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Value-at-Risk on Central and Eastern European Stock Markets: An Empirical Investigation Using GARCH Models

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Abstract:
Using daily return data from the four major Central and Eastern European stock markets including fourteen highly liquid stocks and ATX (Vienna), PX (Prague), BUX (Budapest), and WIG20 (Warsaw) market indices, we model the value-at-risk using a set of univariate GARCH-type models. Our results show that, in both in-sample and out-of-sample value-at-risk estimations, the models based on asymmetric distribution of the error term tend to perform better or at least as well as the models based on symmetric distribution (i.e., Normal or Student) when the left tails of daily return distributions are concerned. Evaluation of the same models is less clear, however, when the right tails of the distribution of daily returns must be modelled. We suggest an asset-specific approach to selecting the correct parametric VaR model that depends not only on the risk level considered but also on the position in the underlying asset.

Keywords: Value-at-Risk, Expected Shortfall, Backtesting

JEL: C14, C32, C52, C53, G12.

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1 Introduction

Careful management of financial risk assumes not only understanding the nature (or types) of financial risk but also the techniques that allow for the measurement of the risk exposure associated with a particular asset or portfolio of assets. Over the last two decades,[1] one tool has proved to be especially useful in the measurement of the (financial) risk exposure: the value at risk (VaR).

VaR summarizes the expected (financial) loss that - under normal market conditions - one can expect to incur on a given asset or a portfolio of assets over a given time horizon within a given confidence interval. Hence, as a single number that measures the market risk in the same units as bank's (or, in general, any other risk management party's) bottom line, VaR provides for a *summary measure* of market risk that is not only easily understood but, also, relatively simple to use. As a result, the use of some form of VaR methodology is nowadays advocated by a majority of regulators while, at the same time, it is thought appropriate by most of the leading practitioners dealing in financial risk management.

Several approaches exist to measuring the VaR. In every case, the focus is on the empirical distribution of returns on the given asset or a portfolio of assets as it is the sample quantile from this distribution that effectively defines the VaR. In the study at hand, we model the VaR by attempting to fit several parametric conditional volatility models to the asset returns data and then use the modeled conditional variance in constructing the sample (VaR) quantiles. In order to make our results more general, we compute both left and right quantiles of the distribution of asset returns, corresponding to the practical cases when the trader or, in general, a risk manager, has a long/short position in the underlying asset and is concerned with a drop/rise in its price.[2]

Further in the analysis, we proceed by assessing the (forecasting) performance of the models in correctly modeling the VaR. We use the models that differ not only in the way they treat the relationship between the conditional variance and the lagged squared error term (i.e., we use both symmetric and asymmetric time-varying volatility models), but also in the assumption they make on the distribution of the error term. In fact, with respect to the latter and, for the purpose of the analysis, we divide the models in two groups based on whether the distribution of the error term is assumed to be symmetric (i.e., normal or Student) or asymmetric (in our case, skewed Student). The division makes sense both *ex-ante* - as both the usual symmetric and asymmetric models were previously shown to have a tough job in modelling correctly the left and right tails of the distribution,[3] as

[1] Two decades have passed since the landmark Basle Accord of 1988. The agreement set the minimum capital requirements that must be met by commercial banks to guard against credit risk.

[2] In other words, the construction of the left (or right) quantile is necessary to obtain the VaR for the negative (or positive) returns on a given asset.

[3] This finding is not at all surprising given that the distribution of assets returns is often not

well as *ex-post* - as later in the study we show that, compared to the models based on asymmetric distribution, both the (conditionally) symmetric and asymmetric models used in the analysis tend to underperform in correctly modelling either the left or right tails of the return distributions when the distribution of the error term is symmetric.[4]

The evaluation of the performance of the models is based on calculating the VaR for long and short trading positions on four major Central and Eastern European (CEE) daily stock market indices (ATX, PX, BUX, and WIG20) and fourteen of the most liquid CEE stocks traded in these markets. We hope this way to extend the previous research concerned with the evaluation of alternative volatility forecasting methods used in VaR modeling via not only a wide class of models employed in the analysis but also through its focus on the CEE markets. The latter aspect of the study seems to be all the more relevant since, as of today, no study has appeared that would analyze the VaR for the same markets to the extent comparable to the present study.

Several studies have attempted to evaluate the performance of various GARCH models in the way similar to the present paper. Giot and Laurent (2003) estimated the daily VaR for both long and short trading positions by employing three symmetric and one asymmetric conditional volatility model (APARCH) concluding that the latter performed better overall compared to the symmetric models. Brooks and Persand (2003) also considered the issue of asymmetry. In their analysis, they found that the models which did not allow for asymmetries either in the unconditional return distribution or in the volatility specification underestimated the *true* VaR. More recently, Bali and Theodossiou (2007) used ten popular variations of the GARCH models based on skewed generalized t-distribution to calculate the VaR. The authors argued that TS-GARCH and EGARCH models provided the best overall performance.[5] Finally, McMillan and Speight (2007) claimed both asymmetric and long memory volatility features to be important considerations in providing improved VaR estimates.

The rest of the paper is organized as follows. In Section 2 we provide a detailed description of the data used in the analysis. In Section 3, we describe basic methodology and develop the estimation techniques employed in the analysis further in the paper. Section 4 describes the results and provides the discussion of the results of estimation. Finally, Section 5 summarizes and concludes the study.

symmetric. See Giot and Laurent (2003) and El Babsiri and Zakoian (1999) for a more detailed analysis of these findings.

[4] El Babsiri and Zakoian (1999) note that, in the (conditionally) asymmetric GARCH models (e.g., EGARCH or TGARCH), the two components of the innovation have - up to a constant - the same volatilities. Given what we said about the distribution of the asset returns, it makes sense to allow an asymmetric confidence interval around the forecasted volatility in the VaR computation.

[5] The TS-GARCH model is due to Taylor (1986) and Schwert (1989). The EGARCH model is discussed in Section 3.2 of this study.

2 Data Description

In the analysis, we compute the VaR for both negative and positive returns on four major CEE stock indices (ATX, PX, BUX, and WIG) as well as on fourteen major CEE stocks including three ATX stocks, four PX stocks, four BUX stocks, and three WIG stocks.[6]

All four market indices are based on the period that starts on 10.04.1995 (April 10, 1995) and ends on 10.04.2008. The data for the fourteen stocks is of variable length although in each case the series ends on 10.04.2008. In the description of the stocks that follows, we include a shortcut that used to refer to the stock in the latter parts of the study as well as the date when the series starts.

The three ATX stocks include Erste Bank der österreichischen Sparkassen (EBS: 10.12. 1997), OMV (OMV: 10.04.1995), and Telekom Austria (TKA: 03.12.2000), the four PX stocks include ČEZ (CEZ: 10.04.1996), Erste Bank (ERS: 10.10.2002), Komerční Banka (KOB: 10.04.1996), and Telefónica O2 Czech Republic (TEF: 10.06.1998), the four BUX stocks include MOL (MOL: 10.04.1996 to 10.04.2008), Magyar Telekom (MTE: 10.12.1997), OTP Bank (OTP: 10.04.1996), Richter Gedeon (RCH: 10.04.1996), and the three WIG stocks include PKO Bank Polski (PKO: 12.07.1998), PKN Orlen (PKN: 12.12.1992), and Telekom Polska (TPS: 10.12.1998).[7]

The combined capitalization of the three ATX stocks (as well as of the three WIG stocks) represents around 40% of the ATX (WIG) market index, while the market capitalizations of the combined four PX stocks and the combined four BUX stocks represent about 80% of their respective market indices (see Table 1). In addition, majority of the stocks just described belongs to the most actively traded stocks on their home stock markets, a fact that makes them especially suitable for the VaR calculations, as well as a reason why we chose these stocks for the analysis in the first place.

[Insert Table 1]

Table 2 gives the descriptive statistics for the daily returns on the four market indices and the fourteen stocks.[8] We notice that both across the indices and the stocks, the returns

[6] The indices mentioned correspond, respectivelly, to the stock exchanges in Vienna (http://en.wienerborse.at), Prague (http://www.pse.cz), Warsaw (http://www.gpw.pl/index.asp), and Budapest (http://www.bse.hu). We used Bloomberg® as source for the data.

[7] We used a standard procedure to adjust the data to account for the stock splits. For example, in case of a 2-for-1 stock split, we cut all of the prices that occured before the stock split in half to ensure continuity in the pricing. A special instance of a stock split (a stock merger, to be exact) occured on Feb 15, 1999, when two previous issues of CEZ shares with face values of CZK 1,000 and CZK 1,100 were merged and subsequently split into shares with the face value of CZK 100. In that case, we split the stocks according to a 10.5-for-1 ratio although it is clear that a more appropriate way would have been to use two different stock-split ratios, weighting the CZK 1,000 and the CZK 1,000 stocks by the corresponding market capitalization existing at the time of the stock split.

[8] We defined the daily returns in usual way as $r_t = 100\,[\ln(p_t) - \ln(p_{t-1})]$, where $\ln(p_t)$ is a natural

exhibit relatively high excess kurtosis. In general, this is a result of relatively large skewness combined with large minimum and maximum values. Especially pronounced in this regard are the cases of BUX stock index or the cases of the TKA, KOB, and RCH stocks - in all these instances, the skewness is smaller than -0.6 and the returns reach easily over 20% in absolute value (KOB). The Ljung-Box Q-statistics computed on the squared return series indicates a high serial correlation in the variance with KOB and RCH giving especially large values of the Q-statistics.

[Insert Table 2]

Figures 1 and 2 provide a graphical way to investigate the empirical properties of the asset return series - namely, those of the four stock indices and four representative stocks. As already noted (see Table 1), the histograms tend to show long stretches on their left sides due to the fact that the minimum (negative) returns are generally larger in absolute value than the maximum (positive) values. Compared to the normal distribution, the returns also show fat tails - note PX and WIG indices as cases that are particularly visible. The QQ-plots present the sample quantiles of the corresponding asset's returns against the quantiles of the $N(0,1)$ distribution. They are indicative not only of the fact that both tails of the empirical distribution are heavier than those of normal distribution but, perhaps more importantly, that they are not symmetric.[9]

[Insert Figures 1 and 2]

The description of the returns provided so far shows that the data series used in the study exhibit several of the stylized features that have been repeatedly found in many similar financial series.[10] Heavy-tailed by nature, classical ARCH (or, GARCH) -type models have been used to model some of these features (e.g., heavy tails, volatility clustering) while a little more recent extensions of these models (e.g., EGARCH and/or fractionally integrated GARCH) have been developed to model the features such as asymmetry and long-range dependence.[11]

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logarithm of the price of the corresponding asset. One way to see why this particular transformation is used is to note that $(p_t - p_{t-1})/r_{t-1} \approx \ln[1 + (p_t - p_{t-1})/p_{t-1}] = \ln(p_t) - \ln(p_{t-1})$.

[9] Moreover, the QQ-plots are informative for identifying the moment condition. When we compared the tails of the empirical distribution of the returns to that of the t-distribution, we again found heavier tails. Moreover, in this case, the tails were relatively lighter in case of t-distribution with 8 degrees of freedom than in case of the t-distribution with 4 degrees, so that we could reasonably assume that $E(r_t^6) = \infty$ and $E(|r_t|^{3-\epsilon}) < \infty$ for any $\epsilon > 0$ (see Fan and Yao, 2005).

[10] These *stylized facts* include heavy tails, volatility clustering, asymmetry, aggregational gaussianity, and long range dependence. We do not (explicitly) present the plots that would depict the volatility clustering in our study. They are, nevertheless, available upon request from the author.

[11] See Bera and Higgins (1993) and Shephard (1996), among others for comprehensive survey on ARCH and extended GARCH models, respectively.

3 Econometric Framework

In all specifications, we model the daily return series as an AR(2) process of the form $(1+\phi_1 B+\phi_2 B^2)(r_t-\mu) = \epsilon_t$ or, $\phi(B)(r_t-\mu) = \epsilon_t$, where $\phi(B)$ is a second order polynomial in B,and ϵ_t is the disturbance term, with $E(\epsilon_t) = 0$ and $E(\epsilon_t\epsilon_s) = 0$, for each $t \neq s$. We note that, while we found the second order autoregressive process to be sufficient to correct for the serial autocorrelation that seems to be (to some degree) present[12] in the conditional mean of all of the daily returns series used in this study, our approach is also similar to that of Giot and Laurent (2003), who also impose a single autoregressive structure on all the data series that they consider.

It follows that the conditional mean for the daily returns, μ_t, can be determined simply as

$$\mu_t = \mu + \phi_1(r_{t-1} - \mu) + \phi_2(r_{t-2} - \mu). \tag{1}$$

In the following paragraphs, we introduce several specifications for the conditional variance of ϵ_t, defined as

$$\epsilon_t = z_t\sigma_t, \tag{2}$$

where z_t is a zero mean and unit variance *i.i.d.* random variable distributed according to some specified distribution. Later, in sections 3.4.1 to 3.4.3, we will discuss two symmetric (Normal and student) and one asymmetric distribution (skewed Student) for z_t. Both the type of distribution assumed for z_t as well as the conditional variance (see eq. 2) are used to calculate the VaR and hence, the performance of the models in correctly modelling the risk summary measure depends directly on the two.

3.1 General GARCH and IGARCH

A generalized autoregressive conditional heteroskedastic (GARCH) model with order $p\,(\geq 1)$ and $q\,(\geq 0)$ defines the conditional variance as

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} a_i\epsilon_{t-i}^2 + \sum_{j=1}^{q} b_j\sigma_{t-j}^2, \tag{3}$$

where $\omega \geq 0$, and a_i $(i = 0, ..., p)$ and b_j $(j = 1, ..., q)$ are assumed to be positive to ensure that the conditional variance, σ_t^2, is always positive.[13]

As already noted, the GARCH model is capable of explaining many of the stylized facts

[12] The reader may refer to the descriptive statistics section of this study for the details.

[13] See Nelson and Cao (1992) for the general conditions on the positivity of conditional variance. We note that positive coefficients represent sufficient but not necessary conditions for the positivity of conditional variance.

characterizing the financial time series,[14] including the heavy tails.[15] In fact, it is its ability to account for large negative (or, positive) returns that is to a large extent responsible for its performance at predicting the VaR for long/short trading positions.

Following the links that exist between GARCH(p, q) and ARMA(p, q) models and the fact that an (invertible) ARMA(p, q) process with finite p and q is equivalent to an AR(∞) process, it is also easy to see how even a lower order GARCH model (e.g., GARCH$(1, 1)$) can provide parsimonious representation of many complex dependence structures of (ϵ_t) that could otherwise be accommodated only by an ARCH(p) model with large p. With this in mind, we again follow the approach of Giot and Laurent (2003) in that we use the same low-order GARCH$(1, 1)$ model for most of the investigated return series to allow for a single benchmark when evaluating the performance of the various parametric models.

In general, the ARMA(p, q) model can be written as $(1 - \sum a_i + \sum b_j)\ \epsilon_t^2$ or, in the lag polynomial form, as $(1 - a(L) - b(L))\ \epsilon_t^2$. Here, whenever the polynomial part contains a unit root, the process has an integrated variance and effectively defines an integrated GARCH (or, IGARCH) process. The **RiskMetrics** model (Morgan, 1996) used in the study is equivalent to an IGARCH model with the weight parameter (also called the *decay* factor) set at $\eta = 0.94$ and the coefficient ϵ_{t-1}^2 equal to η. Formally, the RiskMetrics specification defines the conditional variance as

$$\sigma_t^2 = \eta \sigma_{t-j}^2 + (1 - \eta)\epsilon_{t-i}^2, \tag{4}$$

where ϵ_{t-1}^2 is the first lag of the (squared) disturbance term as defined by (2). Put differently, in the RiskMetrics model, the volatility for time t is given as a weighted average of the previous forecast and of the squared error term.

[14] The ability of the GARCH models to explain many of the stylized facts can be easily understood by following the links that exist between GARCH and ARMA processes. Recall that a GARCH(p, q) model, defined by equation (3), represents a more parsimonious version of an ARCH(p) model that is defined only in terms of the past squared returns (ϵ_{t-i}^2). Just as an ARCH(p) model can be expressed as an AR(p) model, however, a GARCH(p, q) model can be, similarly, expressed as an ARMA(m, q) model, where $m = max(p, q)$. As an example that easily generalizes to any p and q, consider a GARCH$(1, 1)$ model $\sigma_t^2 = \omega + a_1\epsilon_{t-1}^2 + b_1\sigma_{t-1}^2$. Since $E_{t-1}(\epsilon_t^2) = \sigma_t^2$, the same equation can be written as a function of ϵ_t^2 only as $\epsilon_t^2 = \omega + (a_1 + b_1)\epsilon_{t-1}^2 + u_t - b_1 u_{t-1}$, where $u_t = \epsilon_t^2 - E_{t-1}(\epsilon_t^2) \sim WN$, which clearly is an ARMA$(1, 1)$ process.

Given the value of b_1 (found to be around 0.9 for many weekly or daily financial time series; see Zivot and Wang (2006)), we can immediately see that large (small) changes in ϵ_{t-1}^2 will be followed by large (small) changes in ϵ_t^2. Applying the same reasoning to a GARCH$(1, 1)$ process, it follows similarly that large (small) values of σ_{t-1}^2 would be followed by large (small) values of σ_t^2. Hence, a GARCH model implies the volatility clustering. Furthermore, using the ARMA representation, we can show that a GARCH model also implies a volatility mean-reversion. Using again the same example of a stationary GARCH$(1, 1)$ model, for example, one can show that in this model the long-run (mean-reverting) level of volatility of ϵ_t is given by $\omega(1 - a_1 - b_1)^{-1}$.

[15] We note that GARCH models can also replicate the fat tails provided that the condition for the existence of the fourth order moment of the GARCH process is satisfied (He and Terasvirta, 1999).

3.2 EGARCH

In the basic GARCH model (eq. 3), only squared residuals ϵ_{t-i}^2 enter the equation, so that the signs of residuals (or shocks) have no effects on conditional volatility. Still, as already mentioned, the assumption that good or bad shocks have no (or symmetric) effect on the volatility is frequently violated in practice. In fact, in most of the returns series, negative shocks (corresponding to bad news) tend to have larger impact on volatility than positive shocks (or, good news).

Black (1976) was the first one to attribute this effect to the fact that bad news tends to drive down the stock price, thus increasing the leverage (D/E ratio) of the stock causing the stock to be more volatile. Consequently, a number of so-called "second-generation" asymmetric GARCH models have been developed to account for the asymmetric response of volatility to such shocks (McMillan and Speight, 2007). In these models, the impact that the asymmetric news has on the stock price is usually referred to as the *leverage effect*. Nelson (1991) introduced the following *exponential* GARCH (EGARCH) model to allow for leverage effects:

$$h_t = \omega + \sum_{i=1}^{p} a_i \frac{g(\epsilon_{t-i})}{\sigma_{t-i}} + \sum_{j=1}^{q} b_j h_{t-j}, \tag{5}$$

$$\text{where } g_t(\epsilon_{t-i}) = |\epsilon_{t-i}| \ + \gamma_i \epsilon_{t-i} \text{ and } h_t = \log \sigma_t^2. \tag{6}$$

In this model, the value of the function $g_t(\cdot)$ depends on both the sign and the size (or, magnitude) of its argument. As a result, the EGARCH can respond nonsymmetrically to random shocks ϵ_t. Whenever $\epsilon_{t-i} > 0$ (i.e., whenever a good news occurs), the total effect of the shock to ϵ_{t-i} is $(1+\gamma_i)|\epsilon_{t-i}|$; if, on the other hand, $\epsilon_{t-i} < 0$, the total effect is equal to $(1-\gamma_i)|\epsilon_{t-i}|$. As bad news can have a larger impact on volatility, the value of γ_i would be expected to be negative.[16]

In the next section, we introduce a more recent extension of the GARCH model of Bollerslev (1986) that, compared to EGARCH model, allows for more flexibility in modelling the leverage effects.

3.3 APARCH (or PGARCH)

The *asymmetric power* ARCH (APARCH) model, often called *power* GARCH (PGARCH) in the literature, proposed by Ding, Granger and Engle (1993), has the following specification:

[16] It is rather straightforward to identify the properties of h_t. For example, provided that $g(\epsilon_t)$ is *i.i.d.*, then so is h_t, so that h_t is causal linear AR(1) process whenever $|\sum b_j| < 1$. We refer the reader to Bollerslev, Engle, and Nelson (1994), for further discussion of EGARCH models.

$$\sigma_t^d = \omega + \sum_{i=1}^{p} a_i [g(\epsilon_{t-i})]^d + \sum_{j=1}^{q} b_j \sigma_{t-j}^d, \tag{7}$$

$$\text{where } g_t(\epsilon_{t-i}) = |\epsilon_{t-i}| \; + \gamma_i \epsilon_{t-i}. \tag{8}$$

The flexibility of this model compared to that of a generic EGARCH is obvious. The model includes not only the asymmetry coefficient (γ_i) but also a positive exponent d. The asymmetry coefficient plays the same role as in case of the EGARCH model in that it models the leverage effect. In other words, as in the previous case, a positive value of γ_i translates into the past negative shocks having a deeper impact on current conditional volatility than the past positive shocks (again, see Black (1976) or, Pagan and Schwert (1990)).

The PGARCH model is interesting in that it nests at least seven other ARCH specifications as special cases. For example, when $d = 2$ and $\gamma_i = b_j = 0$ (for $\forall i$ and $\forall j$), the model becomes a generic ARCH model. Given the same restrictions but with b_j allowed to vary (for $\forall j$), the same model reduces to the basic GARCH model, while if both b_j and γ_i are allowed to vary and $d = 1$, the model reduces to TARCH of Zakoian (2002).

3.4 Calculation of the VaR

Having provided a brief overview of the empirical specifications of the models used in the study, we are now ready to consider the calculation of the Value-at-Risk for the various models. First, however, we provide the basic methodology behind VaR calculation.

As already noted, a daily VaR is the expected loss that one can expect to incur on a given asset or portfolio of assets over one day within a given confidence interval. Hence, in statistical terms, the VaR corresponds to a high (low) order quantile of the distribution of daily losses when the VaR of long (short) positions is concerned. In general then, we have

$$VaR_p = \inf\{x : F(x) \geq p\} = F^{-1}(p), \tag{9}$$

where x represents daily returns and p defines the corresponding empirical quantile at $p\%$.

To give an example, within a 99% probability level required under the Basle Committee rules, the daily 1% VaR on the long (short) trading position is the 99% (1%) quantile of the distribution of daily returns. In words, there exists a 1% probability that the daily losses will exceed $\text{VaR}_{.99}$ (i.e., a 99% probability that the returns will be larger than the $\text{VaR}_{.99}$) when the VaR for the long trading positions is concerned. Similarly, there exists a 1% probability that the daily losses will exceed $\text{VaR}_{.01}$ (i.e., a 99% probability that the returns will be larger than the $\text{VaR}_{.01}$) when the VaR for the short trading positions is concerned.[17]

[17] Along with the 99% probability level, the Basle Accord also requires the VaR to be based on a

Following the same reasoning, the calculation of VaR_p reduces to the following formula

$$VaR_p = \mu + \sigma q_p(\cdot), \tag{10}$$

where $q_p(\cdot)$ is the $p\%$ quantile of the distribution that the returns are assumed to follow. Given (10), the computation of the VaR requires the knowledge of both the conditional variance (and the conditional mean) as well as of the sample quantiles. Thus, its calculation depends not only on the structure we put on the evolution of the conditional variance but also on the distribution of the error term (see eq. (2)).

We are now ready to discuss the calculation of the VaR for each of the models of conditional variance used in this study. Our goal is to define the (10) in terms of the conditional variance (or, conditional standard error) and the sample quantile as determined by the distribution assumed for the error term. We make use of two symmetric distributions (Normal and Student) and one asymmetric distribution (skewed Student).

3.4.1 Normal VaR

Before we provide the formula for the calculation of the VaR under the assumption that $z_t \sim N(0,1)$, see (2), we shortly describe the procedure used to estimate the conditional variance.[18]

As with the other models used in this study (see equations (5) and (7)), the two (benchmark) formulations that assume the Gaussian errors (i.e., (3), (4)) were estimated using a conditional maximum-likelihood (cMLE) approach. Given the structure of the mean equation (1) and assuming that the error term follows a normal distribution, the prediction error decomposition of the log-likelihood function conditional on initial values is then given by:

$$\log L = -\frac{T}{2}\log(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\log\sigma_t^2 - \frac{1}{2}\sum_{t=1}^{T}\frac{\epsilon_t^2}{\sigma_t^2}, \tag{11}$$

It is immediately clear that as the recursive evaluation of the likelihood function is conditional on unobserved values, its estimation is less then exact. To solve this problem, we set these quantities to their unconditional expected values (see LAURENT AND PETERS, 2002). Once the cMLE estimates of the parameters are found, the estimates of the time-varying volatility σ_t (for $\forall t$) used in calculating (10) can be obtained as a side product (see ZIVOT AND WANG (2006)).

ten day holding period. In general, assuming that the returns are *i.i.d.*, the VaR calculated over one day can be used to obtain the VaR for T days (periods) simply as $\text{VaR}^{(T\ periods)} = \sqrt{T}\ \text{VaR}^{(1\ period)}$.

[18] We do not discuss the maximization process in detail. Instead, we refer the reader to HAMILTON (1994), for example, for further discussion of this topic.

In the Normal (I)GARCH model - represented in this study by two benchmark formulations (4) and (3) - the one-step-ahead VaR computed in $(t-1)$ is then given by $\mu_t + \sigma_t q_p(N)$ (long positions) and by $\mu_t + \sigma_t q_{1-p}(N)$ (short positions), where $q_p(N)$ is the $p\%$ quantile of the standard normal distribution. The one-step-ahead forecasts for the RiskMetrics formulation (IGARCH) are obtained the same way.

3.4.2 Student VaR

Assuming that the error term (2) follows a standardized Student distribution or, $z_t \sim t(0,1,\nu)$, the log-likelihood function conditional on initial values is given by:

$$\log L = T\left\{\log\Gamma\left(\frac{\nu+1}{2}\right) - \log\Gamma\left(\frac{\nu}{2}\right) - \frac{1}{2}\log[\pi(\nu-2)]\right\} - \\ -\frac{T}{2}\sum_{t=1}^{T}\log\left(\sigma_t^2\right) - \frac{(\nu+1)}{2}\sum_{t=1}^{T}\log\left(1+\frac{z_t^2}{\nu-2}\right), \quad (12)$$

where $\Gamma(\cdot)$ is a gamma density function and ν are the degrees of freedom.

In the Student GARCH model - the one-step-ahead VaR computed in $(t-1)$ is given by $\mu_t + \sigma_t q_p(t(\nu))$ (long positions) and by $\mu_t + \sigma_t q_{1-p}(t(\nu))$ (short positions), where $q_p(t(\nu))$ is the $p\%$ quantile of the standardized student distribution with ν degrees of freedom estimated from (12). The one-step-ahead forecasts for the Student EGARCH and PGARCH (or, APARCH) formulations can be obtained similarly.

3.4.3 Skewed Student VaR

Assuming that the error term (2) follows a standardized skewed Student distribution[19] or, $z_t \sim sk\ t(0,1,\nu,\xi)$, the log-likelihood function conditional on initial values is given by:

$$\log L = T\left\{\log\Gamma\left(\frac{\nu+1}{2}\right) - \log\Gamma\left(\frac{\nu}{2}\right) - \frac{1}{2}\log[\pi(\nu-2)] + \log\left(\frac{2}{\xi+\xi^{-1}}\right)\right\} - \\ -\frac{T}{2}\sum_{t=1}^{T}\log\left(\sigma_t^2\right) - \frac{(\nu+1)}{2}\sum_{t=1}^{T}\log\left(1+\frac{(sz_t+m)^2}{\nu-2}\xi^{-2I_t}\right), \quad (13)$$

where $\Gamma(\cdot)$ and ν are defined as in (12), ξ the asymmetry parameter, defined as $\xi^2 = [P(z \geq 0|\xi)] \cdot [P(z < 0)|\xi]^{-1}$, and I_t a time-varying index function that is equal to

[19] The idea of extending the Student distribution by adding a skewness parameter to account for excess skewness (in addition to excess kurtosis) is due to Fernandez and Steel (1998) who expressed the new density in terms of the mode and the dispersion. It was not until Lambert and Laurent (2001), however, who re-expressed the density in terms of the mean and the variance, that the new framework could be applied in the GARCH framework. Most of the results in this section are due to these two authors.

1 whenever $z_t \geq -ms^{-1}$ and to -1 otherwise. Also, m is the mean and s the standard deviation of the (non=standardized) skewed Student distribution and can be expressed in terms of the degrees of freedom and the asymmetry parameters as follows:

$$m = \Gamma\left(\frac{\nu+1}{2}\right)\left[\Gamma\left(\frac{\nu}{2}\right)\right]^{-1}\sqrt{\left(\frac{\nu-2}{\pi}\right)\left(\xi-\xi^{-1}\right)} \tag{14}$$

$$s = \sqrt{\left(\xi^2+\xi^{-2}-1\right)-m^2} \tag{15}$$

To calculate the corresponding quantiles, a quantile function is needed. For the standardized skewed Student case, the function looks like

$$q_p(t(\nu,\xi)) = \frac{1}{\xi s}q_p(t(\nu))[\frac{p}{2}\left(1+\xi^2\right)] - \frac{m}{s} \qquad \text{if } p < \left(1+\xi^2\right)^{-1}$$

$$q_p(t(\nu,\xi)) = -\frac{\xi}{s}q_p(t(\nu))[\frac{1-p}{2}\left(1+\xi^{-2}\right)] - \frac{m}{s} \quad \text{if } p \geq \left(1+\xi^2\right)^{-1}, \tag{16}$$

where $q_p(t(\nu))$ is the $p\%$ quantile of the standardized student distribution with ν degrees of freedom.

In order to compute the one-step-ahead VaR for the GARCH, EGARCH, and PGARCH models, we will also need the stationary solutions to these models in the presence of skewed Student distribution.[20] We focus on the more interesting cases of EGARCH and PGARCH models here. For the EGARCH formulation, the solution to eq. (5) depends on the expected value of $|z_t|$ or,

$$E\left(|z_t|\right) = \frac{4\xi^2\sqrt{\nu-2}\Gamma\left[0.5(1+\nu)\right]}{\xi+\xi^{-1}\sqrt{\pi}\left(\nu-1\right)\Gamma\left(0.5\nu\right)}, \tag{17}$$

where $\xi = 1$ implies the case of the symmetric Student density (Laurent and Peters, 2002). In PGARCH case, the solution to eq. (7) is less straightforward as it requires finding the expected value of $\left(|z_t| - \gamma_i z\right)^d$. A closed-form solution to this expectation was first derived by Ding, Granger and Engle (1993) for the Gaussian case and by Lambert and Laurent (2001) for the standardized skewed Student case

$$E\left(|z_t|\right) = \left\{\xi^{-(1+d)}\left(1+\gamma_i\right)^d + \xi^{(1+d)}\left(1-\gamma_i\right)^d\right\}\frac{\left(\nu-2\right)^{0.5(1+d)}\Gamma\left(0.5(1+d)\right)\Gamma\left(0.5(\nu-d)\right)}{\left(\xi+\xi^{-1}\right)\sqrt{\pi\left(\nu-2\right)}\Gamma\left(0.5\nu\right)}, \tag{18}$$

where, as in case of (17), $\xi = 1$ implies the case of the symmetric Student density.

In the skewed Student GARCH model - the one-step-ahead VaR computed in $(t-1)$ is then given by $\mu_t + \sigma_t q_p(t(\nu,\xi))$ (long positions) and by $\mu_t + \sigma_t q_{1-p}(t(\nu,\xi))$ (short positions),

[20] We have not focused on the stationary solutions to our models in neither the Gaussian nor the symmetric Student density cases as their derivation is rather straightforward.

where $q_p(t(\nu,\xi))$ is the $p\%$ quantile of the standardized skewed student distribution with the degrees of freedom (ν) and the asymmetry coefficient (ξ) both estimated from (13).[21] It is important to note that if $\xi < 1$, the third moment of the density of z_t will be negative. As a result, the density of z_t will be skewed to the left and the VaR for the long trading positions (corresponding to large negative returns) will be larger than for the short trading positions since $|q_p(t(\nu,\xi))| > |q_{1-p}(t(\nu,\xi))|$. The same reasoning, of course, holds for the case when $\xi < 1$.

3.5 Model Estimation

The calculation of the VaR is based on estimating the models specified above (see the mean equation (1) combined with different specifications for the conditional variance (2) described in sections 3.1-3.3). To recall, in addition to estimating the GARCH (3) and (4) specifications as benchmarks (in which case both models are based on normal distribution of the error term), we also estimate the GARCH, EGARCH (5), and PGARCH (7) formulations under the assumption that the error term has either a Student or an (asymmetric) skewed Student distribution described in sections 3.4.2 and 3.4.3.

As the ultimate goal of this study is not the analysis of how well the different parametric models can model the time-varying variance (instead, the focus is on how well the different models can compute the VaR), we do not report the estimation results for the various models in this study.[22]

3.6 VaR Model: In-Sample and Out-of-Sample Evaluation

The internally generated VaR models are nowadays a commonplace in risk-management departments around the world.[23] The rule of the game, however, is that the validity of these models be evaluated or, *back-tested*, by comparing the (internal) VaR estimates with the actual observations. In this section, we discuss a common framework used in this study to examine the accuracy of the VaR models, the *failure rate* methodology.

Before we proceed, it is important to mention the difference between in-sample and out-of-sample VaR estimation.[24] While in the former case we are simply comparing the sample

[21] The one-step-ahead forecasts for the Student EGARCH and PGARCH formulations can, once again, be obtained similarly.

[22] Except where otherwise stated, the estimations were performed using a console version of Ox 4.0 (see DOORNIK (2007)) and G@RCH 4.10 Ox Package (see LAURENT AND PETERS (2002). Complete estimation results are available from the author upon request.

[23] In April 1995, Basle Committee on Banking Supervision (see the references), recognized "[] *that risk management models in use by major banks are far more advanced than anything they could propose*" (JORION, 1997, pp 41).

[24] The same failure rate testing methodology is, of course, applied the same way in both in- and out-of-sample VaR estimations.

observations with the VaR measures based on the fitted values of the volatility estimates, in the out-of-sample estimation, we are much closer to the real-time estimation faced by the risk-managers in that we are comparing the sample observations for the period T forward with the VaR measures based on the volatility estimates forecasted for a sample of daily returns up to time T. As a result, the out-of-sample forecasts are not only much more demanding to construct from the computational point-of-view than the in-sample forecasts but, even more importantly, the out-of-sample testing is also closer to reality and hence a better evaluator of the accuracy of the different types of the parametric VaR models used in our study.

3.6.1 The Failure Rate

Perhaps the simplest approach to verifying the accuracy of the VaR models is to record the *failure rate* (Jorion, 1997), which is defined as the proportion of times that the (calculated) VaR is exceeded in a given sample of returns. Specifically, given a sample of observations, we are interested in counting the number of times that the observed negative returns are smaller (that is, the actual losses are larger), than the one-step-ahead VaR for the long positions or, conversely, in counting the number of times that the observed positive returns are larger than the one-step-ahead VaR for the short positions.

In either case, if the (internal) VaR model is well specified, the failure rate or, the proportion of such returns in the given sample, should equal to $(1-p)$, where p is the specified probability level, otherwise defined by (9). In the study, we set the probability level p to 99% (as required by the Basle Committee rules), as well as to 95% and 97.5% (in order to make it comparable with other studies that used the same probabilities), and to 99.5% (in order to evaluate the performance of the models with respect to their ability to fit the extreme events).

In order to test the null hypothesis that the estimated failure rate equals to $(1-p)$, we use a test proposed by Kupiec (1995).[25] The confidence regions for this test are defined by the tail points of the likelihood ratio

$$l_K = 2\log\left[\left(1-\frac{N}{T}\right)^{T-N}\left(\frac{N}{T}\right)^{N}\right] - 2\log\left[(p)^{T-N}(1-p)^{N}\right], \qquad (19)$$

where $(1-p)$ is the *correct* failure rate, T the sample size, and N the number of deviations that we would expect to observe at a given confidence level. Under the null hypothesis that the observed frequency of deviations, N/T, equals to $(1-p)$, this statistics has a $\chi^2(1)$ distribution.

[25] Kupiec (1995) LR test is commonly used in the literature to test the accuracy of VaR models.

Kupiec (1995) gives *example* confidence regions for various probability levels when the confidence level (or, the level of significance) is 5%. The table below (based on Kupiec, 1995) reproduces some of the confidence regions both in levels and - explicitly in the parentheses - also in percents (calculated as N/T) for the sample size equal to the average number of daily return observations in one year, 255, as well as to $1,000$.

$(1-p)$	$T = 255$		$T = 1000$	
99.0%	N < 7	(n/m : 2.8)	4 < N < 17	(3.7 : 6.5)
99.0%	6 < N < 21	(2.4 : 8.2)	15 < N < 36	(1.5 : 3.6)
95.0%	16 < N < 36	(6.3 : 14.1)	37 < N < 65	(3.7 : 6.5)

The table shows two facts. The first - a good one - tells us that the longer the sample period, the easier it is to reject the VaR model if it is false (notice that the percentage intervals become smaller as T becomes larger). The second - a disturbing one - shows that the smaller the probability level $(1-p)$, the more difficult it is to confirm the deviations. In fact, when the confidence region is N < 7, it becomes difficult if impossible to tell whether N is just small or whether the model systematically overestimates the risk.

The problem of small $(1-p)$ will become clear in the next section when, in many instances, we will have to comment on the unsatisfactory VaR results obtained using various GARCH-type models for the case of $p = 99\%$. This makes sense intuitively as the higher the value of p, the more rare are the observations we try to model and, hence, the more difficult it is to recognize correctly the systematic biases.

Before we proceed, we mention that while the Kupiec test can be used to examine whether the model systematically overestimates or underestimates the correct VaR, it cannot be used to determine whether the deviations are randomly distributed across time. A test that jointly investigates the failure rate of the model and the independence of deviations was developed by Christoffersen (1998). The test is based on the likelihood ratio statistic - distributed as $\chi^2(2)$ under the null hypothesis - of the form

$$l_C = 2\log\left[(1-\pi_{01})^{n_{01}}\,\pi_{01}^{n_{01}}\,(1-\pi_{11})^{n_{10}}\,\pi_{11}^{n_{11}}\right] - 2\log\left[p^{T-N}\,(1-p)^N\right], \qquad (20)$$

where $(1-p)$ is the probability level defined the same way as in (19), n_{jk} the number of observations such that, for example, n_{11} represents the number observations that can be classified as deviations that are followed by another deviation, and π_{jk} the appropriate probabilities defined as $\pi_{jk} = n_{jk}(\sum_k n_{jk})$.[26]

[26] We do not apply Christoffersen's test in the current version of the study.

3.6.2 Out-of-Sample VaR Methodology

As already noted, in the out-of-sample forecast, we are comparing the sample observations for the period T forward with the VaR measures based on the volatility estimates forecasted for a sample of daily returns up to time T.

The sample size (i.e., the number of daily return observations available) for the four indices as well as for the eighteen stocks considered in this study are presented in Table 2 in the Appendix. In the same table we also present the size/proportion of the sample used in the initial parameter estimation (see the column marked "OoS obs (%)").

As the sample size differs across the stocks (and with respect to the indices), we make it a rule that for each stock and each index the number of observations used in the initial estimation represents about 60% of the whole sample. Given the average length of one trading year (252), this leaves 5 years of observations $(1,260)$ that we could use for the volatility forecasts in case of all four stock indices (ATX, PX, BUX, WIG), as well as in case of OMV, CEZ, KOB, MOL, OTP, and RCH stocks. Similarly, we are left with 4 years of observations $(1,008)$ in case of EBS, TEF, MTE, PKO, and TPS, 3 years (756) in case of TKA and PKN, and 2 years (504) in case of ERS.[27]

Based on the initial estimation, we construct a one-day-ahead volatility forecast/estimate for $(T+1)$, and use it to obtain the (one-day-ahead) VaR measure that we can compare to the actual observed return at the same period. We follow the same procedure when computing the one-day-ahead VaR for either large negative or positive returns (long or short positions, respectively), saving the results along the way. The same estimation is then repeated at period $(T+2),(T+3),...,(T+\tau_k-1)$, where $\tau_k = 1260$, 1008, 756, or 504, based on the size of the sample left for the volatility forecast for the particular index or stock. At each step, the size of the sample increases by one observation.

The last note concerns an update of the model parameters. We surpass the Basle Committee requirement in this regard as we re-estimate the model parameters every fifty observations effectively creating a total of $\tau_k/50$ $(\tau_k = 1260, 1008, 756, 504)$ sub-samples for each particular index/stocks over which the model is estimated and hence assessed.[28]

[27] There is no particular reason for why we choose sixty percent as a cut-off for the number of observations usedin the inital forecast other than to comply with the two goals we had in this regard: a) to be consistent across the data for the reason of VaR evaluation across the models and, b) to comply with the Basel Committee one year requirement (discussed further in the text). Aside, we note that various lengths for parameter estimation have been previously tried in the literature. We refer the reader to Kupiec (1995) or Berkowitz (2001), among others).

[28] According to the rules set forth by the Basel Committee, the computation of the VaR estimates should be based on (at least) one year of historical return data, the probability level of $p = 99$ percent, and the horizon of ten trading days. In addition, the VaR estimates should be updated (at least) once per quarter (i.e., about 60 days).

3.6.3 Expected Shortfall

To provide the risk-managers with more information about the VaR, several measures have been suggested in the literature. In this study, we characterize the VaR models with the used of a sensitivity measure called *expected shortfall* (ES), defined as the expected value of the losses that are greater than the VaR estimate(s). Put differently, ER calculates the average of the negative returns smaller than the VaR for the short positions and, similarly, the average of the positive returns greater than the VaR for the long positions.[29]

4 Empirical Results

The estimation results are presented in Tables 3 to 14 in the Appendix. We first discuss the results based on in-sample estimation (Tables 3, 5, 7, 9, and 15 for the stock indices and the selected stocks and Tables 11-14 for the other stocks). The out-of-sample results (Tables 4, 6, 8, 10, and 16 for the stock indices and the selected stocks) are discussed in the paragraphs that follow. We will focus on the stock indices and the selected stocks as the results for the other stocks are very similar.

Tables 3 and 5 suggest that - in an in-sample estimation - the models based on skewed Student distribution (overall) perform generally better then the models based on symmetric (either Normal or Student) distributions in case of ATX and PX indices and at least as well as the models based on the Student distribution in case of BUX and WIG indices and most of the selected stocks. The results for the remaining series (see Tables 11 to 14) are similar: that is, the models based on skewed Student distribution do not tend to perform any better than the skewed Student models. The performance of the VaR models is summarized in Table 15 that shows the number of times (out of a hundred) that the p-value of the Kupiec test is smaller than five percent for the combined four possible values of the confidence levels used in the analysis. We can immediately notice a relatively worse performance of the models when the short positions are concerned. This is especially evident in case of the CEZ stock.

Before we proceed with the out-of-sample estimation results, we analyze the in-sample estimation from the risk level point of view. As could be expected, the higher the confidence level, the more difficult it is for the conditional volatility models to pick-up the extremely large (in absolute value) returns, a fact that we already discussed in [Section 3.6.1. Failure

[29] Note that this average can be expected to be larger for the models based on the Student distribution. The reason for this lies in the properties of the Student distribution: namely, fatter tails compared to those of the Normal distribution. As a result, when the model based on the Student distribution fails, the returns affected (i.e., those that drive the calculation of the sensitivity measure) are smaller/larger for long/short positions than if the distribution were *normal*, implying a larger measure in absolute value.

Rate]. Still, it is impossible to select a single model that would perform better than all the other ones across all assets when the extreme VaR quantiles are at stake. Consider the case of the 99.5% risk level, for example. If we are only interested in modeling the short positions, Normal GARCH model seems to perform at least as well as most of the asymmetric density models in case of the ATX index and still as well or better than all other models in case of PX and WIG indexes and one in three stocks (OMV, TKA, MTE, PKO: see Tables 11 to 14). The reasons for this performance lie clearly in the shape of the distribution of extreme returns. Perhaps more important, however, is the conclusion that the parametric VaR modeling requires an specific-asset approach, including the right choice of the model for a given risk level and a type of position in the underlying asset.

Turning our attention to the out-of-sample estimation results, Tables 4 and 6 suggest that the models based on Student or skewed Student distributions perform once again generally better then the models based on symmetric (either Normal or Student) distributions.[30] Once again, the choice of the most appropriate model depends on both the underlying asset and the confidence level. For example, while all the Student based GARCH models show superior performance in case of the PX index and EBS and PKN stocks across all risk levels, only the Student GARCH model seems to deliver desirable results for the BUX index (excluding, of course, the case of the 99% probability level on short positions).

The out-of-sample results are summarized in Table 16. In case of the stock indices, the combined success rates for the (skewed) Student models seems to be worse than for the stock indices than for the stocks, making it clear that the models do not perform equally well for types of assets. This only confirms the need for asset-specific approach to applied VaR modeling using parametric conditional volatility models.

5 Conclusions

In this study, we further extend previous research concerned with the empirical evaluation of the value-at-risk models by focusing on four Central and Eastern European (CEE) stock markets.

Comparing a wide range of univariate conditional variance models, we show that the models based on asymmetric distribution of the error term tend to perform better than the models based on symmetric distributions both in in-sample and out-of-sample (one-day-ahead) VaR forecasts. The optimal results, however, call for an asset-specific approach in which the superior performance of the different models depends not only on the choice of the confidence level but also on whether (extremely) small or large values be modeled.

[30] Again, we note that the results for the other stocks (namely OMV, TKA, ERS, KOB, TEF, MOL, MTP, RCH, PKO, and TPS) are very similar. The tables with the results for these stocks are available upon request from the author.

In fact, with regard to our findings, the VaR modeling of extremely small/large returns deserves a standalone comment. While traditional (normal) GARCH models represent a simple and efficient approach[31] to applied VaR modeling when it comes to *basic* probability levels (up to around 95%) - a point clearly demonstrated in the present study - our results also demonstrate another (empirically well-established) fact: that for most of the assets, the advanced class of GARCH models based on non-normal distributions of the error term and including EGARCH and APARCH models, seriously fails at higher probability levels. In our study, none of the models based on asymmetric distribution of the error term, including the asymmetric time-varying volatility models, can reasonably model the VaR for the probability levels above 99%. One way out of this problem would be to use the tools of the extreme value theory (EVT), obtaining the corresponding VaR quantiles using the extreme values of the distribution of asset returns only.[32]

There are several other ways in which the current study could be extended. For example, instead of evaluating the performance of the parametric GARCH models for one-day time horizons only, one could assess the performance of the same models over longer time periods. Also, on more general level, one could compare the performance of the parametric VaR models to the performance of the non-parametric (historical simulation) approaches.[33] Last but not least, one could employ a (fully-parametrized) multivariate framework and assess the performance of various multivariate conditional volatility models in the portfolio settings.

[31] Normal GARCH class of models may account for more than 80% of all risk models in practical use today (see Ong (2005), p. 511).

[32] The statistical theory behind EVT tells us that if we are interested in the analysis of (only) the smallest/largest observations from the distribution of asset returns, then - regardless of the overall shape of this distribution - the tails of this distribution will asymptotically resemble one of the following three distributions: finite (*Weibull*), exponential (*Gumbel*), or fat (*Fréchet*). Consequently, the EVT approach simplifies the traditional VaR analysis by focusing only on the extreme values while discarding the rest of the distribution. Modeling the VaR by focusing on the behavior of extreme return values is the focus of the next study by the author.

[33] Possibly the simplest method of forecasting the future volatility is to use a sample estimate of the unconditional variance of the data. This method is also referred to as the an equally weighted moving average model (EWMA).

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Appendix

Table 1: Weights of Stocks in the Indices

ATX		PX		BUX		WIG	
stock	% ATX	stock	% PX	stock	% BUX	stock	% WIG
OMV	18.028	ERS	27.885	MOL	32.570	PKO	15.254
EBS	17.212	CEZ	25.149	OTP	29.334	PKN	12.071
TKA	8.844	TEF	15.006	RCH	15.852	TPS	10.285
		KOB	14.512	MTE	13.312		
Total	44.084	Total	82.552	Total	77.756	Total	37.610

Notes: The stocks used in the study sorted according to their weights in the indices to which they belong. The data is as of May 1, 2008. Source: Bloomberg.

Table 2: Summary Statistics for Daily Returns

	obs	AVG	ASD	MIN	MAX	SK	EK	$Q^2(10)$
Indices								
ATX	3,219	0.043	17.336	-8.699	5.359	-0.739	4.790	767.0
PX	3,252	0.040	19.248	-7.077	8.084	-0.230	3.134	667.9
BUX	3,246	0.088	27.351	-17.90	13.62	-0.858	12.554	756.9
WIG	3,247	0.043	27.912	-10.32	7.647	-0.154	2.194	863.3
Stocks								
EBS	2,558	0.052	29.909	-11.52	8.701	-0.102	2.427	399.4
OMV	3,220	0.059	30.641	-10.23	8.374	-0.200	2.087	593.5
TKA	1,820	0.030	28.780	-21.68	7.647	-1.613	19.59	18.7*
CEZ	3,016	0.083	34.122	-23.26	13.11	-0.321	7.756	250.1
ERS	1,382	0.066	25.415	-8.610	6.899	-0.105	2.203	183.6
KOB	3,016	0.022	41.016	-24.09	20.05	-0.739	12.60	1,907
TEF	2,474	0.007	34.488	-15.60	12.03	-0.189	4.158	309.2
MOL	2,996	0.090	35.807	-22.10	13.19	-0.341	6.211	353.7
MTE	2,579	-0.004	33.323	-11.33	11.99	-0.133	3.272	283.4
OTP	2,996	0.131	39.629	-25.13	18.68	-0.299	8.282	490.1
RCH	2,996	0.062	41.492	-22.10	21.78	-0.617	14.31	2,034
PKN	2,088	0.027	31.083	-9.298	8.456	0.074	0.906	63.01
PKO	2,445	0.052	35.733	-10.54	11.92	0.031	2.629	419.7
TPS	2,337	0.011	36.684	-8.622	10.18	0.206	1.324	537.0

Notes: Descriptive statistics for the daily returns on the four stock indices (ATX, PX, BUX, WIG) and the fourteen stocks analyzed in the study. AVG stand for mean, ASD for annual standard deviation, SK for skewness, EK for excess kurtosis, and Q(10) for Ljung-Box Q-statistic of order 10 based on the squared returns. We note that, for the TKA stock, the Q-statistic is significant at 5 percent (p-value = 0.044). All values are computed using R.

Table 3: VaR Failure Rates (in %): Stock Indices (In-Sample Results)

	(Long Positions)				(Short Positions)			
conf. level (%)	95.0	97.5	99.0	99.5	95.0	97.5	99.0	99.5
	ATX Index							
RiskMetrics	*5.93*	**3.85**	**2.27**	**1.55**	4.29	2.24	1.21	*0.78*
N GARCH	5.25	**3.42**	**1.71**	**1.24**	**3.51**	*1.90*	0.84	0.44
ST GARCH	5.65	*3.17*	1.27	**0.90**	**3.82**	**1.68**	**0.56**	**0.16**
ST EGARCH	5.44	*3.11*	1.30	0.75	4.32	**1.71**	**0.53**	0.28
ST APARCH	5.50	*3.08*	1.30	*0.84*	4.38	**1.68**	**0.53**	0.34
skST GARCH	$5.03^{.93}$	$2.67^{.54}$	$1.21^{.24}$	*0.81*	$4.66^{.37}$	$2.18^{.23}$	$0.93^{.69}$	$0.34^{.18}$
skST EGARCH	$4.75^{.52}$	$2.67^{.54}$	$1.12^{.51}$	$0.68^{.16}$	$5.13^{.84}$	$2.18^{.23}$	$0.81^{.26}$	$0.44^{.59}$
skST APARCH	$4.72^{.47}$	$2.70^{.47}$	$1.12^{.51}$	$0.68^{.16}$	$5.10^{.81}$	$2.18^{.23}$	$0.75^{.13}$	$0.44^{.59}$
	PX Index							
RiskMetrics	5.32	*3.17*	**2.06**	**1.38**	4.34	2.89	**1.54**	**0.95**
N GARCH	4.92	2.74	**1.69**	**1.11**	*4.06*	2.21	0.98	0.74
ST GARCH	5.35	2.64	1.20	0.68	4.37	*1.91*	0.74	*0.28*
ST EGARCH	5.29	2.68	1.20	0.52	4.40	*1.97*	0.77	**0.22**
ST APARCH	5.23	2.58	1.17	0.55	4.49	2.00	0.74	*0.25*
skST GARCH	$5.14^{.72}$	$2.46^{.88}$	$1.11^{.55}$	$0.58^{.51}$	$4.71^{.43}$	$2.12^{.16}$	$0.83^{.32}$	$0.34^{.16}$
skST EGARCH	$5.01^{.97}$	$2.43^{.80}$	$1.14^{.44}$	$0.52^{.86}$	$4.64^{.35}$	$2.15^{.19}$	$0.89^{.53}$	$0.31^{.09}$
skST APARCH	$5.07^{.85}$	$2.52^{.94}$	$1.14^{.44}$	$0.52^{.86}$	$4.61^{.30}$	$2.21^{.29}$	$0.89^{.53}$	*0.28*
	BUX Index							
Riskmetrics	5.08	2.77	**1.48**	**1.17**	5.05	2.80	**1.69**	**0.96**
N GARCH	4.31	2.40	1.32	**1.05**	4.28	2.16	1.20	*0.77*
ST GARCH	4.84	2.37	1.02	0.62	5.05	2.16	0.77	0.34
ST EGARCH	4.68	2.09	0.99	0.68	4.68	2.43	0.96	0.34
ST APARCH	4.68	2.25	0.99	0.59	5.21	2.31	0.92	0.37
skST GARCH	$4.68^{.40}$	$2.25^{.35}$	$0.96^{.80}$	$0.59^{.50}$	$5.15^{.71}$	$2.19^{.24}$	$0.86^{.42}$	$0.37^{.27}$
skST EGARCH	$4.59^{.28}$	$2.00^{.06}$	$0.99^{.94}$	$0.68^{.17}$	$4.78^{.55}$	$2.47^{.90}$	$1.02^{.92}$	$0.34^{.17}$
skST APARCH	$4.56^{.24}$	$2.16^{.20}$	$0.96^{.80}$	$0.59^{.50}$	$5.24^{.53}$	$2.37^{.63}$	$0.96^{.80}$	$0.37^{.27}$
	WIG Index							
Riskmetrics	5.33	**3.33**	**1.79**	**1.17**	5.48	**3.27**	**1.60**	*0.83*
N GARCH	4.80	2.93	**1.48**	**1.08**	5.08	2.83	1.36	0.65
ST GARCH	4.93	2.83	1.23	0.62	5.24	2.68	0.83	*0.25*
ST EGARCH	4.80	2.56	1.23	0.68	5.36	2.90	0.96	*0.28*
ST APARCH	4.84	2.68	1.23	0.62	5.39	2.68	0.92	*0.25*
skST GARCH	$5.05^{.89}$	$2.90^{.16}$	$1.23^{.20}$	$0.65^{.26}$	$5.14^{.71}$	$2.50^{.98}$	$0.71^{.08}$	*0.25*
skST EGARCH	$4.99^{.98}$	$2.56^{.84}$	$1.23^{.20}$	$0.71^{.11}$	$5.14^{.71}$	$2.74^{.39}$	$0.80^{.24}$	**0.22**
skST APARCH	$5.02^{.96}$	$2.77^{.33}$	$1.26^{.15}$	$0.65^{.26}$	$5.33^{.40}$	$2.59^{.75}$	$0.89^{.53}$	*0.25*

Notes: Percentage of negative daily returns smaller than one-step-ahead VaR (left column, long positions) and the percentage of positive daily returns larger than one-step-ahead VaR (right column, short positions) corresponding to 95, 97.5, 99, and 99.5 percent confidence levels. The (percentage) values found significant at 1 percent (5 percent) level are shown in bold (italics). The p-values for the Kupiec LR test are shown explicitly (viz superscripts) only at the models estimated with skewed Student density. The results are from in-sample estimation.

Table 4: VaR Failure Rates (in %): Stock Indices (Out-of-Sample Results)

	(Long Positions)				(Short Positions)			
conf. level (%)	95.0	97.5	99.0	99.5	95.0	97.5	99.0	99.5
	ATX Index							
RiskMetrics	5.32	*3.49*	**1.90**	**1.27**	4.84	2.94	*1.67*	0.87
N GARCH	5.24	*3.65*	**1.90**	**1.27**	4.05	2.30	1.19	0.48
ST GARCH	5.32	*3.57*	1.27	*1.03*	4.37	2.22	0.56	*NA*
ST EGARCH	6.11	*3.41*	1.59	*0.95*	5.48	2.46	*0.40*	**0.08**
ST APARCH	6.11	*3.41*	1.19	*1.03*	5.16	2.30	*0.40*	*0.16*
skST GARCH	$4.76^{.70}$	$2.94^{.33}$	$1.19^{.51}$	*1.03*	$5.24^{.70}$	$2.70^{.66}$	$0.95^{.86}$	*0.16*
skST EGARCH	$5.24^{.70}$	$3.10^{.19}$	$1.19^{.51}$	$0.87^{.09}$	$6.03^{.10}$	$2.78^{.53}$	$0.71^{.28}$	$0.24^{.14}$
skST APARCH	$5.48^{.44}$	$3.02^{.26}$	$1.19^{.51}$	$0.87^{.09}$	$5.87^{.17}$	$2.62^{.79}$	$1.03^{.91}$	$0.24^{.14}$
	PX Index							
Riskmetrics	5.16	3.25	**2.06**	**1.67**	4.21	2.54	1.51	**1.03**
N GARCH	4.76	2.70	**1.83**	**1.59**	3.81	2.22	1.19	*0.95*
ST GARCH	5.00	2.70	1.59	0.87	4.37	2.06	0.95	0.48
ST EGARCH	5.08	2.54	1.59	0.79	4.29	1.75	0.95	0.24
ST APARCH	4.92	2.54	1.75	0.87	4.92	2.14	0.87	0.48
skST GARCH	$5.00^{1.0}$	$2.70^{.66}$	$1.59^{.05}$	*0.95*	$4.13^{.14}$	$1.98^{.22}$	$0.95^{.86}$	$0.40^{.59}$
skST EGARCH	$4.76^{.70}$	$2.54^{.93}$	$1.59^{.05}$	$0.87^{.09}$	$4.29^{.23}$	$1.91^{.16}$	$0.95^{.86}$	$0.24^{.14}$
skST APARCH	$5.08^{.90}$	$2.54^{.93}$	**1.90**	**1.11**	$4.76^{.70}$	$2.14^{.41}$	$0.79^{.45}$	$0.40^{.59}$
	BUX Index							
Riskmetrics	5.16	2.62	1.27	0.87	4.84	2.70	*1.67*	0.71
N GARCH	*3.65*	1.83	1.11	0.79	*3.73*	1.91	0.87	0.40
ST GARCH	4.76	1.75	0.71	0.24	4.52	1.83	*0.40*	0.24
ST EGARCH	4.21	*1.59*	0.71	0.24	*3.65*	1.75	*0.40*	0.24
ST APARCH	4.84	*1.51*	0.79	0.24	4.84	1.91	*0.40*	0.24
skST GARCH	$4.68^{.60}$	*1.67*	$0.71^{.28}$	$0.24^{.14}$	$4.84^{.79}$	$1.91^{.16}$	*0.40*	$0.24^{.14}$
skST EGARCH	$3.89^{.06}$	**1.43**	$0.71^{.28}$	$0.24^{.14}$	*3.81*	$1.98^{.22}$	*0.48*	$0.24^{.14}$
skST APARCH	$4.76^{.70}$	*1.51*	$0.79^{.44}$	$0.24^{.14}$	$4.52^{.43}$	$1.91^{.16}$	*0.48*	$0.24^{.14}$
	WIG Index							
Riskmetrics	4.68	2.86	*1.75*	**1.35**	*6.51*	**3.81**	1.35	0.48
N GARCH	**3.33**	1.98	1.35	*0.95*	4.68	2.46	0.56	**0.08**
ST GARCH	**3.49**	1.83	1.19	0.56	4.92	2.38	**0.08**	*NA*
ST EGARCH	*3.57*	1.75	1.27	0.79	4.84	2.62	**0.24**	*NA*
ST APARCH	*3.57*	1.83	1.19	0.63	4.92	2.62	**0.16**	*NA*
skST GARCH	*3.65*	$1.98^{.22}$	$1.19^{.51}$	$0.63^{.51}$	$4.84^{.79}$	$2.38^{.79}$	**0.08**	*NA*
skST EGARCH	*3.57*	$1.83^{.11}$	$1.27^{.36}$	$0.79^{.17}$	$4.52^{.43}$	$2.30^{.65}$	**0.16**	*NA*
skST APARCH	*3.65*	$1.90^{.16}$	$1.35^{.24}$	$0.71^{.31}$	$4.76^{.70}$	$2.22^{.52}$	**0.08**	*NA*

Notes: Percentage of negative daily returns smaller than one-step-ahead VaR (left column, long positions) and the percentage of positive daily returns larger than one-step-ahead VaR (right column, short positions) corresponding to 95, 97.5, 99, and 99.5 percent confidence levels. The (percentage) values found significant at 1 percent (5 percent) level are shown in bold (italics). The p-values for the Kupiec LR test are shown explicitly (viz superscripts) only at the models estimated with skewed Student density. The results are from out-of-sample estimation.

Table 5: VaR Failure Rates (in %): Selected Stocks (In-Sample Results)

	(Long Positions)				(Short Positions)			
conf. level (%)	95.0	97.5	99.0	99.5	95.0	97.5	99.0	99.5
	EBS							
Riskmetrics	5.86	**3.44**	**1.80**	**1.06**	5.36	**3.68**	**2.03**	**1.29**
N GARCH	5.31	2.85	1.41	0.78	5.08	2.93	**1.64**	**1.21**
ST GARCH	5.51	2.46	0.70	0.55	5.55	2.82	1.17	0.39
ST EGARCH	5.43	2.31	0.74	0.39	5.71	2.97	1.17	0.51
ST APARCH	5.32	2.38	0.74	0.39	5.83	2.82	1.10	0.51
skST GARCH	$5.79^{.07}$	$2.62^{.70}$	$0.78^{.25}$	$0.55^{.74}$	$5.37^{.41}$	$2.66^{.61}$	$0.98^{.91}$	$0.35^{.26}$
skST EGARCH	$5.79^{.07}$	$2.54^{.89}$	$0.86^{.47}$	$0.47^{.82}$	$5.51^{.24}$	$2.70^{.53}$	$1.02^{.93}$	$0.31^{.15}$
skST APARCH	$5.75^{.09}$	$2.54^{.89}$	$0.74^{.17}$	$0.51^{.95}$	$5.51^{.24}$	$2.66^{.61}$	$0.98^{.91}$	$0.35^{.26}$
	CEZ							
Riskmetrics	5.47	**3.58**	**2.19**	**1.62**	5.21	**3.42**	**1.89**	**1.26**
N GARCH	4.87	2.95	**1.76**	**1.29**	*4.15*	2.62	1.36	**0.90**
ST GARCH	5.50	2.92	1.19	0.53	4.91	2.35	0.73	*0.23*
ST EGARCH	5.27	2.82	1.19	0.43	4.81	2.32	0.86	*0.23*
ST APARCH	5.31	2.85	1.09	0.56	4.84	2.35	0.73	*0.27*
skST GARCH	$5.31^{.45}$	$2.72^{.45}$	$1.09^{.61}$	$0.50^{.98}$	$5.34^{.40}$	$2.42^{.78}$	$0.80^{.24}$	*0.23*
skST EGARCH	$5.11^{.79}$	$2.72^{.45}$	$1.13^{.49}$	$0.43^{.58}$	$5.07^{.85}$	$2.49^{.96}$	$1.00^{.98}$	*0.23*
skST APARCH	$5.21^{.61}$	$2.65^{.60}$	$0.99^{.98}$	$0.56^{.63}$	$5.04^{.92}$	$2.42^{.78}$	$0.76^{.17}$	$0.33^{.16}$
	OTP							
Riskmetrics	5.17	*3.14*	**1.77**	**1.30**	5.51	**3.37**	**1.84**	**1.14**
N GARCH	4.64	2.74	**1.67**	*0.83*	4.51	*3.10*	*1.44*	**0.90**
ST GARCH	4.97	2.54	0.87	0.50	5.01	2.80	1.00	0.37
ST EGARCH	5.04	2.37	0.80	0.47	5.21	2.77	1.07	0.43
ST APARCH	4.91	2.54	0.90	0.50	4.97	2.84	1.04	0.40
skST GARCH	$5.17^{.66}$	$2.70^{.48}$	$1.13^{.47}$	$0.50^{1.0}$	$4.71^{.46}$	$2.47^{.92}$	$0.90^{.58}$	$0.33^{.17}$
skST EGARCH	$5.21^{.61}$	$2.47^{.92}$	$1.00^{.99}$	$0.47^{.80}$	$4.74^{.51}$	$2.54^{.90}$	$0.94^{.72}$	$0.40^{.42}$
skST APARCH	$5.07^{.85}$	$2.60^{.72}$	$1.07^{.71}$	$0.50^{1.0}$	$4.77^{.57}$	$2.47^{.92}$	$1.04^{.85}$	$0.40^{.42}$
	PKN							
Riskmetrics	5.12	*3.26*	*1.48*	**1.05**	5.13	*3.26*	**1.77**	**1.29**
N GARCH	4.45	2.78	1.05	0.72	4.69	3.02	*1.49*	*0.86*
ST GARCH	4.65	2.54	0.77	0.53	4.89	2.97	1.15	0.48
ST EGARCH	4.60	2.54	0.91	0.29	5.27	3.02	1.15	0.58
ST APARCH	4.60	2.54	0.91	0.29	5.22	2.92	1.15	0.53
skST GARCH	$4.98^{.97}$	$2.78^{.42}$	$0.96^{.85}$	$0.62^{.44}$	$4.69^{.52}$	$2.87^{.29}$	$0.96^{.85}$	$0.38^{.43}$
skST EGARCH	$5.03^{.95}$	$2.68^{.60}$	$1.01^{.98}$	$0.43^{.65}$	$5.03^{.95}$	$2.78^{.42}$	$0.91^{.67}$	$0.43^{.65}$
skST APARCH	$5.12^{.79}$	$2.68^{.60}$	$1.05^{.81}$	$0.53^{.86}$	$4.93^{.89}$	$2.78^{.42}$	$0.96^{.85}$	$0.43^{.65}$

Notes: Percentage of negative daily returns smaller than one-step-ahead VaR (left column, long positions) and the percentage of positive daily returns larger than one-step-ahead VaR (right column, short positions) corresponding to 95, 97.5, 99, and 99.5 percent confidence levels. The (percentage) values found significant at 1 percent (5 percent) level are shown in bold (italics). The p-values for the Kupiec LR test are shown explicitly (viz superscripts) only at the models estimated with skewed Student density. The results are from in-sample estimation.

Table 6: VaR Failure Rates (in %): Selected Stocks (Out-of-Sample Results)

	(Long Positions)				(Short Positions)			
conf. level (%)	95.0	97.5	99.0	99.5	95.0	97.5	99.0	99.5
	EBS Stock							
Riskmetrics	6.05	3.37	1.59	*1.09*	5.36	3.73	**2.18**	**1.39**
N GARCH	4.46	2.68	1.49	**1.19**	5.46	2.58	*1.69*	0.89
ST GARCH	5.65	2.28	0.79	0.50	5.26	2.68	1.09	0.30
ST EGARCH	5.46	2.48	0.60	0.20	4.76	2.48	0.99	0.20
ST APARCH	5.06	2.48	0.60	0.30	5.36	2.28	1.29	0.30
skST GARCH	$6.25^{.08}$	$2.78^{.58}$	$0.79^{.49}$	$0.49^{.99}$	$4.86^{.84}$	$2.68^{.72}$	$0.89^{.73}$	$0.30^{.32}$
skST EGARCH	$5.85^{.23}$	$2.58^{.87}$	$0.60^{.16}$	$0.40^{.63}$	$4.27^{.27}$	$1.98^{.28}$	$0.60^{.16}$	*0.10*
skST APARCH	$5.85^{.23}$	$2.58^{.87}$	$0.69^{.30}$	$0.40^{.63}$	$4.56^{.51}$	$2.08^{.38}$	$0.79^{.49}$	$0.20^{.12}$
	CEZ Stock							
Riskmetrics	4.92	3.10	**1.83**	**1.43**	6.11	*3.65*	**2.14**	**1.19**
N GARCH	*3.73*	2.22	1.35	**1.27**	*3.73*	2.30	1.11	0.79
ST GARCH	4.37	1.90	0.95	0.40	4.29	1.75	0.64	**0.08**
ST EGARCH	*3.81*	1.90	0.95	0.24	4.68	1.75	0.56	*NA*
ST APARCH	4.05	1.83	0.79	0.32	4.21	*1.67*	0.56	**0.08**
skST GARCH	$4.29^{.23}$	$1.83^{.11}$	$0.95^{.86}$	$0.39^{.59}$	$5.08^{.90}$	$1.98^{.22}$	$0.64^{.16}$	**0.08**
skST EGARCH	*3.65*	$1.83^{.11}$	$0.95^{.86}$	$0.24^{.14}$	$4.92^{.90}$	$1.91^{.16}$	$0.71^{.28}$	**0.08**
skST APARCH	$3.97^{.08}$	$1.83^{.11}$	$0.79^{.45}$	$0.32^{.32}$	$4.52^{.43}$	*1.67*	$0.56^{.08}$	**0.08**
	OTP Stock							
Riskmetrics	5.71	3.25	*1.67*	**1.35**	5.87	2.94	1.51	0.87
N GARCH	4.13	2.06	1.11	0.32	*3.57*	2.14	0.87	0.48
ST GARCH	4.52	1.83	**0.32**	0.24	4.13	1.75	*0.48*	**0.08**
ST EGARCH	4.21	1.75	**0.32**	**0.08**	4.21	1.91	*0.48*	*NA*
ST APARCH	4.76	1.90	**0.32**	0.24	4.21	1.91	*0.48*	**0.08**
skST GARCH	$5.00^{1.0}$	$2.06^{.31}$	$0.63^{.16}$	$0.24^{.14}$	*3.73*	*1.59*	**0.24**	**0.08**
skST EGARCH	$4.60^{.51}$	$1.83^{.11}$	**0.32**	*0.16*	$3.89^{.06}$	*1.59*	*0.40*	*NA*
skST APARCH	$5.08^{.90}$	$1.98^{.22}$	*0.48*	$0.24^{.14}$	$3.97^{.08}$	*1.51*	*0.40*	**0.08**
	PKN Stock							
Riskmetrics	5.16	3.31	1.59	1.06	5.03	3.04	1.46	0.93
N GARCH	5.42	*3.70*	1.72	1.06	5.69	3.04	1.72	0.79
ST GARCH	5.82	3.17	1.32	0.79	6.09	3.04	1.32	0.40
ST EGARCH	5.69	3.31	1.46	0.79	5.95	3.04	1.19	0.40
ST APARCH	5.69	3.17	1.46	0.79	5.95	2.91	1.19	0.40
skST GARCH	$6.35^{.10}$	*3.70*	$1.59^{.13}$	$1.06^{.06}$	$5.56^{.49}$	$2.51^{.98}$	$0.79^{.55}$	$0.27^{.31}$
skST EGARCH	$6.35^{.10}$	$3.57^{.08}$	$1.59^{.13}$	$0.79^{.29}$	$5.29^{.72}$	$2.65^{.80}$	$0.93^{.84}$	$0.27^{.31}$
skST APARCH	$6.08^{.19}$	$3.44^{.12}$	$1.59^{.13}$	$0.79^{.29}$	$5.16^{.84}$	$2.65^{.80}$	$0.79^{.55}$	$0.27^{.31}$

Notes: Percentage of negative daily returns smaller than one-step-ahead VaR (left column, long positions) and the percentage of positive daily returns larger than one-step-ahead VaR (right column, short positions) corresponding to 95, 97.5, 99, and 99.5 percent confidence levels. The (percentage) values found significant at 1 percent (5 percent) level are shown in bold (italics). The p-values for the Kupiec LR test are shown explicitly (viz superscripts) only at the models estimated with skewed Student density. The results are from out-of-sample estimation.

Table 7: Sensitivity Analysis: Expected Shortfall: Indices (In-Sample)

	(Long Positions)				(Short Positions)			
conf. level (%)	95.0	97.5	99.0	99.5	95.0	97.5	99.0	99.5
	ATX Index							
Riskmetrics	-2.26	-2.60	-3.07	-3.38	1.98	2.34	2.41	2.66
N GARCH	-2.42	-2.84	-3.35	-3.74	2.15	2.34	2.75	2.92
ST GARCH	-2.39	-2.91	-3.69	-3.95	2.13	2.44	2.95	2.77
ST EGARCH	-2.42	-2.94	-3.66	-4.07	2.08	2.41	2.83	2.83
ST APARCH	-2.39	-2.90	-3.60	-3.96	2.06	2.44	2.84	2.80
skST GARCH	-2.49	3.05	-3.77	-3.99	2.06	2.35	2.76	2.78
skST EGARCH	-2.52	-3.01	-3.80	-4.09	1.99	2.35	2.70	2.87
skST APARCH	-2.53	-2.95	-3.80	-3.91	2.01	2.27	2.69	2.87
	PX Index							
Riskmetrics	-2.42	-2.76	-3.07	-3.30	2.33	2.46	2.86	3.10
N GARCH	-2.52	-2.90	-3.24	-3.53	2.41	2.74	3.21	3.52
ST GARCH	-2.43	-2.88	-3.43	-3.98	2.31	2.80	3.41	4.20
ST EGARCH	-2.46	-2.86	-3.49	-4.30	2.34	2.72	3.49	4.24
ST APARCH	-2.44	-2.88	-3.56	-4.25	2.33	2.70	2.30	4.01
skST GARCH	-2.46	-2.96	-3.50	-4.16	2.27	2.73	3.34	3.86
skST EGARCH	-2.48	-2.95	-3.53	-4.30	2.30	2.67	3.34	3.78
skST APARCH	-2.46	-2.90	-3.57	-4.29	2.30	2.60	3.14	4.09
	BUX Index							
Riskmetrics	-3.34	-4.08	-5.21	-5.51	3.22	3.65	4.18	4.46
N GARCH	-3.71	-4.38	-5.58	-5.91	3.35	3.91	4.55	4.41
ST GARCH	-3.55	-4.42	-6.30	-6.96	3.23	3.96	4.61	5.13
ST EGARCH	-3.67	-5.01	-6.47	-7.20	3.27	3.79	4.73	5.07
ST APARCH	-3.59	-4.64	-6.40	-7.06	3.25	3.88	4.41	4.96
skST GARCH	-3.60	-4.49	-6.52	-7.03	3.21	3.94	4.44	5.14
skST EGARCH	-3.71	-5.11	-6.47	-7.20	3.26	3.77	4.64	5.07
skST APARCH	-3.63	-4.75	-6.49	-7.06	3.24	3.85	4.35	4.96
	WIG Index							
Riskmetrics	-3.50	-3.86	-4.33	-4.70	3.35	3.77	4.21	4.24
N GARCH	-3.70	-4.09	-4.66	-4.98	3.60	4.02	4.56	4.52
ST GARCH	-3.67	-4.10	-5.00	-5.80	3.57	4.00	4.54	5.16
ST EGARCH	-3.73	-4.24	-5.02	-5.53	3.58	4.00	4.53	5.18
ST APARCH	-3.71	-4.16	-4.95	-5.68	3.55	4.02	4.53	5.16
skST GARCH	-3.64	-4.07	-5.00	-5.66	3.68	4.09	4.45	5.16
skST EGARCH	-3.69	-4.24	-5.02	-5.59	3.61	3.98	4.63	5.26
skST APARCH	-3.66	-4.12	-4.90	-5.77	3.56	4.07	4.60	5.16

Notes: Expected values of the losses greater than VaR for long (left column) and short (right column) positions for the ATX, PX, BUX, and WIG stock indices. The results are based on in-sample estimations.

Table 8: Sensitivity Analysis: Expected Shortfall: Indices (Out-of-Sample)

	(Long Positions)				(Short Positions)			
conf. level (%)	95.0	97.5	99.0	99.5	95.0	97.5	99.0	99.5
	ATX Index							
Riskmetrics	-2.44	-2.82	-3.43	-4.00	2.00	2.30	2.37	2.75
N GARCH	-2.59	-2.92	-3.51	-3.91	2.19	2.37	2.89	2.71
ST GARCH	-2.57	-2.99	-4.00	-4.20	2.14	2.43	3.09	*NA*
ST EGARCH	-2.38	-2.97	-3.69	-4.27	2.00	2.33	2.75	2.68
ST APARCH	-2.41	-2.98	-3.71	-4.21	2.05	2.39	2.79	2.59
skST GARCH	-2.68	-3.12	-4.03	-4.20	2.06	2.35	3.00	2.82
skST EGARCH	-2.58	-3.07	-4.14	-4.21	1.98	2.32	3.19	2.72
skST APARCH	-2.54	-3.06	-4.03	-4.21	1.98	2.31	2.83	2.72
	PX Index							
Riskmetrics	-2.49	-2.94	-3.38	-3.55	2.28	2.58	3.08	3.40
N GARCH	-2.59	-3.11	-3.46	-3.58	2.28	2.74	3.25	3.48
ST GARCH	-2.52	-3.07	-3.58	-3.83	2.16	2.67	3.48	3.80
ST EGARCH	-2.55	-3.17	-3.62	-3.99	2.24	2.80	3.48	4.94
ST APARCH	-2.55	-3.24	-3.63	-4.31	2.24	2.70	3.39	3.58
skST GARCH	-2.52	-3.07	-3.58	-3.77	2.20	2.78	3.48	4.26
skST EGARCH	-2.57	-3.17	-3.62	-3.95	2.27	2.82	3.48	4.94
skST APARCH	-2.52	-3.26	-3.51	-3.85	1.98	2.34	2.41	2.66
	BUX Index							
Riskmetrics	-2.55	-3.11	-3.71	-4.00	2.63	2.83	3.00	3.42
N GARCH	-2.79	-3.32	-3.81	-4.10	2.73	3.02	3.48	3.75
ST GARCH	-2.64	-3.39	-4.18	-4.59	2.66	3.11	3.75	4.15
ST EGARCH	-2.70	-3.51	-4.15	-5.29	2.71	3.07	3.75	4.15
ST APARCH	-2.63	-3.50	-4.16	-4.76	2.65	3.06	3.63	4.15
skST GARCH	-2.67	-3.45	-4.18	-4.59	2.65	3.13	3.75	4.15
skST EGARCH	-2.78	-3.56	-4.15	-5.29	2.68	3.00	3.50	4.15
skST APARCH	-2.64	-3.50	-4.16	-4.76	2.65	3.06	3.63	4.15
	WIG Index							
Riskmetrics	-2.97	-3.24	-3.70	-3.96	2.67	2.98	3.36	2.83
N GARCH	-3.42	-3.76	-4.09	-4.58	3.02	3.34	3.99	3.70
ST GARCH	-3.36	-3.81	-4.24	-5.06	2.97	3.36	3.70	*NA*
ST EGARCH	-3.29	-3.88	-4.16	-4.37	2.97	3.25	3.54	*NA*
ST APARCH	-3.33	-3.81	-4.26	-4.83	2.94	3.29	3.84	*NA*
skST GARCH	-3.31	-3.71	-4.24	-4.83	2.98	3.36	3.70	*NA*
skST EGARCH	-3.29	-3.85	-4.16	-4.37	3.01	3.25	3.32	*NA*
skST APARCH	-3.30	-3.79	-4.09	-4.70	2.97	3.38	3.70	*NA*

Notes: Expected values of the losses greater than VaR for long (left column) and short (right column) positions for the ATX, PX, BUX, and WIG stock indices. The results are based on out-of-sample estimations.

Table 9: Sensitivity Analysis: Expected Shortfall: Selected Stocks (In-Sample)

	(Long Positions)				(Short Positions)			
conf. level (%)	95.0	97.5	99.0	99.5	95.0	97.5	99.0	99.5
	EBS Stock							
Riskmetrics	-3.73	-4.17	-4.78	-5.44	3.80	4.13	4.59	4.84
N GARCH	-3.95	-4.67	-5.52	-6.36	3.99	4.53	5.03	5.06
ST GARCH	-3.91	-4.78	-6.50	-6.90	3.90	4.55	5.11	5.97
ST EGARCH	-3.90	-4.86	-6.51	-6.96	3.85	4.40	5.06	5.50
ST APARCH	-3.91	-4.84	-6.37	-6.63	3.80	3.39	4.90	5.40
skST GARCH	-3.84	-4.69	-6.24	-6.90	3.93	4.60	5.09	6.02
skST EGARCH	-3.83	-4.76	-6.32	-6.48	3.90	4.49	5.07	5.44
skST APARCH	-3.83	-4.75	-6.37	-6.95	3.80	4.56	5.01	5.77
	CEZ Stock							
Riskmetrics	-4.42	-4.91	-5.56	-6.02	4.41	4.97	5.72	6.26
N GARCH	-3.50	-3.81	-4.37	-4.76	3.45	3.83	4.52	4.94
ST GARCH	-4.39	-5.25	-6.37	-7.51	4.51	5.38	6.63	7.01
ST EGARCH	-4.52	-5.29	-6.40	-7.71	4.53	5.58	6.91	6.95
ST APARCH	-4.43	-5.28	-6.40	-7.33	4.53	5.44	6.43	7.42
skST GARCH	-4.43	-5.35	-6.45	-7.73	4.43	5.34	6.83	7.01
skST EGARCH	-4.57	-5.37	-6.47	-7.71	4.44	5.52	6.61	6.95
skST APARCH	-4.47	-5.45	-6.58	-7.33	4.48	5.41	6.60	6.87
	OTP Stock							
Riskmetrics	-4.81	-5.60	-6.66	-7.33	4.88	5.64	6.53	7.09
N GARCH	-5.07	-5.95	-6.93	-8.93	5.38	5.98	6.88	7.72
ST GARCH	-5.04	-6.14	-8.82	-10.3	5.24	6.20	7.55	9.57
ST EGARCH	-5.07	-6.22	-8.95	-10.4	5.12	6.15	7.39	9.66
ST APARCH	-5.06	-6.14	-8.74	-10.3	5.23	6.16	7.47	9.44
skST GARCH	-5.00	-5.99	-7.90	-10.3	5.34	6.39	7.72	9.77
skST EGARCH	-5.06	-6.21	-8.22	-10.4	5.25	6.26	7.60	9.57
skST APARCH	-5.00	-6.09	-8.20	-10.3	5.30	6.32	7.47	9.44
	PKN Stock							
Riskmetrics	-3.95	-4.35	-5.06	-5.36	4.12	4.66	4.87	4.97
N GARCH	-4.17	-4.67	-5.60	-6.14	4.38	4.83	5.30	5.71
ST GARCH	-4.13	-4.75	-6.06	-6.43	4.33	4.84	5.57	6.12
ST EGARCH	-4.14	-4.77	-5.76	-6.92	4.24	4.83	5.49	5.91
ST APARCH	-4.14	-4.77	-5.76	-6.92	4.25	4.87	5.49	5.74
skST GARCH	-4.06	-4.67	-5.71	-6.32	4.37	4.89	5.68	6.49
skST EGARCH	-4.03	-4.72	-5.65	-6.44	4.27	4.88	5.72	5.92
skST APARCH	-4.03	-4.72	-5.59	-6.28	4.31	4.90	5.66	5.92

Notes: Expected values of the losses greater than VaR for long (left column) and short (right column) positions for the EBS, CEZ, OTP, and PKN stocks. The results are based on in-sample estimations.

Table 10: Sensitivity Analysis: Expected Shortfall: Selected Stocks (Out-of-Sample)

	(Long Positions)				(Short Positions)			
conf. level (%)	95.0	97.5	99.0	99.5	95.0	97.5	99.0	99.5
	EBS Stock							
Riskmetrics	-3.63	-4.04	-4.46	-4.74	3.73	4.19	4.40	4.69
N GARCH	-3.87	-4.49	-4.92	-5.08	3.99	4.59	4.91	4.76
ST GARCH	-3.83	-4.60	-5.22	-5.60	3.82	4.59	4.82	6.35
ST EGARCH	-3.89	-4.58	-5.69	-6.09	3.93	4.41	4.86	6.88
ST APARCH	-3.92	-4.57	-5.69	-5.88	3.80	4.55	4.81	6.35
skST GARCH	-3.69	-4.39	-5.22	-5.60	3.89	4.59	5.01	6.35
skST EGARCH	-3.81	-4.56	-5.69	-5.44	3.98	4.62	5.24	5.05
skST APARCH	-3.72	-4.49	-5.39	-5.44	3.92	4.60	5.12	6.88
	CEZ Stock							
Riskmetrics	-3.91	-4.31	-5.07	-5.40	3.66	4.19	4.72	5.34
N GARCH	-4.21	-4.79	-5.45	-5.56	4.17	4.66	5.43	6.02
ST GARCH	-3.98	-4.98	-5.74	-6.21	3.88	4.69	6.41	6.32
ST EGARCH	-4.18	-4.92	-5.97	-5.98	3.90	4.84	6.63	*NA*
ST APARCH	-4.12	-4.94	-5.92	-5.91	3.92	4.96	5.66	6.32
skST GARCH	-3.94	-4.99	-5.74	-6.21	3.72	4.58	6.41	6.32
skST EGARCH	-4.20	-4.99	-5.97	-5.98	3.84	4.74	6.26	6.32
skST APARCH	-4.15	-4.94	-5.92	-5.91	3.85	4.96	5.66	6.32
	OTP Stock							
Riskmetrics	-3.89	-4.33	-4.95	-5.26	4.02	4.52	5.14	5.20
N GARCH	-4.26	-4.81	-5.31	-6.57	4.53	4.95	5.23	5.63
ST GARCH	-4.25	-4.98	-6.57	-6.99	4.42	5.13	5.63	7.53
ST EGARCH	-4.19	-4.89	-6.57	-6.87	4.31	4.95	5.63	*NA*
ST APARCH	-4.20	-4.95	-6.57	-6.99	4.39	5.09	5.63	7.53
skST GARCH	-4.16	-4.83	-5.81	-6.99	4.51	5.21	6.08	7.53
skST EGARCH	-4.08	-4.83	-6.57	-6.47	4.41	5.10	5.85	*NA*
skST APARCH	-4.16	-4.88	-6.13	-6.99	4.45	5.18	5.86	7.53
	PKN Stock							
Riskmetrics	-4.40	-4.90	-5.83	-6.37	4.27	4.75	5.00	5.22
N GARCH	-4.48	-4.97	-5.84	-6.56	4.26	4.83	5.38	6.08
ST GARCH	-4.39	-5.19	-6.18	-6.83	4.21	4.83	5.52	6.14
ST EGARCH	-4.42	-5.13	-6.02	-6.83	4.24	4.83	5.57	6.14
ST APARCH	-4.43	-5.19	-6.02	-6.83	4.21	4.88	5.57	6.14
skST GARCH	-4.26	-4.97	-5.92	-6.56	4.30	5.04	6.08	6.73
skST EGARCH	-4.28	-5.02	-5.88	-6.83	4.35	4.99	5.88	6.73
skST APARCH	-4.33	-5.06	-5.92	-6.83	4.37	4.99	6.08	6.73

Notes: Expected values of the losses greater than VaR for long (left column) and short (right column) positions for the EBS, CEZ, OTP, and PKN stocks. The results are based on out-of-sample estimations.

Table 11: VaR Failure Rates (in %): ATX (In-Sample)

ATX Index	(Long Positions)				(Short Positions)			
conf. level (%)	95.0	97.5	99.0	99.5	95.0	97.5	99.0	99.5
	EBS							
Riskmetrics	5.86	**3.44**	**1.80**	**1.06**	5.36	**3.68**	**2.03**	**1.29**
N GARCH	5.31	2.85	1.41	0.78	5.08	2.93	**1.64**	**1.21**
ST GARCH	5.51	2.46	0.70	0.55	5.55	2.82	1.17	0.39
ST EGARCH	5.43	2.31	0.74	0.39	5.71	2.97	1.17	0.51
ST APARCH	5.32	2.38	0.74	0.39	5.83	2.82	1.10	0.51
skST GARCH	5.79$^{.07}$	2.62$^{.70}$	0.78$^{.25}$	0.55$^{.74}$	5.37$^{.41}$	2.66$^{.61}$	0.98$^{.91}$	0.35$^{.26}$
skST EGARCH	5.79$^{.07}$	2.54$^{.89}$	0.86$^{.47}$	0.47$^{.82}$	5.51$^{.24}$	2.70$^{.53}$	1.02$^{.93}$	0.31$^{.15}$
skST APARCH	5.75$^{.09}$	2.54$^{.89}$	0.74$^{.17}$	0.51$^{.95}$	5.51$^{.24}$	2.66$^{.61}$	0.98$^{.91}$	0.35$^{.26}$
	OMV							
Riskmetrics	5.75	**3.35**	**1.96**	**1.37**	5.25	3.01	**1.52**	**1.09**
N GARCH	5.09	*3.23*	**1.80**	**1.27**	4.53	2.67	1.18	0.75
ST GARCH	5.53	2.76	1.15	0.65	4.69	2.45	0.84	0.34
ST EGARCH	5.43	2.80	1.12	0.53	5.00	2.27	0.68	0.37
ST APARCH	5.37	2.89	1.18	0.56	5.03	2.42	0.78	0.34
skST GARCH	5.43$^{.26}$	2.67$^{.54}$	1.12$^{.51}$	0.53$^{.82}$	4.85$^{.68}$	2.48$^{.95}$	0.84$^{.34}$	0.37$^{.28}$
skST EGARCH	5.19$^{.63}$	2.64$^{.61}$	1.02$^{.89}$	0.47$^{.78}$	5.37$^{.34}$	2.39$^{.69}$	0.81$^{.26}$	0.44$^{.59}$
skST APARCH	5.09$^{.81}$	2.61$^{.69}$	1.09$^{.62}$	0.47$^{.78}$	5.25$^{.52}$	2.52$^{.96}$	0.84$^{.34}$	0.37$^{.28}$
	TKA							
Riskmetrics	4.78	3.08	**1.81**	**1.37**	4.95	*3.46*	**1.92**	**1.43**
N GARCH	4.23	2.36	*1.59*	**1.04**	4.07	2.64	1.26	0.82
ST GARCH	4.78	2.47	1.10	0.55	5.39	2.69	0.77	0.39
ST EGARCH	4.18	2.14	0.93	0.44	5.39	2.80	0.88	0.39
ST APARCH	4.23	2.25	0.88	0.44	5.33	2.53	0.82	0.33
skST GARCH	4.89$^{.83}$	2.69$^{.60}$	1.10$^{.68}$	0.55$^{.77}$	5.06$^{.91}$	2.42$^{.82}$	0.60$^{.07}$	0.33$^{.30}$
skST EGARCH	4.73$^{.59}$	2.42$^{.82}$	1.04$^{.85}$	0.55$^{.77}$	4.95$^{.91}$	2.47$^{.94}$	0.77$^{.30}$	0.33$^{.27}$
skST APARCH	4.73$^{.59}$	2.58$^{.82}$	1.04$^{.85}$	0.49$^{.97}$	4.89$^{.83}$	2.42$^{.82}$	0.71$^{.20}$	0.33$^{.27}$

Notes: Percentage of negative daily returns smaller than one-step-ahead VaR (left column, long positions) and the percentage of positive daily returns larger than one-step-ahead VaR (left column, short positions) corresponding to 95, 97.5, 99, and 99.5 percent confidence levels. The (percentage) values found significant at 1 percent (5 percent) level are shown in bold (italics). The p-values for the Kupiec LR test are shown explicitly (viz superscripts) only at the models estimated with skewed Student density. The results are from in-sample estimation.

Table 12: VaR Failure Rates (in %): WIG Stocks (In-Sample)

WIG Index	(Long Positions)				(Short Positions)			
conf. level (%)	95.0	97.5	99.0	99.5	95.0	97.5	99.0	99.5
	PKN							
Riskmetrics	5.12	*3.26*	*1.48*	**1.05**	5.13	*3.26*	**1.77**	**1.29**
N GARCH	4.45	2.78	1.05	0.72	4.69	3.02	*1.49*	*0.86*
ST GARCH	4.65	2.54	0.77	0.53	4.89	2.97	1.15	0.48
ST EGARCH	4.60	2.54	0.91	0.29	5.27	3.02	1.15	0.58
ST APARCH	4.60	2.54	0.91	0.29	5.22	2.92	1.15	0.53
skST GARCH	4.98$^{.97}$	2.78$^{.42}$	0.96$^{.85}$	0.62$^{.44}$	4.69$^{.52}$	2.87$^{.29}$	0.96$^{.85}$	0.38$^{.43}$
skST EGARCH	5.03$^{.95}$	2.68$^{.60}$	1.01$^{.98}$	0.43$^{.65}$	5.03$^{.95}$	2.78$^{.42}$	0.91$^{.67}$	0.43$^{.65}$
skST APARCH	5.12$^{.79}$	2.68$^{.60}$	1.05$^{.81}$	0.53$^{.86}$	4.93$^{.89}$	2.78$^{.42}$	0.96$^{.85}$	0.43$^{.65}$
	PKO							
Riskmetrics	4.70	*3.19*	**1.88**	**1.15**	5.69	*3.35*	**1.68**	**1.06**
N GARCH	4.54	2.74	1.43	0.86	5.15	2.90	*1.43*	0.78
ST GARCH	4.62	2.58	0.90	0.45	5.77	2.86	0.78	0.41
ST EGARCH	4.66	2.58	0.90	0.49	5.52	2.95	0.86	0.45
ST APARCH	4.54	2.54	0.94	0.41	*5.89*	2.70	0.86	0.45
skST GARCH	4.79$^{.62}$	2.74$^{.45}$	1.02$^{.91}$	0.45$^{.72}$	5.40$^{.37}$	2.66$^{.62}$	0.78$^{.25}$	0.41$^{.51}$
skST EGARCH	4.95$^{.91}$	2.78$^{.38}$	0.94$^{.77}$	0.53$^{.83}$	5.15$^{.73}$	2.62$^{.71}$	0.78$^{.25}$	0.45$^{.72}$
skST APARCH	4.74$^{.56}$	2.78$^{.38}$	0.94$^{.77}$	0.45$^{.72}$	5.56$^{.21}$	2.58$^{.81}$	0.78$^{.25}$	0.41$^{.51}$
	TPS							
Riskmetrics	5.01	2.61	*1.45*	**1.07**	5.73	**3.68**	**2.01**	**1.24**
N GARCH	4.54	2.40	1.28	0.86	5.61	*3.34*	*1.54*	**0.94**
ST GARCH	4.62	2.14	0.11	0.26	5.95	3.25	*1.24*	*0.47*
ST EGARCH	4.32	2.27	0.94	0.26	*5.95*	*3.25*	1.28	0.51
ST APARCH	4.49	2.10	1.11	0.30	5.91	*3.21*	1.24	0.56
skST GARCH	5.39$^{.39}$	2.52$^{.94}$	1.28$^{.19}$	0.64$^{.35}$	5.18$^{.70}$	2.70$^{.55}$	0.98$^{.94}$	0.34$^{.25}$
skST EGARCH	5.35$^{.44}$	2.40$^{.75}$	1.24$^{.26}$	0.68$^{.23}$	5.22$^{.63}$	2.53$^{.94}$	0.90$^{.62}$	0.34$^{.25}$
skST APARCH	5.26$^{.56}$	2.57$^{.84}$	1.33$^{.13}$	0.68$^{.23}$	5.19$^{.70}$	2.87$^{.27}$	0.90$^{.62}$	0.3$^{.14}$

Notes: Percentage of negative daily returns smaller than one-step-ahead VaR (left column, long positions) and the percentage of positive daily returns larger than one-step-ahead VaR (left column, short positions) corresponding to 95, 97.5, 99, and 99.5 percent confidence levels. The (percentage) values found significant at 1 percent (5 percent) level are shown in bold (italics). The p-values for the Kupiec LR test are shown explicitly (viz superscripts) only at the models estimated with skewed Student density. The results are from in-sample estimation.

Table 13: VaR Failure Rates (in %): BUX Stocks (In-Sample)

BUX Index	(Long Positions)				(Short Positions)			
conf. level (%)	95.0	97.5	99.0	99.5	95.0	97.5	99.0	99.5
	MOL							
Riskmetrics	4.94	*3.10*	**1.74**	**0.93**	5.47	**3.44**	**2.07**	**1.34**
N GARCH	4.14	2.17	1.17	0.93	4.74	*3.14*	**1.57**	**1.17**
ST GARCH	4.84	2.14	0.90	0.50	5.54	2.97	1.24	0.53
ST EGARCH	4.61	2.24	0.80	0.50	5.07	2.90	1.14	0.43
ST APARCH	4.67	2.14	0.87	0.47	5.47	2.84	1.27	0.53
skST GARCH	5.14$^{.73}$	2.34$^{.56}$	0.97$^{.86}$	0.57$^{.61}$	5.17$^{.66}$	2.70$^{.48}$	1.07$^{.71}$	0.47$^{.80}$
skST EGARCH	4.97$^{.95}$	2.47$^{.92}$	0.93$^{.72}$	0.67$^{.22}$	4.91$^{.81}$	2.57$^{.81}$	1.14$^{.47}$	0.3$^{.09}$
skST APARCH	5.01$^{.99}$	2.44$^{.82}$	0.93$^{.72}$	0.57$^{.61}$	5.24$^{.55}$	2.67$^{.55}$	1.14$^{.47}$	0.43$^{.60}$
	MTE							
Riskmetrics	4.81	2.91	**1.90**	**1.40**	5.20	3.10	*1.44*	**0.93**
N GARCH	4.27	2.25	*1.51*	**1.01**	4.58	2.40	1.20	0.78
ST GARCH	4.77	2.09	1.05	*0.81*	5.08	2.25	0.85	0.31
ST EGARCH	4.46	2.09	1.05	*0.81*	4.96	2.33	0.97	0.43
ST APARCH	4.46	2.13	1.09	*0.81*	5.08	2.25	0.85	0.43
skST GARCH	4.69$^{.47}$	2.06$^{.14}$	1.05$^{.81}$	*0.81*	5.12$^{.78}$	2.29$^{.48}$	0.89$^{.57}$	0.31$^{.14}$
skST EGARCH	4.38$^{.14}$	2.09$^{.17}$	1.05$^{.81}$	*0.81*	4.96$^{.93}$	3.27$^{.66}$	0.89$^{.57}$	0.47$^{.80}$
skST APARCH	4.46$^{.20}$	2.13$^{.22}$	1.09$^{.67}$	*0.81*	5.08$^{.85}$	2.25$^{.41}$	0.85$^{.44}$	0.43$^{.59}$
	OTP							
Riskmetrics	5.17	*3.14*	**1.77**	**1.30**	5.51	**3.37**	**1.84**	**1.14**
N GARCH	4.64	2.74	**1.67**	*0.83*	4.51	*3.10*	*1.44*	**0.90**
ST GARCH	4.97	2.54	0.87	0.50	5.01	2.80	1.00	0.37
ST EGARCH	5.04	2.37	0.80	0.47	5.21	2.77	1.07	0.43
ST APARCH	4.91	2.54	0.90	0.50	4.97	2.84	1.04	0.40
skST GARCH	5.17$^{.66}$	2.70$^{.48}$	1.13$^{.47}$	0.50$^{1.0}$	4.71$^{.46}$	2.47$^{.92}$	0.90$^{.58}$	0.33$^{.17}$
skST EGARCH	5.21$^{.61}$	2.47$^{.92}$	1.00$^{.99}$	0.47$^{.80}$	4.74$^{.51}$	2.54$^{.90}$	0.94$^{.72}$	0.40$^{.42}$
skST APARCH	5.07$^{.85}$	2.60$^{.72}$	1.07$^{.71}$	0.50$^{1.0}$	4.77$^{.57}$	2.47$^{.92}$	1.04$^{.85}$	0.40$^{.42}$
	RCH							
Riskmetrics	5.17	2.97	**2.14**	**1.50**	5.14	**3.37**	**1.77**	**1.24**
N GARCH	4.41	2.94	**1.67**	**1.13**	4.84	2.70	*1.47*	**0.97**
ST GARCH	4.74	2.74	1.10	0.40	5.27	2.74	0.94	*0.27*
ST EGARCH	4.81	2.67	1.00	0.50	5.37	2.60	0.90	0.40
ST APARCH	4.64	2.74	1.00	0.43	5.17	2.77	0.97	0.40
skST GARCH	4.97$^{.95}$	2.77$^{.35}$	1.13$^{.47}$	0.40$^{.42}$	5.14$^{.73}$	2.60$^{.72}$	0.87$^{.46}$	*0.27*
skST EGARCH	4.84$^{.69}$	2.70$^{.48}$	1.00$^{.99}$	0.50$^{1.0}$	5.41$^{.31}$	2.64$^{.63}$	0.87$^{.46}$	0.37$^{.28}$
skST APARCH	4.77$^{.57}$	2.84$^{.25}$	1.03$^{.85}$	0.47$^{.80}$	5.11$^{.79}$	2.74$^{.41}$	0.94$^{.72}$	0.40$^{.42}$

Notes: Percentage of negative daily returns smaller than one-step-ahead VaR (left column, long positions) and the percentage of positive daily returns larger than one-step-ahead VaR (left column, short positions) corresponding to 95, 97.5, 99, and 99.5 percent confidence levels. The (percentage) values found significant at 1 percent (5 percent) level are shown in bold (italics). The p-values for the Kupiec LR test are shown explicitly (viz superscripts) only at the models estimated with skewed Student density. The results are from in-sample estimation.

Table 14: VaR Failure Rates (in %): PX Stocks (In-Sample)

PX Index	(Long Positions)				(Short Positions)			
conf. level (%)	95.0	97.5	99.0	99.5	95.0	97.5	99.0	99.5
	CEZ							
Riskmetrics	5.47	**3.58**	**2.19**	**1.62**	5.21	**3.42**	**1.89**	**1.26**
N GARCH	4.87	2.95	**1.76**	**1.29**	*4.15*	2.62	1.36	**0.90**
ST GARCH	5.50	2.92	1.19	0.53	4.91	2.35	0.73	*0.23*
ST EGARCH	5.27	2.82	1.19	0.43	4.81	2.32	0.86	*0.23*
ST APARCH	5.31	2.85	1.09	0.56	4.84	2.35	0.73	*0.27*
skST GARCH	5.31$^{.45}$	2.72$^{.45}$	1.09$^{.61}$	0.50$^{.98}$	5.34$^{.40}$	2.42$^{.78}$	0.80$^{.24}$	*0.23*
skST EGARCH	5.11$^{.79}$	2.72$^{.45}$	1.13$^{.49}$	0.43$^{.58}$	5.07$^{.85}$	2.49$^{.96}$	1.00$^{.98}$	*0.23*
skST APARCH	5.21$^{.61}$	2.65$^{.60}$	0.99$^{.98}$	0.56$^{.63}$	5.04$^{.92}$	2.42$^{.78}$	0.76$^{.17}$	0.33$^{.16}$
	ERS							
Riskmetrics	5.79	*3.62*	**2.24**	**1.37**	4.92	3.18	**2.10**	**1.30**
N GARCH	5.07	*3.55*	**1.88**	**1.09**	4.49	2.68	1.45	**1.09**
ST GARCH	5.35	3.18	0.87	0.58	5.07	2.46	1.01	0.65
ST EGARCH	5.35	3.18	1.16	0.29	4.85	2.68	1.01	0.72
ST APARCH	5.07	3.18	1.01	0.43	5.07	2.46	0.94	0.58
skST GARCH	5.21$^{.72}$	3.04$^{.21}$	0.80$^{.43}$	0.51$^{.97}$	5.07$^{.92}$	2.61$^{.80}$	1.23$^{.41}$	0.65$^{.45}$
skST EGARCH	5.35$^{.55}$	3.04$^{.21}$	0.94$^{.82}$	0.22$^{.09}$	5.07$^{.91}$	2.68$^{.68}$	1.09$^{.75}$	0.80$^{.15}$
skST APARCH	4.99$^{.99}$	2.89$^{.36}$	1.01$^{.96}$	0.43$^{.72}$	5.07$^{.91}$	2.61$^{.80}$	1.01$^{.96}$	0.58$^{.69}$
	KOB							
Riskmetrics	5.11	**3.45**	**2.19**	**1.62**	4.68	3.05	**1.92**	**1.26**
N GARCH	4.91	3.02	**1.69**	**1.26**	**3.75**	2.59	*1.49*	*0.83*
ST GARCH	5.27	2.95	1.13	0.66	4.48	2.35	0.70	0.37
ST EGARCH	5.27	2.75	1.13	0.56	4.87	2.32	0.86	0.37
ST APARCH	5.24	2.82	1.09	0.60	4.81	2.42	0.80	0.40
skST GARCH	5.21$^{.61}$	2.69$^{.52}$	1.03$^{.88}$	0.60$^{.46}$	4.84$^{.69}$	2.62$^{.68}$	0.86$^{.44}$	0.37$^{.29}$
skST EGARCH	5.11$^{.79}$	2.69$^{.52}$	0.90$^{.56}$	0.50$^{.98}$	5.14$^{.73}$	2.59$^{.76}$	1.06$^{.74}$	0.43$^{.58}$
skST APARCH	5.14$^{.73}$	2.65$^{.60}$	0.99$^{.98}$	0.50$^{.98}$	5.11$^{.79}$	2.62$^{.67}$	0.90$^{.56}$	0.40$^{.41}$
	TEF							
Riskmetrics	4.93	**3.35**	**1.86**	**1.25**	4.61	2.79	**1.62**	**1.17**
N GARCH	4.16	2.38	1.33	1.05	**3.44**	2.10	*1.46*	*0.85*
ST GARCH	4.89	2.22	0.93	0.32	5.09	2.26	0.93	0.49
ST EGARCH	**3.60**	**1.21**	**0.36**	*0.20*	**3.84**	**1.29**	**0.45**	*0.24*
ST APARCH	4.93	2.18	0.89	0.40	4.89	2.39	1.05	0.69
skST GARCH	4.89$^{.80}$	2.22$^{.37}$	0.93$^{.72}$	0.32$^{.18}$	5.09$^{.83}$	2.26$^{.44}$	0.93$^{.72}$	0.49$^{.92}$
skST EGARCH	4.73$^{.53}$	2.38$^{.71}$	0.97$^{.88}$	0.44$^{.69}$	4.81$^{.66}$	2.26$^{.44}$	0.97$^{.88}$	0.53$^{.86}$
skST APARCH	5.13$^{.76}$	2.22$^{.37}$	0.97$^{.88}$	0.40$^{.48}$	4.81$^{.66}$	2.34$^{.62}$	1.05$^{.80}$	0.69$^{.21}$

Notes: Percentage of negative daily returns smaller than one-step-ahead VaR (left column, long positions) and the percentage of positive daily returns larger than one-step-ahead VaR (left column, short positions) corresponding to 95, 97.5, 99, and 99.5 percent confidence levels. The (percentage) values found significant at 1 percent (5 percent) level are shown in bold (italics). The p-values for the Kupiec LR test are shown explicitly (viz superscripts) only at the models estimated with skewed Student density. The results are from in-sample estimation.

Table 15: VaR Failure Rates (H_0 view): Indices and Selected Stocks (In-Sample)

Stock Indices	(Long Positions)				(Short Positions)			
	ATX	PX	BUX	WIG	ATX	PX	BUX	WIG
RiskMetrics	0	25	50	25	75	50	50	25
N GARCH	25	50	75	50	50	75	75	75
ST GARCH	50	100	100	100	0	50	100	75
ST EGARCH	75	100	100	100	50	50	100	75
ST APARCH	50	100	100	100	50	75	100	75
skST GARCH	75	100	100	100	100	100	100	75
skST EGARCH	100	100	100	100	100	100	100	75
skST APARCH	100	100	100	100	100	75	100	75

Stocks	(Long Positions)				(Short Positions)			
	EBS	CEZ	OTP	PKN	EBS	CEZ	OTP	PKN
RiskMetrics	25	25	25	25	25	25	50	25
N GARCH	100	50	50	100	50	50	25	50
ST GARCH	100	100	100	100	100	75	100	100
ST EGARCH	100	100	100	100	100	75	100	100
ST APARCH	100	100	100	100	100	75	100	100
skST GARCH	100	100	100	100	100	75	100	100
skST EGARCH	100	100	100	100	100	75	100	100
skST APARCH	100	100	100	100	100	100	100	100

Notes: Number of times out of 100 that the p-value of the Kupiec LR test is smaller than five percent [that is, the null hypothesis that the failure rate is equal to (1 - appropriate confidence level) cannot be rejected], for the combined four possible values of the confidence levels used in the analysis. The results for the ATX, PX, BUX, and WIG stock indices (top) and EBS, CEZ, OTP, and PKN stocks (bottom) are based on in-sample estimations.

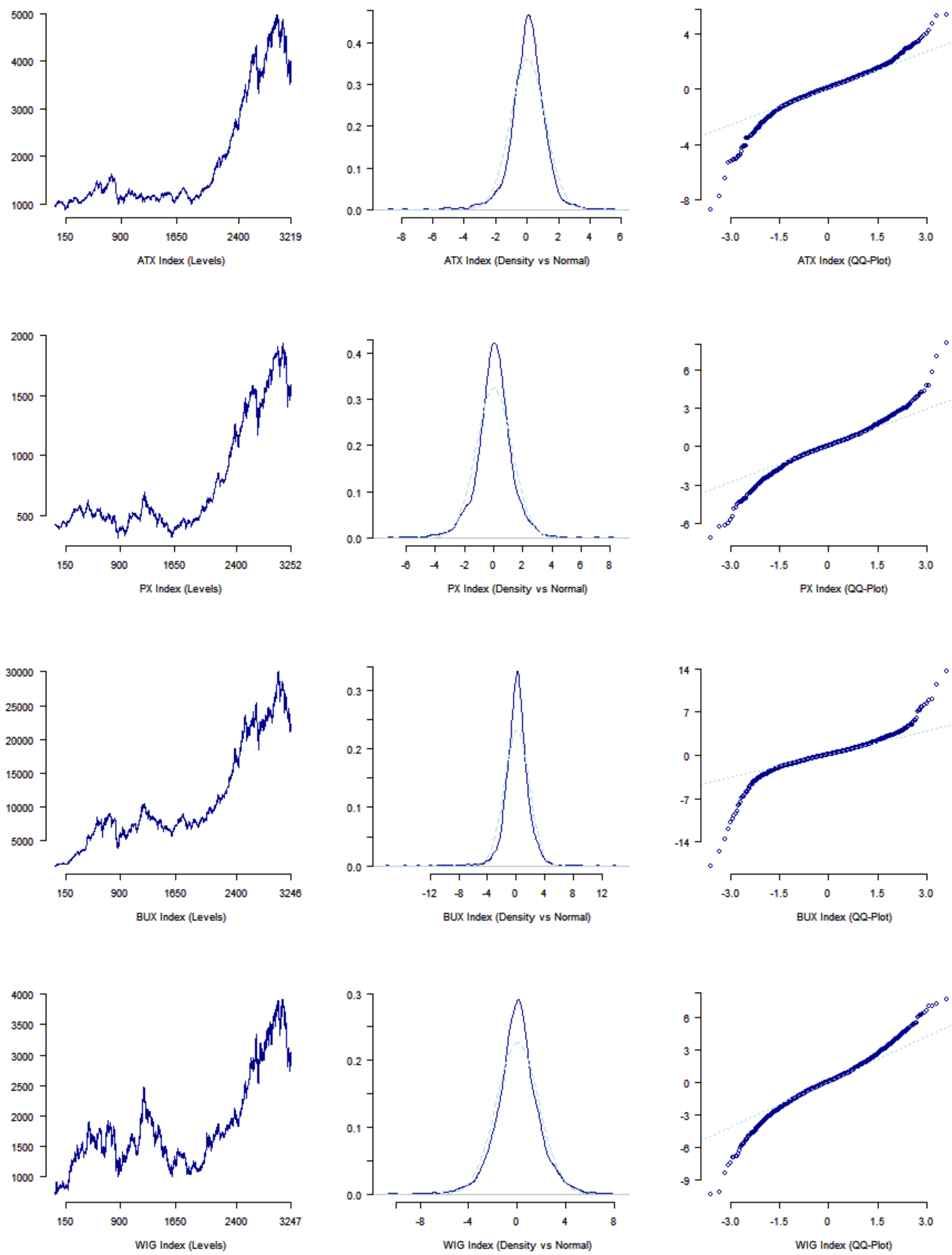
Table 16: VaR Failure Rates (H_0 view): Indices and Selected Stocks (Out-of-Sample)

Stock Indices	(Long Positions)				(Short Positions)			
	ATX	PX	BUX	WIG	ATX	PX	BUX	WIG
RiskMetrics	25	50	100	50	75	75	75	50
N GARCH	25	50	75	50	100	75	75	75
ST GARCH	50	100	100	75	75	100	75	50
ST EGARCH	50	100	75	75	50	100	50	50
ST APARCH	50	100	75	75	50	100	75	50
skST GARCH	75	75	75	75	75	100	75	50
skST EGARCH	100	100	75	75	100	100	50	50
skST APARCH	100	50	75	75	100	100	75	50

Stocks	(Long Positions)				(Short Positions)			
	EBS	CEZ	OTP	PKN	EBS	CEZ	OTP	PKN
RiskMetrics	25	25	25	25	25	25	25	25
N GARCH	100	50	50	100	50	50	25	50
ST GARCH	100	100	100	100	100	75	100	100
ST EGARCH	100	100	100	100	100	75	100	100
ST APARCH	100	100	100	100	100	75	100	100
skST GARCH	100	100	100	100	100	75	100	100
skST EGARCH	100	100	100	100	100	75	100	100
skST APARCH	100	100	100	100	100	75	100	100

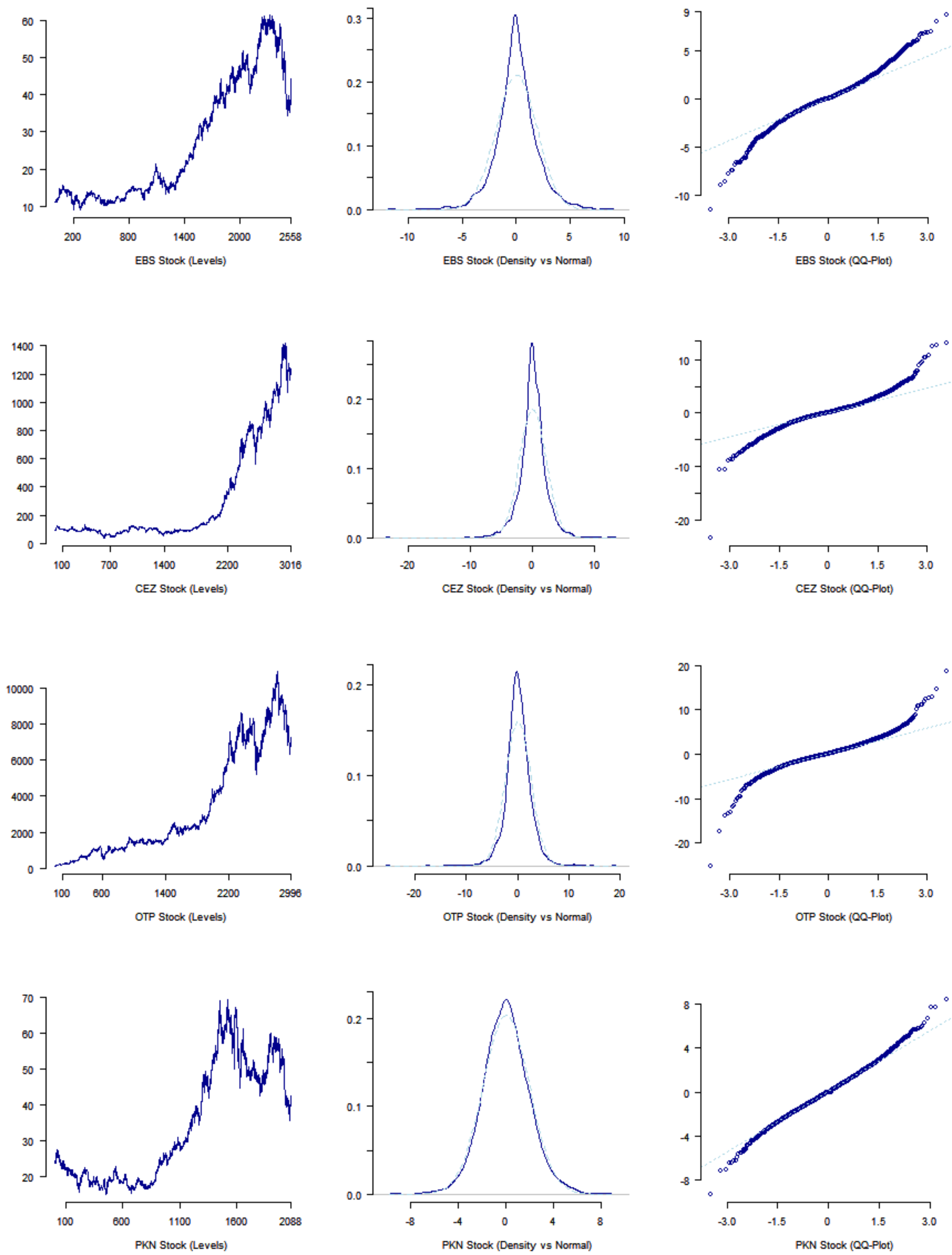
Notes: Number of times out of 100 that the p-value of the Kupiec LR test is smaller than five percent [that is, the null hypothesis that the failure rate is equal to (1 - appropriate confidence level) cannot be rejected], for the combined four possible values of the confidence levels used in the analysis. The results for the ATX, PX, BUX, and WIG stock indices (top) and EBS, CEZ, OTP, and PKN stocks (bottom) are based on out-of-sample estimations.

Figure 1. ATX, PX, BUX, and WIG Stock Indices



Note: Density estimates for ATX, PX, BUX and WIG stock indices. The normal distribution is shown with a dashed line. The densities are estimated using non-parametric density estimators due to Scott (1985) with h = 0.35.

Figure 2. EBS, CEZ, OTP, and PK Stock Returns



Note: Density estimates for EBS, CEZ, OTP and PKN stock indices. The normal distribution is shown with a dashed line. The densities are estimated using non-parametric density estimators due to Scott (1985) with h = 0.35.

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