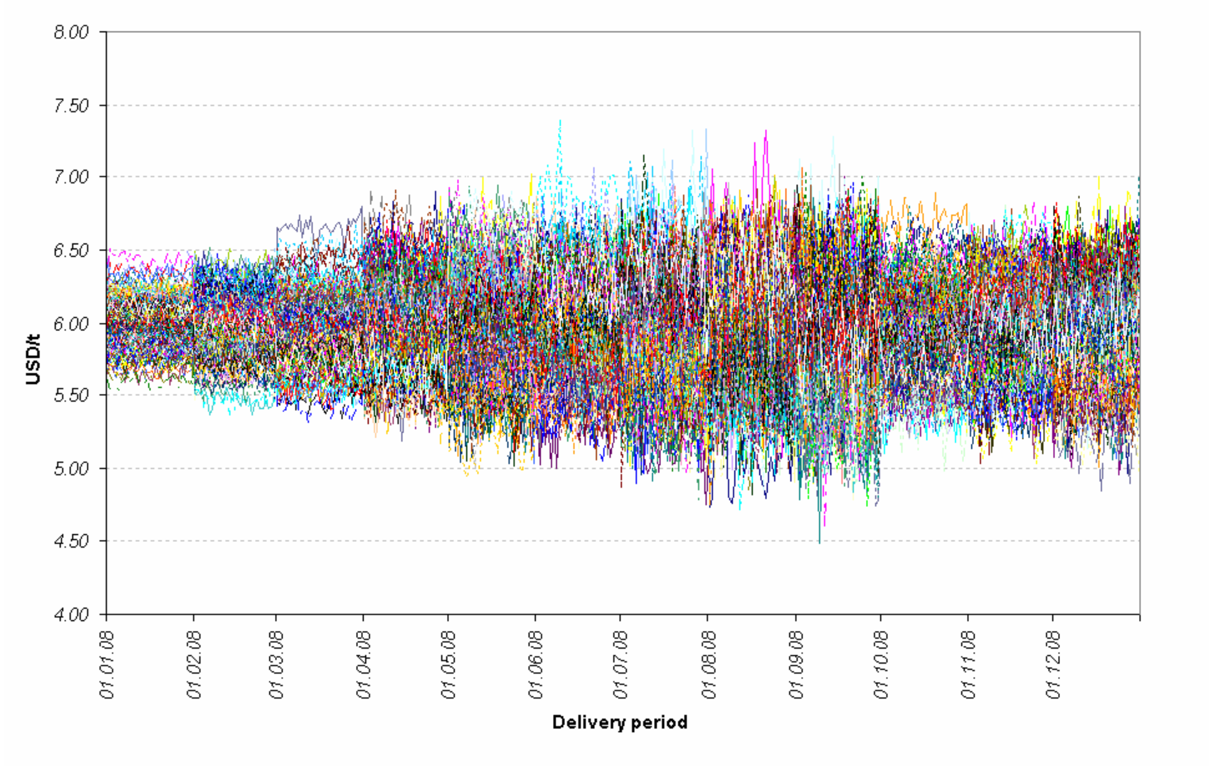
[1] http://www.ecb.int/stats/money/yc/html/index.en.html

commodities as given by equations (5.2). The Monte Carlo methods then seem to be the right means.[1]

5.4.1. Monte Carlo simulation

We generate n random paths (n = 254) of the underlying process lnS_1, and the same number of paths of the underlying process lnS_2, over the interval [0, T] and from the standardized normal distribution N(0,1) (see equations (5.2)). The start point (time 0) is the signing date of the contract (Oct 1, 2007) and the end point (time T) corresponds to the day the contract expires (Dec 31, 2008). See graphs 9 and 10 for the random paths over the delivery period [t_0, T], where t_0 stands for its first day (January 1, 2008).

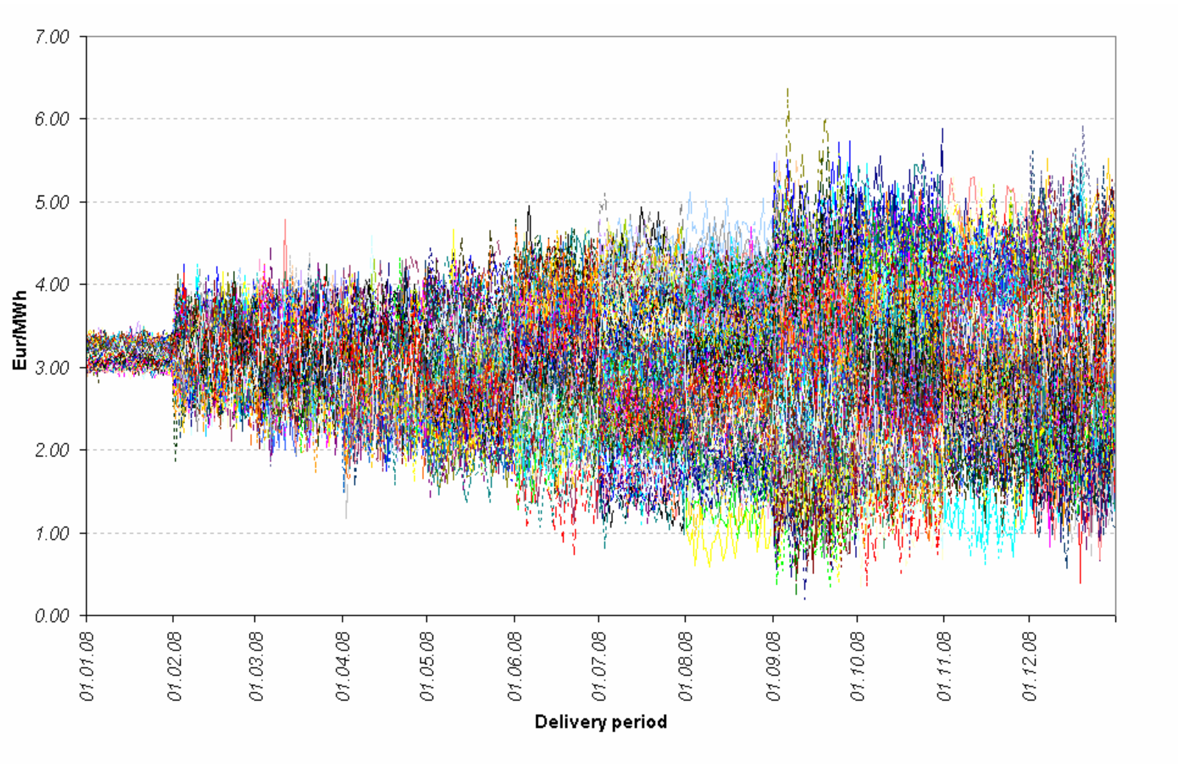
Graph 9: Random paths of process lnS_1



Source: Author

[1] We run the MTS in the Microsoft Office Excel. However, it can be done in any other software that is able to generate pseudo-random numbers.

Graph 10: Random paths of process lnS_2



Source: Author

Now we have a set of 254 values of lnS_{1t} and a set of 254 values of lnS_{2t} for each day t of the period [0, T]. We modify them to the values of variables S_{1t} and S_{2t} using the exponential function. These values are then filled in equation (4.17) to get the estimates $\hat{E}_t\{[q-(bS_{2t}-aS_{1t})]^+\}$, or $\hat{E}_t\{[q-(S_t-kLSFO_t)]^+\}$ since $a=k=0.072117$, $b=1$ and commodities 1 and 2 are fuel oil and natural gas, respectively, for each day $t\in[t_0,T]$.

See appendix B for the results.

5.4.2. Exercise days

The number of options the buyer owns is given by the equation (3.3). In such case, the contractual volumes take the values as defined in subchapter 5.1, the buyer owns fifty-five American put options with fifty-five various expiration dates. Since they are of an American style, they can be exercised by the buyer any time up to their expiration dates. It is thus necessary to disclose the days on which the options are supposed to be exercised.

There are some American style spread options for which holds that it is never optimal to exercise them early. Specifically, American calls on non-dividend paying stocks and calls or puts on forward contracts are two examples (see James, 2003). Prices of these options are equal to the prices of the corresponding European calls or puts. However, it is not the case of spread options on commodities due to convenience yields adjusting the cost of carry model for forward prices of these commodities. More precisely, when the convenience yields of underlying commodities are unequal, the price of an American style spread option before its expiration is always higher than the price of the corresponding European spread option. However, the energy markets are a special case regarding this due to the absence of non-operating inventories as mentioned previously. In addition, there is no liquid market for swing options of the form examined in this thesis. Despite these facts, one is able to conclude that it can be profitable to exercise some of the options the buyer owns early.

We assume that the buyer's choice is a result of a dynamic optimization. Specifically, on each day of the delivery period the buyer is supposed to make a decision on the source of his purchase based on a spot value of the spread and his expectation of the future spot spreads. More precisely, every time the spread is greater than the strike price the buyer is not supposed to exercise any of the options. In case the spread is less than the strike price, it is profitable to exercise one of the options. However, we assume that the buyer wants to maximize his total payoff from the swing option. This means that every time he expects there is going to be next fifty-five days with a greater payoff than the one present on the day of his decision making, he is not supposed to exercise any of the options. Regarding his expectation we assume that he relies on estimates shaped by the market, i.e. forward prices.

Following these optimizing criteria with respect to the present forward prices means to choose the fifty-five days of the contract year that offer the greatest estimated payoffs. For results see appendix B (red marked values). Discounting these unit payoffs $\hat{E}_i$ ($i = 1,2,\ldots,55$) at the risk-free interest rates r_{m+3} (see subchapter 5.3. for their values), summing them and multiplying them by the DCQ, we get a resulting value of the swing option premium. It is equal to 61,974 Euro. Since the premium is the sum of the intrinsic and extrinsic value and the intrinsic value equals 8,182 Euro, the extrinsic value is estimated at 53,792 Euro on the signing day of the contract (October 1, 2007).

5.4.3. Confidence interval

Instead of estimating the swing option premium by a single value one can prefer interval estimation for its reliability indication. We can derive the confidence interval for the premium using confidence limits on the values of spreads $q-(S_i - kLSFO_i)$, for i = 1,2,…,55. Denoting $q-(S_i - kLSFO_i)$ by X_i, for i = 1,2,…,55, the estimate of the standard deviation of X_i is given by $\hat{s}_i$ where

$$\hat{s}_i^2 = \hat{E}\{X_i^2\} - \left(\hat{E}\{X_i\}\right)^2 . \qquad (5.9)$$

The distribution of

$$\frac{\hat{E}\{X_i\} - E\{X_i\}}{\hat{s}_i}$$

then tends to a standardized normal distribution with increasing *n*, i.e. amount of random paths of the processes lnS_{1t} and lnS_{2t} (see subchapter 5.4.1.). For n = 254 considered in this thesis, the distribution can be regarded as normal and confidence limits on $E\{X_i\}$ can be obtained on this basis. Specifically, the confidence interval for $E\{X_i\}$ is given by

$$\hat{E}\{X_i\} - u_{1-\alpha/2}\hat{s}_i < E\{X_i\} < \hat{E}\{X_i\} + u_{1-\alpha/2}\hat{s}_i \text{, for i = 1,2,…,55,} \qquad (5.10)$$

where $u_{1-\alpha/2}$ stands for a quantile of the standardized normal distribution N(0,1) and α denotes a confidence level. We construct such an interval for each chosen exercise day i. The resulting intervals can then be interpreted in the following way: with (1-α)% probability $E\{X_i\}$, where i = 1,2,…,55, will lie in these intervals. Since $E\{X_i\}$ stands for $E\{q-(S_i - kLSFO_i)\}$ we can say that the value of spread between the contractual price and market price of gas on day *i* will, with (1-α)% probability, lie in the derived interval for $E\{X_i\}$. However, these intervals still need to be modified as we are interested in the option payoffs, not spreads taking both positive and negative values. Specifically, in case the value of spread $q-(S-kLSFO)$ is negative, the option payoff is equal to zero. In our case, it means that we only need to replace the negative lower confidence limits with zero values.

Now it is not a problem to construct the interval estimate of the swing option premium. Confidence limits (lower and upper) of such estimate are given as sums of the discounted confidence limits on $E\{X_i\}$, where $i = 1,2,\ldots,55$, multiplied by the DCQ. See table 12 for the results under various confidence levels *α*.

Table 12: Interval estimates of the swing option premium

α	Lower limit Euro	Upper limit Euro
0.2	0	289,698
0.1	0	387,591
0.05	0	472,073
0.01	0	637,284

Source: Author

5.5. Comparison with the ex-post value

Using the real values of spot prices of the underlying commodities observed in the calendar year 2008, we can calculate the maximum profit the buyer could realize by exercising his right stemming from the DQT provision. In appendix C one can see the payoff values for all days of the contract year. As we are interested in the maximum ex-post swing option payoff we choose the fifty-five largest ones of them (red marked up). Discounting these payoffs at the risk-free interest rates r_{m+3}, and summing them, we get a value of 129,186 Euro.

When we compare the theoretical value of the swing option premium on the date October 1, 2007 (61,974 Euro) with its ex-post value (129,186 Euro), we can conclude that the theoretical premium is undervalued. This can be explained by the specific nature of the year 2008 given by the global financial crisis. The crisis has induced more volatile prices of energy commodities. Specifically, prices of such fuels as gas oil, fuel oil, Brent crude oil or natural gas rose steeply in the first half of the year 2008, and subsequently started falling even steeper. However, the contractual price of gas given by the formula described in subchapter 5.1. has followed such a trend with a certain delay. This is due to the fact that prices of competing fuels entering into the formula have been defined by GSA as lagged. It means that there were months when the market price of gas was falling and at the same time the contractual price was steeply growing, which resulted in unanticipated values of spreads between contractual and market prices.

Comparing the ex-post value of the swing option with the interval estimates summarized in table 12, one can see that the value falls into all of them.

Now, it is important to emphasize that the ex-post value of the option represents the maximum realizable profit/loss of the buyer/seller from the option purchase/sale in case the buyer/seller does not hedge against his price risk. However, if we assume that either the buyer or the seller are not speculators they are supposed to hedge against their price risks. Then if the implicit forecast of future price volatilities of the underlying commodities turns out to be correct, the option premium should equal the cost of delta hedging the position. It means that by the dynamic delta hedging the buyer/seller hedges against his price risk and takes volatility risk. Since our pricing model is based on parameters derived from historical data of the periods unaffected by any crisis it does not allow for such price volatilities as observed at time of the global financial crisis.

6. Conclusion

In this thesis we tried to find a fair value price of an option (gas swing option) embedded in an arbitrarily chosen gas sales agreement. A gas swing option is a set of several spread options on energy commodities. More precisely, it consists of several American put options on a spread between the market price of gas and the market prices of two or more competing fuels. Pricing of the gas swing option thus represents a multifactor valuation problem. For the sake of simplicity, but without loss of generality, we focused on a two-factor option pricing, i.e. the case when the puts are written on natural gas and only one competing fuel.

Before the model derivation we went through the theory on spread option pricing. We recognized that the existing analytic approximations for spread option pricing are hardly applicable to spread options written on energy commodities. This is due to the specific nature of the energy markets. They are fundamentally different compared to the traditional financial security markets. We thus employed the widely used numerical method, the Monte Carlo method, instead of an analytic approximation. In addition, the future spot prices of the underlying commodities were modeled through two stochastic processes, chosen as to capture these specific features and at the same time to be consistent with the observed forward prices.

In this thesis, two data sets were examined: spot and forward prices of the underlying commodities observed in the gas years 2005/06 and 2006/07 (in-sample data) and spot prices of these commodities for the calendar year 2008 (out-of-sample data). We used the in-sample data for the estimation of the model parameters, such as volatilities and correlations of the underlying prices. The out-of-sample data was then used at the close of this thesis for the model testing.

Our model was based on the concept of risk-neutral expectations. More precisely, an option price was considered as an expectation of discounted future cash flows for a probability structure called risk-neutral. Using the Monte Carlo method, we simulated values of the future cash flows for each day of the delivery period (calendar year 2008). After calculating the means of such values it was still necessary to decide on the exercise days since the options were of an American style. We concluded that it was profitable for the buyer to exercise some of the options early. In accordance with the concept of risk-neutral expectations, we finally estimated the fair value price of the gas swing option embedded in the arbitrarily chosen gas sales agreement. In addition to the single value estimate of the premium, we calculated an interval estimate (confidence interval) for various confidence levels.

Finally, the estimates were compared with the ex-post value of the option computed for the out-of-sample data. The single value estimate of the premium was undervalued in comparison to the ex-post option value. This has been explained by extremely high price volatilities of energy commodities observed during the global financial crisis in 2008. Since our pricing model has been based on parameters derived from historical data of periods unaffected by any crisis, the model has not allowed for such high price volatilities. Despite this fact, the ex-post option value has fallen into the computed confidence interval.

This thesis has tried to contribute to the field where the evaluation methods have not yet been fully developed and, at the same time, their development has been highly requested. Specifically, the changing nature of energy markets stemming from the process of their liberalization, has asked for new evaluation approaches. Even if this thesis has brought some useful contributions it needs to be further extended, mainly by the means of dynamic delta hedging.

References

Alaton P., Djehiche B. and Stillberger D. (2002): *On modelling and pricing weather derivatives*, Appl. Math. Finance, 9(1), 1-20

Alexander C. and Venkatramanan A. (2007): *Analytic Approximations for Spread Options*, ICMA Centre Discussion Papers in Finance DP2007-11, 15th August 2007

Asche F., Osmundsen P. and Tveteras R. (2002a): *European market integration for gas? Volume flexibility and political risk*, Energy Economics 24, pp. 249-265

Asche F., Osmundsen P. and Tveteras R. (2002b): *Tax shifting in long-term gas sales contracts*, Working paper no. 3/02, SNF-project no. 7220, Institute for research in economics and business administration, Bergen, January 2002

Boyle P. P. (1977): *Options: A Monte Carlo Approach*, Journal of Financial Economics 4, pp. 323-38

Boyle P., Broadie M. and Glasserman P. (1997): *Monte Carlo Methods for Security Pricing*, Journal of Economic Dynamics and Control, Vol. 21, Nos. 8-9, pp. 1267-1322

Brennan M. and Schwartz E. (1985): *Evaluating Natural Resource Investments*, Journal of Business, 58, No. 2, pp. 135-157

Broadie M. and Glasserman P. (1997): *Monte Carlo Methods for Pricing High-Dimensional American Options: An Overview*, Net Exposure, Issue 3, pp. 15-37

Carmona R. and Durrleman V. (2003): *Pricing and Hedging Spread Options*, SIAM Review, Vol. 45, No. 4, pp. 627–685

Creti A. and Villeneuve B. (2003): *Long-term Contracts and take-or-pay clauses in natural gas markets*, Energy Studies Review, 13, 75-94

Cronshaw I. and col. (2008): Development of competitive gas trading in continental Europe, How to achieve workable competition in European gas markets?, IEA Information paper, International Energy Agency, May 2008

Dornier F. and Queruel M. (2000): *Caution to the wind*, EPRM, 30, August 2000

Gaylord G.L. (1989): *Assigning responsibility for long-term gas supply: The gas inventory charge*, Public Utilities Fortnightly, 124(4), pp. 9-14

Haase N. (2008): *European gas market liberalisation: Are regulatory regimes moving towards convergence?*, Oxford Institute for Energy Studies, NG 24, May 2008 (ISBN 978-1-901795-73-8)

Hubbard D. (2007): *How to Measure Anything: Finding the Value of Intangibles in Business*, John Wiley & Sons, pp. 46

Hull J. and White A. (1987): *The pricing of options on assets with stochastic volatilities*, Journal of Finance 42, pp. 281-300

Hull J. (2006): Options, Futures and other Derivatives, Sixth Edition, Prentice Hall, New Jersey

Hurt J., Dupacova J. and Stepan J. (2003): *Stochastic Modeling in Economics and Finance*, New York: Kluwer Academic Publishers, pp. 386, 1st ed.

James P. (2003): Option Theory, Wiley New York

Johnson H. and Shanno D. (1987): *Option pricing when the variance is changing*, Journal of Financial and Quantitative Analysis 22, pp. 143-151

Kirk E. (1995): *Correlation in the energy markets, in managing energy price risk*, Risk Publications and Enron

Klein B., Crawford R. and Alchian A. (1978): *Vertical Integration, Appropriable Rents, and the Competitive Contracting Process*, Journal of Law and Economics, 21(2), 297-326

Margrabe W. (1978): *The Value of an Option to Exchange One Asset for Another*, The Journal of Finance, Vol. 33, pp. 177–186

Masten S. E. and Crocker K. (1985): *Efficient Adaptation in Long-Term Contracts: Take-or-Pay Provisions for Natural Gas*, American Economic Review, December 1985, 1083-93

Meinshausen N. and Hambly B.M. (2004): *Monte Carlo methods for the valuation of multiple exercise options*, Math. Finance 14, 557–583

Merton R. C. (1973): *The theory of rational option pricing*, Bell J. Econ. Manag. Sci., 4, pp. 141–183

Miltersen K. R. (2003): *Commodity price modeling that matches current observables: A new approach*, Quant. Finance, 3, pp. 51–58

Neumann A. and C. von Hirschhausen (2005): *Long-Term Contracts for Natural Gas - An Empirical Analysis*, Globalization of Natural Gas Markets Working Papers, WP-GG-13, Paper presented at the 9th ISNIE Conference, Barcelona (Spain), September 2005

Poitras G. (1998): *Spread options, exchange options and arithmetic Brownian motion*, J. Futures Markets, 18, pp. 487–517

Santa-Clara P. and Sornette D. (2001): *The dynamics of the forward interest rate curve with stochastic string shocks*, Review of Financial Studies 14, 149-85

Schaefer M. P. (2002): *Pricing and Hedging European Options On Futures Spreads Using the Bachelier Spread Option Model*, Presented at the NCR-134 Conference on Applied Commodity Price Analysis, Forecasting, and Market Risk Management St. Louis, Missouri, April 22-23, 2002

Schultz C. R. (1997): *Modeling Take-or-Pay Contract Decisions*, Decision Sciences, Volume 28 Number I

Schwartz E. S. (1997): *The stochastic behavior of commodity prices: Implications for pricing and hedging*, J. Finance, 52, pp. 923–973

Shimko D. (1994): *Options on future spreads*, Journal of Futures Markets 14, No. 2, 183-213

Smith C. (1976): *Option Pricing: A Review*, Journal of Financial Economics 3, pp. 3–51

Sobczyk K. (1991): *Stochastic differential equations*, Kluwer Academic Publishers, Dordrecht

Svec J. and Stevenson M. (2007): *Modelling and forecasting temperature based weather derivatives*, Global Finance Journal 18, 185–204

Uhlenbeck G.E. and Ornstein L.S. (1930): *On the Theory of Brownian Motion*, Physical Review vol. 36, September 1930

Wilcox D. (1990): *Energy futures and options: Spread options in energy markets*, Goldman Sachs & Co., New York

EFET Gas Committee (2003): *Creating successful transparent and liquid gas trading hubs - the operational and commercial essentials*, February 2003

“*Ruhrgas Gas Release Programme 2004*”, Information Memorandum, 31 March 2004

• Appendix A

(A1) Wiener process

Wiener process is a stochastic process with the following features:

1. $W_0 = 0$
2. W_t is almost surely[1] continuous
3. W_t has independent increments with a distribution $W_t - W_s \sim \Phi(0, t-s)$, $0 \le s < t$,

where $\Phi(\mu, \sigma^2)$ denotes the normal cumulative distribution function with an expected value μ and a variance σ^2. The independent increments mean that for $0 \le s_1 \le t_1 \le s_2 \le t_2$, $W_{t_1} - W_{s_1}$ and $W_{t_2} - W_{s_2}$ are independent random variables (see e.g. Hurt et al. (2003)).

(A2) Itô's lemma

Itô's lemma is used in Itô stochastic calculus to find the differential of a function of a particular type of stochastic process. Let us assume that the random variable *x* follows a stochastic process given by a stochastic differential equation of the form

$$dS(t) = \alpha(S,t) + \beta(S,t)dW(t)$$

where $\alpha(S,t)$ denotes a drift term, $\beta(S,t)$ refers to a volatility function and $dW(t)$ is a Wiener process. A stochastic differential of the process $f(S(t),t)$ is then given as

$$df(S(t),t) = \left(\alpha(S,t)\frac{\partial f}{\partial S} + \frac{1}{2}\beta(S,t)^2\frac{\partial^2 f}{\partial S} + \frac{\partial f}{\partial t}\right)dt + \beta(S,t)\frac{\partial f}{\partial S}dW(t).$$

See e.g. Hurt et al. (2003) or Hull (2006).

(A3) Black-Scholes formula

The Black-Scholes formula for the price of a European call option on a non-dividend-paying asset is

[1] Almost surely means: Let (Ω, A, P) be a probability space. Then an event *x* in A happens almost surely if P(*x*) = 1.

$$p = S_0\Phi(d_1) - qe^{-rT}\Phi(d_2),$$

where

$$d_1 = \frac{\ln\left(S_0 e^{rt} / q\right)}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T} \qquad \text{and} \qquad d_2 = d_1 - \sigma\sqrt{T}\ .$$

Again, $\Phi(.)$ denotes the cumulative distribution function of the standard normal distribution, S_0 is the spot price of the asset at time zero, q stands for a strike price, r denotes a continuously compounded risk-free interest rate, σ is the price volatility of the asset, and T refers to time to maturity of the option.

For a derivation and proof of the formula see Black and Scholes (1973).

• Appendix B

Estimated option payoffs

Month	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8	Day 9	Day 10	Day 11	Day 12	Day 13	Day 14	Day 15
1	0.138	0.148	0.158	0.173	0.136	0.164	0.15	0.145	0.146	0.142	0.154	0.155	0.129	0.146	0.152
2	0.973	1.164	1.192	1.039	1.096	0.967	1.204	1.178	0.968	1.313	1.18	1.281	1.131	0.948	1.022
3	2.114	2.223	2.255	1.914	2.099	1.716	2.01	1.971	1.974	2.272	1.932	2.001	2.088	2.229	1.785
4	2.892	3.045	3.078	3.136	2.811	3.104	3.035	3.238	3.19	3.107	3.245	2.961	3.084	3.296	3.053
5	3.417	3.107	3.438	3.34	3.192	3.207	3.182	3.309	3.32	3.321	3.285	3.157	3.127	3.342	3.122
6	4.147	3.98	3.991	4.245	4.191	4.227	4.225	4.11	4.069	4.069	4.042	4.178	4.393	4.1	4.214
7	4.694	**5.0009**	**4.8324**	**5.027**	4.766	**4.978**	4.769	**4.8928**	**4.9761**	**4.9072**	**5.0931**	**4.9725**	**5.0923**	**4.9307**	**4.8315**
8	4.46	4.349	4.298	4.437	4.417	4.218	4.444	4.527	4.396	4.477	4.41	4.524	4.231	4.385	4.449
9	4.672	**4.850**	4.622	4.796	4.784	4.822	**5.031**	**4.874**	**4.9712**	4.667	4.800	**4.9223**	**4.996**	4.822	**4.955**
10	**5.132**	**5.0207**	**4.851**	4.763	4.773	4.752	**4.893**	4.710	4.607	**5.065**	4.527	**5.147**	**5.181**	**5.0389**	**4.9623**
11	3.237	3.155	3.218	3.399	3.106	3.127	3.247	3.069	3.324	3.243	3.01	3.187	3.345	3.283	3.259
12	3.67	3.277	3.388	3.829	2.982	3.825	3.069	3.468	4.13	3.58	3.501	3.278	3.309	3.558	3.542

Day 16	Day 17	Day 18	Day 19	Day 20	Day 21	Day 22	Day 23	Day 24	Day 25	Day 26	Day 27	Day 28	Day 29	Day 30	Day 31
0.136	0.149	0.159	0.137	0.146	0.154	0.155	0.136	0.142	0.155	0.15	0.141	0.16	0.139	0.147	0.165
1.007	0.994	1.305	1.308	1.148	1.023	1.136	1.114	1.041	0.997	1.177	0.939	1.19	1.253		
2.068	2.077	2.281	1.99	2.125	2.001	2.001	2.144	1.927	2.125	2.019	1.688	2.002	1.954	2.091	1.947
3.043	2.823	2.891	3.019	2.939	2.966	3.245	3.037	3.033	3.087	2.923	3.025	2.878	2.821	3.104	
3.15	3.165	2.98	3.121	3.383	3.034	3.315	3.173	3.289	3.237	2.936	3.178	2.886	3.375	3.1	3.055
4.015	4.047	4.21	4.086	4.148	4.114	4.283	4.131	4.169	4.183	4.368	4.199	4.096	4.292	4.115	
4.9133	**4.9217**	4.804	4.819	**4.8804**	**4.9495**	4.753	**4.8589**	4.775	4.736	**4.9198**	**5.003**	**4.887**	**4.9042**	**5.0489**	**4.9433**
4.445	4.432	4.105	4.356	4.544	4.377	4.454	4.455	4.604	4.383	4.42	4.579	4.39	4.351	4.193	**4.498**
4.608	4.809	**5.102**	4.578	4.667	4.683	4.815	**4.980**	4.730	**4.979**	4.694	**4.973**	4.68	4.754	4.487	
5.329	4.73	**5.204**	**4.942**	**4.9806**	4.349	**4.992**	**5.057**	**4.9456**	**5.013**	**5.031**	**4.874**	**5.2249**	**5.031**	4.416	**4.690**
2.999	3.314	3.255	3.137	3.422	3.183	2.798	3.178	2.995	3.139	3.231	3.183	3.086	3.124	3.155	
3.421	3.486	3.407	3.539	3.386	3.574	3.244	3.451	3.458	3.788	3.359	3.474	3.666	3.319	3.469	3.695

• Appendix C

Ex-post option payoffs

Month	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8	Day 9	Day 10	Day 11	Day 12	Day 13	Day 14	Day 15
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0.375	0.327	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	**7.7005**	**9.432**	**9.432**	**9.432**	**10.191**	**10.418**	**10.162**	**9.942**	6.777	6.777	6.777	4.451	4.966	6.229	**7.074**
9	5.524	4.331	2.988	3.079	3.737	3.757	3.757	3.757	3.413	3.031	2.677	2.721	2.617	2.617	2.617
10	4.826	**6.8395**	**7.717**	**7.987**	**7.987**	**7.987**	**8.620**	**8.885**	**9.418**	**11.083**	**12.068**	**12.068**	**12.068**	**13.739**	**9.244**
11	3.243	3.243	3.243	3.182	3.944	5.015	6.095	**7.050**	**7.050**	**7.050**	**9.701**	**13.780**	**13.419**	**14.855**	**14.202**
12	0.496	1.108	3.035	3.215	3.278	3.418	3.418	3.418	3.296	3.264	2.743	1.793	1.633	1.633	1.633

Day 16	Day 17	Day 18	Day 19	Day 20	Day 21	Day 22	Day 23	Day 24	Day 25	Day 26	Day 27	Day 28	Day 29	Day 30	Day 31
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0		
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	0	0	0.206	0.206	0.206	3.133	6.016	**7.493**
6.839	6.839	6.839	6.58	6.273	5.182	4.318	3.521	3.521	3.521	4.51	4.51	2.839	2.476	2.811	2.811
2.35	1.614	1.019	1.755	1.451	1.451	1.451	2.154	1.674	1.922	1.916	1.71	1.71	1.71	1.231	
12.600	**12.126**	**14.191**	**14.191**	**14.191**	**10.782**	**10.754**	**9.469**	**8.143**	6.398	6.398	6.398	4.07	3.284	4.493	4.945
14.202	**14.202**	**11.555**	**11.103**	**10.182**	**9.436**	**7.861**	**7.861**	**7.861**	**9.077**	**9.470**	**9.342**	**8.940**	**8.552**	**8.552**	
2.763	3.099	3.631	4.9	4.764	4.764	4.764	5.558	5.859	4.788	1.859	1.859	1.859	1.859	1.859	2.514