Empirical Example - The Model

- gasoline demand consumption from Baltagi

- model: $\ln \frac{\text{Gas}}{\text{Car}} = \alpha + \beta_1 \ln \frac{Y}{N} + \beta_2 \ln \frac{P_{\text{MG}}}{P_{\text{GDP}}} + \beta_3 \ln \frac{\text{Car}}{N} + u$

- data: annual observations among 4 countries (Canada, Denmark, France, Germany)

Remarks

- we can estimate this model country by country (4 OLS equations)

- but: residuals among countries can be correlated (non-diagonality of Σ matrix) - then an estimator based on Zellner's SUR can be more efficient than estimating by OLS equation-by-equation

- why can the correlation appear?
     - the countries might experience similar shocks that would cause their error terms to be related
     - the error term will contain the influence on gasoline demand of any omitted factors, and to the extend that these omitted factors tend to have similar effects on gasoline demands in all   countries, their error terms will be correlated
     - if there is a significant correlation of disturbances across equations, then when the equations are estimated jointly, and the contemporaneous correlation is taken into account, better   (i.e. more efficient) estimates can be achieved than those from estimating the regressions separately

- we need to test the diagonality of Σ matrix

Testing diagonality of Σ matrix

(i) Breusch-Pagan Lagrange Multiplier test:

$$\text{LM} = T \sum_{i=2}^{M} \sum_{j=1}^{i-1} r_{ij}^2,$$

where M denotes the number of equations and $r_{ij} = \hat{s}_{ij} \Big/ \left(\hat{s}_{ii}\, \hat{s}_{jj}\right)^{1/2}$ is the correlation among estimated residuals. This statistics is asymptotically distributed as $\chi^2_{M(M-1)/2}$.

(ii) Likelihood ration can be used too:

$$\lambda_{\text{LR}} = T\left(\sum_{i=1}^{M} \log \hat{s}_{ii} - \log \left| \hat{\Sigma} \right|\right)$$

where $\hat{s}_{ii}$ is the restricted MLE of σ_{ii} obtained from the OLS residuals. $\hat{\Sigma}$ is unrestricted MLE of Σ.

Bibliography

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