Frictions in the Interbank Market and Uncertain Liquidity Needs: Implications for Monetary Policy Implementation

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Abstract

This paper shows that depending on the distribution of banks' uncertain liquidity needs and on how monetary policy is implemented, frictions in the interbank market may reinforce the effectiveness of monetary policy. These frictions imply that with its lending and deposit facilities the central bank has an additional effective instrument at hand to impose an impact on bank loan supply. While lowering the rate on the lending facility has, taken for itself, an expansionary effect, lowering the rate on the deposit facility has a contractionary effect. This result has interesting implications for monetary policy implementation at the zero lower bound.

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1 Introduction

The interbank market for overnight loans is important for monetary policy implementation. By steering the interest rate in this market, the central bank aims to influence short-term nominal interest rates, and thereby, through various channels, the price level and maybe aggregate output. In the euro area, the interest rate channel and the credit channel play an important role.[1] Both transmission mechanisms rest on the central bank's ability to influence bank lending.

During the recent financial crisis, euro area interbank markets seized up. This led to concerns about the Eurosystem's ability, or the lack thereof, to actually control bank lending in times of malfunctioning interbank markets, and it triggered a heated debate of whether the transmission mechanism of monetary policy might be impaired. Our paper aims to contribute to this debate by studying in how far frictions in the interbank market for overnight loans influence the impact of monetary policy on bank loan supply and by discussing the implications for monetary policy implementation.

We develop a theoretical model that has two central features. First, it accounts for interbank market frictions. Second, it captures main elements of the Eurosystem's[2] operational framework. Frictions in the interbank market emerge in the form of transaction costs. We broadly interpret these transaction costs as search costs. Banks must find suitable transaction partners with first, matching liquidity needs and second, a willingness to conclude mutual agreements for trade. The former may be costly as, for example, banks have to split large transactions into small ones to work around credit lines (Bartolini, Bertola, and Prati, 2001). The latter may be costly because lenders in the overnight interbank market are typically unwilling to expose themselves to any counterparty credit risk (Hauck and Neyer, 2013). Consequently, they engage in costly checks of the creditworthiness of potential borrowers who in turn must provide costly signals of their creditworthiness.[3] The main elements of the Eurosystem's operational framework captured by

[1] For respective empirical analyses for the interest rate channel see, for example, Čihák, Harjes, and Stavrev (2009); Angeloni, Kashyap, Mojon, and Terlizzese (2003). In an empirical analysis referring, for example, to the Spanish credit market, Jiménez, Ongena, Peydró, and Saurina (2012) confirm a high relevance of the credit channel.

[2] The term "Eurosystem" stands for the institution which is responsible for monetary policy in the euro area, namely the ECB and the national central banks in the euro area. For the sake of simplicity, the terms "ECB" and "Eurosystem" are used interchangeably throughout this paper.

[3] One of the first papers dealing explicitly with interbank market transaction costs is the one by Bartolini, Bertola, and Prati (2001). They argue that interbank market transaction costs are responsible for the relatively high federal funds rate usually observed at the end of a reserve maintenance period. Transaction costs also play a crucial role in Hauck and Neyer (2013). They argue that transaction costs, or participation

our model are the main refinancing operations and the two standing facilities. The main refinancing operations are credit operations with a maturity of one week by which the Eurosystem provides reserves to the banking sector. The two standing facilities, a deposit facility and a lending facility, allow banks to balance their overnight liquidity needs. The interest rates on the facilities form a corridor around the rate on the main refinancing operations with the rate on the deposit facility to be lower and the rate on the lending facility to be higher than the main policy rate.[4] It is worth mentioning that although our model focuses on the Eurosystem's operational framework, our results apply to other operational frameworks as well, as long as they allow commercial banks to balance uncertain liquidity needs by using a deposit facility and a lending facility offered by the central bank.

The results of our model replicate several stylized facts and imply interesting implications for monetary policy implementation. The replicated stylized facts observed before and during the recent financial crisis are:

- If there are no interbank market frictions, the interbank market rate will equal the main policy rate, reserves provided by the central bank to the banking sector through its main refinancing operations will correspond to the benchmark allotment,[5] and the standing facilities will not be used. This is exactly what was observed in the euro area before the outbreak of the financial crisis in 2007.

- Introducing interbank market frictions in our model leads to results which are broadly consistent with the stylized facts observed during the financial crisis. The interbank market rate falls below the main policy rate, reserves provided by the central bank exceed the benchmark allotment and the standing facilities are used whereas the use of the deposit facility outweighs by far the use of the lending facility.

From our model results the following implications for monetary policy implementation can be drawn:

costs, can explain several stylized facts observed in the euro area interbank market during the financial crisis. Models explicitly considering a costly search process in the interbank market can be found for example in Furfine (2004) and Ashcraft and Duffie (2007). Furfine analyzes the effectiveness of standing facilities offered by a central bank at reducing the volatility of the overnight interbank rate. Ashcraft and Duffie show how the search process in a decentralized interbank market influences intraday allocation and the pricing of federal funds.

[4] For a detailed description of the Eurosystem's operational framework see European Central Bank (2012).

[5] The Eurosystem's benchmark allotment is generally understood as the allotment in a main refinancing operation that will allow the banks to smoothly fulfil their reserve requirements taking into account future liquidity needs from reserve requirements and autonomous factors. For details see European Central Bank (2014).

- Irrespectively of interbank market frictions, the central bank can influence banks' expected funding costs, and therefore bank loan supply, by changing its main policy rate even if the frictions imply a total interbank market freeze.

- If interbank market frictions are sufficiently high, they will reinforce the effect of a sole change of the main policy rate on bank loan supply. The reinforcing effect increases in the extent of uncertainty about banks' actual liquidity needs.

- The reinforcing effect will be avoided if the central bank changes all its interest rates (rate on its main refinancing operations and on its standing facilities) to the same extent. Obviously, this may not be possible if the rate on the deposit facility is fixed at the zero lower bound.

- If interbank market frictions are sufficiently high, the central bank can use the rates on the facilities as an additional effective monetary policy instrument by changing the width or the asymmetry of the interest rate corridor. A decrease of the rate on the deposit facility corresponds to a contractionary monetary policy. Consequently, using this instrument for conducting a contractionary monetary policy will not be possible if the zero lower bound becomes binding.

To illustrate the main idea behind the implications of our model results, let us assume that the central bank conducts an expansionary monetary policy by lowering solely the rate on its main refinancing operations. Then, borrowing reserves from the main refinancing operations becomes cheaper which implies that also the price for reserves in the interbank market, the interbank rate, decreases. This means that banks' expected marginal funding costs decline which has a positive impact on their loan supply. However, frictions in the form of transaction costs in the interbank market imply that the interbank rate deviates from the central bank's main policy rate. If these costs are that high, that the interbank rate will be already at its lower bound, which is determined by the rate on the central bank's deposit facility, the above described price mechanism will not work anymore. Therefore, borrowing reserves from the central bank's refinancing operations remains to be relatively cheaper as compared with an interbank market loan. As a consequence, banks increase their borrowing from the refinancing operations. The price effect (lower interbank rate) is replaced by a quantity effect (increased borrowing from the refinancing operations). This quantity effect implies that the expansionary effect of the initial monetary policy impulse is reinforced as the sensitivity of banks' funding costs to the monetary policy impulse is higher. The central bank can steer the extent of the reinforcing effect by changing the rate on its deposit facility.

This rest of this paper is organized as follows. Section 2 presents related literature. Section 3 describes the framework of the model. Sections 4 and 5 derive the optimal behavior of commercial banks. Section 6 discusses the equilibrium of the model. Taking a closer look at this equilibrium in Section 7, we analyze the impact of monetary policy on bank loan supply and discuss the consequences for monetary policy implementation. Section 8 briefly summarizes the paper.

2 Related Literature

Our paper contributes to three strands of literature. The first strand focuses on the influence of monetary policy on bank lending. A huge part of this literature considers asymmetric information in credit markets and argues that these frictions amplify the effects of monetary policy on bank lending and, therefore, on aggregate demand.[6] This credit view of monetary policy can be divided into the balance sheet channel and the bank lending channel. With respect to the balance sheet channel, the crucial point is that a monetary policy impulse changes the borrowers' net worth. A contractionary monetary policy decreases the borrowers' net worth which implies an increase in adverse selection and moral hazard problems leading to a decline in bank lending. Seminal papers are those by Bernanke and Gertler (1989) and Bernanke, Gertler, and Gilchrist (1999). With respect to the bank lending channel, the driving force is that monetary policy has an impact on bank deposits. A contractionary monetary policy reduces bank deposits implying a decline in bank loan supply. Important papers dealing with this traditional bank lending channel are, for example, those by Gertler and Gilchrist (1993) and Kashyap and Stein (1995). This traditional approach of the bank lending channel has been criticized as it neglects, for example, that banks can replace deposits by market-based funding. However, Disyatat (2011) shows that a greater reliance on market-based funding creates a new approach of the bank lending channel. Crucial is that a stronger reliance on market-based funding increases the sensitivity of banks' funding costs to monetary policy as the banks' health, in terms of leverage, asset quality and in perception of risk, becomes more important.

[6] For a short survey on this so called credit view of monetary policy see, for example, Boivin, Kiley, and Mishkin (2010) and Peek and Rosengreen (2012).

The second strand of literature deals with frictions in the interbank market. Until the outbreak of the financial crisis in 2007, the interbank market was typically regarded as frictionless, also in the theoretical literature. As a consequence, the interbank rate was assumed to be identical with the monetary policy rate or the interbank market was entirely neglected. The financial crisis challenged this view and inspired a growing literature dealing with interbank market imperfections, primarily focusing on asymmetric information about credit risks. Freixas and Jorge (2008) consider the impact of this interbank market friction for the transmission of monetary policy. They show that private information in the interbank market with respect to credit risks may induce rationing of firms in credit markets. With respect to the transmission mechanism of monetary policy this implies that asymmetric information in the interbank market may be responsible for a) a magnitude effect, i.e. the aggregate impact of monetary policy may be large given the small interest elasticity of investment, and b) a liquidity effect, i.e. that the impact of monetary policy is stronger for banks with less liquid balance sheets. Heider, Hoerova, and Holthausen (2009) argue that banks' informational disadvantage with respect to counterparty credit risks induces them to hold more liquidity. Depending on the risk dispersion, this may result in either adverse selection or a dry-up of the interbank market. However, banks may learn about counterparty credit risks by repeatedly trading with each other so that the asymmetric information problem may be mitigated. In an empirical analysis of the German unsecured overnight money market, Bräuning and Fecht (2012) determine the impact of such relationship lending for banks' ability to access liquidity. The causes of a possible dry-up of the interbank market are also analyzed by Allen, Carletti, and Gale (2009). They show that banks will start to hoard liquidity if they are unable to hedge idiosyncratic liquidity shocks.

The third strand of literature deals with monetary policy implementation, bank behavior, and consequences for the conditions in the overnight interbank market. This literature can be divided into three groups. The first group focuses on the U.S. before the outbreak of the financial crisis in 2007. Considering major institutional characteristics of the federal funds market, Ho and Saunders (1985) as well as Clouse and Dow (2002) analyze banks' reserve management and draw conclusions for the conditions in the interbank market for reserves. However, the largest part of the literature dealing with the federal funds market focuses on why the federal funds rate fails to follow a martingale within the reserve maintenance period.[7]

The second group of the literature refers to the euro area in the pre-crisis period. A bulk of this literature deals with the under- and overbidding behavior in the Eurosystem's main refinancing operations which could be observed in the first years of the European Monetary Union.[8] Apart from this, there are papers analyzing the consequences of alternative monetary policy implementations. Nautz (1998) shows that the central bank can influence the interbank market rate by being more or less vague about its future monetary policy. Välimäki (2001) analyzes the effects of alternative tender procedures with respect to the Eurosystem's refinancing operations. Neyer and Wiemers (2004) refer to the collateral framework. They show that differences in banks' opportunity costs of holding collateral form a rationale for the existence of an interbank market for reserves. Neyer (2009) demonstrates that remunerating required reserves in a specific way increases the flexibility of monetary policy. Pérez-Quirós and Rodríguez-Mendizábal (2006) show that the two standing facilities offered by the Eurosystem in combination with its minimum reserve system are an effective instrument to stabilize the interbank market rate. Whitesell (2006), although not explicitly referring to the euro area, looks at a minimum reserve system and standing facilities as two alternative regimes for controlling overnight interest rates. Also focusing on the standing facilities, Berentsen and Monnet (2008) develop a general equilibrium framework and show that changing the rates on these facilities may be used actively as a monetary policy instrument. Also Goodhart (2013) points out that by changing the rates on the standing facilities the central bank has an additional instrument at hand. Beaupain and Durré (2008) examine the interday and intraday dynamics of the euro area overnight interbank market and argue that specific features of the Eurosystem's operational framework, as its minimum reserve system, can explain observed regular patterns.

The third group of this third strand of literature comprises papers regarding changes in monetary policy implementation in response to the financial crisis. Eisenschmidt, Hirsch, and Linzert (2009) analyze the relatively aggressive bidding behavior of banks in the Eurosystem's main refinancing operations at the beginning of the financial turmoil. Also

[7] See Hamilton (1996), Clouse and Dow (1999), Furfine (2000), and Bartolini, Bertola, and Prati (2001, 2002).

[8] Under- and overbidding behavior refers to a bidding behavior in which total bids significantly exceed or remain under the Eurosystem's benchmark allotment. Analyses with respect to this under- and overbidding behavior can be found in Ayuso and Repullo (2001, 2003), Ewerhart (2002), Nautz and Oechssler (2003, 2006), and Bindseil (2005).

referring to the first part of the financial crisis (until 2008), Cassola and Huetl (2010) assess the effectiveness of monetary policy implementation during that time. Borio and Disyatat (2009) describe main characteristics of unconventional monetary policies adopted during the financial crisis. They point out that an important feature of these policies is that the central bank also uses its balance sheet to influence prices and conditions in the interbank market. Cheun, von Köppen-Mertes, and Weller (2009) analyze changes to the collateral frameworks of the Eurosystem, the Federal Reserve System and the Bank of England. Lenza, Pill, and Reichlin (2010) describe the way in which these three central banks generally conducted monetary policy during the financial crisis and point to the importance of their influence on money market spreads. Hauck and Neyer (2013) develop a theoretical model considering main institutional features of the Eurosystem's operational framework which has been in place since September 2008 to explain several stylized facts observed during the financial crisis.

Our paper combines all three strands of this literature by analyzing the consequences of frictions in the overnight interbank market, in the form of broadly defined transaction costs, for the impact of monetary policy on bank loan supply. With respect to monetary policy implementation, we point out the crucial role the central bank's standing facilities play for the effectiveness of monetary policy in the presence of interbank market frictions and uncertain liquidity needs.

3 Framework

In our model, we distinguish between three types of agents. We consider a continuum of measure one of price-taking commercial banks with a large number of bank customers and a central bank.

Each commercial bank i grants a loan volume L_i to its customers at a given interest rate i^L. This generates net revenues of

$$i^L L_i - \frac{1}{2}\lambda L_i^2 \tag{1}$$

for bank i. The second term of (1) reflects the costs of managing loans. The quadratic form of this cost function captures the idea that loans differ in their complexity so that the bank adds the least complex loans to its portfolio first.

Bank i credits the loan volume L_i to its customers' demand deposit accounts. The bank's customers can use this newly created money to make payments. They pay by cash or by transferring deposits. Due to the cash payments, each bank i experiences cash withdrawals cL_i, with the currency ratio c reflecting the share of the newly created money used for cash payments. Due to the payments made by deposit transfers, a share χt_i of the remaining deposits $(1-c)L_i$ is transferred to customers of other banks. Crucial is that the net deposit transfer differs across banks. This is reflected by the bank individual variable t_i. For banks facing a net deposit inflow $t_i < 0$, and for those facing a net deposit outflow $t_i > 0$. For a single bank i, its net deposit transfer is uncertain as t_i is the realization of a random variable T. Across all banks, t_i is distributed in the interval $[t^{min}, t^{max}]$ according to the density function $g(t_i) = G'(t_i)$. Note that

$$E[T] = \int_{t^{min}}^{t^{max}} t_i g(t_i)\, dt_i = 0. \tag{2}$$

The parameter χ with $\chi \in (0, \frac{1}{t^{max}}]$ is a scale parameter which determines the dispersion of the distribution of T.[9] If χ increases, the distribution will exhibit a higher dispersion. Accordingly, we will use χ as a measure for uncertainty about a bank's net deposit transfer. Considering both, cash withdrawals and the net deposit transfer, bank i's remaining deposits are given by

$$D_i = L_i - cL_i - (1-c)\chi t_i L_i. \tag{3}$$

This implies that a single bank i may face a liquidity surplus or deficit. Bank i can balance its individual liquidity needs by transacting with the central bank or in the interbank market. However, the cash withdrawals imply that the banking sector as a whole faces a structural liquidity deficit which can only be covered by the central bank being the monopoly producer of currency.

To obtain liquidity from the central bank, each bank i can participate in the central bank's refinancing operations and borrow the amount RO_i at the rate i^{RO}. Moreover, it can use a lending facility to borrow LF_i at the rate i^{LF}.[10] However, it can also place an

[9] As χt_i as a *share* cannot exceed one, χ is restricted to $\frac{1}{t^{max}}$.

[10] Generally, credit operations with the central bank require adequate collateral. In our setting a bank's loan volume L_i serves as collateral, and therefore, limits its central bank borrowing. The central bank may impose a haircut on these loans when accepting them as collateral, like in Bindseil and König (2011). In this setting, however, we assume that such a hair cut is not binding and neglect the collateralization of central bank loans. See in this context also our remarks on this aspect made in the introduction.

amount DF_i of liquidity in a deposit facility offered by the central bank at the rate i^{DF}. Accordingly, transactions with the central bank imply net costs of

$$i^{RO}RO_i + i^{LF}LF_i - i^{DF}DF_i. \tag{4}$$

The rates on the facilities form a corridor around the rate on the refinancing operations with $i^{LF} > i^{RO} > i^{DF}$.

A single bank can also borrow and lend liquidity in the interbank market. Bank i's position in this market is B_i. If $B_i > 0$, bank i will borrow the amount B_i at the rate i^{IBM}. Conversely, $B_i < 0$ indicates that bank i will lend the amount $|B_i|$ at this rate. Independently of whether bank i borrows or lends in the interbank market, transaction costs $\gamma |B_i|$ accrue, with $\gamma \geq 0$. Therefore, net costs in the interbank market account for

$$i^{IBM}B_i + \gamma |B_i| . \tag{5}$$

Considering the described costs and revenues, each bank i aims to maximize its profit Π_i. Banks are risk neutral so that by combining equations (1), (4), and (5), the objective function of bank i simply reads

$$\Pi_i = i^L L_i - \frac{1}{2}\lambda L_i^2 - i^{RO}RO_i - i^{LF}LF_i + i^{DF}DF_i - i^{IBM}B_i - \gamma |B_i| \tag{6}$$

$$\text{s.t.} \qquad L_i + DF_i = RO_i + LF_i + D_i + B_i, \tag{7}$$

where (7) describes bank i's balance sheet constraint. The assets consist of bank i's loans L_i and its deposits held at the central bank DF_i. Its liabilities comprise its central bank borrowing, RO_i+LF_i, and its customers' deposits D_i. The bank's position in the interbank market B_i might constitute an item on the asset or liabilities side of the balance sheet, depending on whether the bank borrows from or lends in the interbank market.

When solving this optimization problem, we have to consider the sequence of moves. First, bank i decides on its optimal loan supply L_i and its optimal borrowing from the central bank's refinancing operations RO_i. When making these decisions, the bank cannot observe the realization t_i of the random variable T. The bank thus faces uncertainty about its future liquidity needs. After bank i observed t_i so that uncertainty about its liquidity needs is resolved, it decides on its transactions in the interbank market B_i and on its use of the central bank's facilities DF_i and LF_i. This sequence of moves implies that the optimization problem can be split up into two stages. Solving this optimization problem by backward induction, we first investigate the second stage of the model and determine a bank's optimal behavior in the interbank market and its optimal use of the central bank's facilities. Then, we analyze the first stage of the model and determine a bank's optimal lending to the non-banking sector and its optimal borrowing from the central bank's refinancing operations.

4 Optimal Behavior at the Second Stage

At the second stage each bank learns its respective share of transferred deposits t_i and, therefore, its actual liquidity needs. Accordingly, banks face no uncertainty at this stage. Using (3) we define bank i's actual individual liquidity needs as

$$N_i := L_i - RO_i - D_i = L_i \left(c + (1-c)\chi t_i\right) - RO_i. \tag{8}$$

If $N_i \geq 0$, bank i will inherit a liquidity deficit. In this case, the bank compares marginal costs of borrowing from the interbank market given by $i^{IBM} + \gamma$ with those of using the lending facility which are simply i^{LF}. As the two marginal costs are constant, the bank will cover its total liquidity deficit by borrowing from the lending facility if $i^{IBM} + \gamma > i^{LF}$. If $i^{IBM} + \gamma < i^{LF}$, it will borrow from the interbank market only. In case both marginal costs are identical, the bank is essentially indifferent between interbank borrowing and the usage of the lending facility.

If $N_i < 0$, bank i will inherit a liquidity surplus so that the bank decides analogously. If the marginal revenues in the interbank market $i^{IBM} - \gamma$ are higher (lower) than the marginal revenues of the central bank's deposit facility i^{DF}, it will place its total surplus in the interbank market (in the deposit facility). In case marginal revenues are identical, the bank will again be indifferent.

As banks will only trade liquidity in the interbank market if this is more beneficial than using the central bank's facilities, the interbank rate in equilibrium will be $i^{IBM*} \in [i^{DF}+\gamma, i^{LF}-\gamma]$. Whether banks prefer the interbank market thus crucially depends on the magnitude of γ. If transaction costs are too high, each bank will prefer to use the central bank's facilities instead of trading in the interbank market. As a result, the interbank market breaks down. This will be the case if $i^{IBM*} - \gamma < i^{DF}$ and $i^{IBM*} + \gamma > i^{LF}$, i.e. if

$$\gamma > \frac{i^{LF} - i^{DF}}{2} =: \bar{\bar{\gamma}}. \tag{9}$$

We thus obtain

Proposition 1: *If $\gamma \leq \bar{\bar{\gamma}}$, the interbank market will be active and we will have to distinguish between three cases regarding the interbank rate:*

$$
\begin{aligned}
i^{IBM*} &= i^{LF} - \gamma && \text{if } RO < cL, \\
i^{IBM*} &\in \left[i^{DF} + \gamma, i^{LF} - \gamma\right] && \text{if } RO = cL, \\
i^{IBM*} &= i^{DF} + \gamma && \text{if } RO > cL.
\end{aligned} \tag{10}
$$

If $\gamma > \bar{\bar{\gamma}}$, the interbank market will be inactive.

Proof: Omitted.

The proposition states that the interbank rate depends crucially on the aggregate liquidity position of the banking sector. Denoting aggregate borrowing from the refinancing operations by RO and aggregate lending to the non-banking sector by L, an aggregate liquidity deficit will arise if banks' cash withdrawals cL are larger than the aggregate amount obtained in the refinancing operations RO. In this case, competition for scarce liquidity brings the interbank rate to its upper limit $i^{LF} - \gamma$. A higher interest rate would not be accepted by the liquidity deficit banks, since then they would prefer to borrow from the central bank's lending facility instead. If an aggregate liquidity surplus occurs, as cash withdrawals are lower than the aggregate amount of liquidity obtained in the refinancing operations, competition for limited lending possibilities in the interbank market brings the interbank rate to its lower limit $i^{DF} + \gamma$. If there is neither an aggregate liquidity deficit nor surplus, neither market side possesses market power. In consequence, any rate within the lower and the upper limit depicts a possible equilibrium.

5 Optimal Behavior at the First Stage

5.1 A Bank's Optimization Problem

At the first stage of the model, bank i must decide on its loan volume L_i and on its borrowing from the refinancing operations offered by the central bank RO_i. The decision problem of bank i that aims to maximize its expected profit $E[\pi_i]$ reads

$$
\begin{aligned}
\max_{L_i, RO_i} E\left[\pi_i\right] = i^L L_i - \tfrac{1}{2}\lambda L_i^2 - i^{RO} RO_i - \max\left\{i^{IBM} - \gamma, i^{DF}\right\} \int_{t^{min}}^{\bar{t}_i} N_i g(t_i)\, dt_i \\
- \min\left\{i^{IBM} + \gamma, i^{LF}\right\} \int_{\bar{t}_i}^{t^{max}} N_i g(t_i)\, dt_i.
\end{aligned} \tag{11}
$$

The first term on the right hand side of (11) reflects the interest revenues of granting loans to the non-banking sector. The second term expresses the management costs associated with these loans. The last three terms reflect expected funding costs. Funding costs occur as due to certain cash withdrawals and the uncertain net deposit transfer there may be a loss of non-interest bearing deposits. The third term on the right hand side of (11) reflects the certain first-stage funding costs. They accrue due to bank i's borrowing from the central bank's refinancing operations. The last two terms show expected second-stage funding costs. Funding needs at the beginning of the second stage are given by (8). The currency ratio c and the scale parameter χ are certain and identical for all banks. In addition, the amounts L_i and RO_i are certain once they are chosen at the first stage. Therefore, the net deposit transfer t_i is the only source of uncertainty of bank i at the first stage regarding its funding needs N_i. From (8) we can infer that bank i will face neither a liquidity deficit nor a surplus at the second stage, i.e. $N_i = 0$, if

$$t_i = \tfrac{RO_i - cL_i}{(1-c)\chi L_i} =: \bar{t}_i. \tag{12}$$

It follows directly from (12) that bank i's critical net deposit transfer $\bar{t}_i$ increases in RO_i and decreases in L_i. If bank i's net deposit transfer is smaller than the critical value ($t_i < \bar{t}_i$), the bank faces a liquidity surplus at the beginning of the second stage ($N_i < 0$). Considering the distribution of T, the bank's expected liquidity surplus is given by $\int_{t^{min}}^{\bar{t}_i} N_i g(t_i)\, dt_i / G(\bar{t}_i)$. The bank will lend its excess liquidity in the interbank market or will place it in the deposit facility, depending on which option is more profitable. Consequently, expected revenues in case of a liquidity surplus are

$$-\max\left\{i^{IBM} - \gamma, i^{DF}\right\} \frac{\int_{t^{min}}^{\bar{t}_i} N_i g(t_i)\, dt_i}{G(\bar{t}_i)}.$$

Analogously, expected costs in case of a liquidity deficit are

$$\min\left\{i^{IBM} + \gamma, i^{LF}\right\} \frac{\int_{\bar{t}_i}^{t^{max}} N_i g(t_i)\, dt_i}{1 - G(\bar{t}_i)}.$$

As the former case occurs with probability $G(\bar{t}_i)$, and the latter with probability $1 - G(\bar{t}_i)$, the last two terms of (11) show expected second-stage funding costs resulting from transactions in the interbank market or from using the central bank's facilities.

Note that uncertainty with respect to net deposit transfer exists only at the individual level. At the aggregate level, the net deposit transfer must be zero so that $E[T] = 0$ (see equation (2)). This has the following implications. First, for any given loan volume L_i and any amount RO_i of liquidity obtained in the refinancing operations, each bank has the same expectations about its subsequent liquidity needs. Second, banks form the same expectations about the interbank rate that will prevail in equilibrium. The interbank rate will only depend on the aggregate liquidity position in the banking sector. Once all banks have granted their loans and borrowed from the refinancing operations, this aggregate liquidity position is certain. This implies that an individual bank takes the aggregate liquidity needs and, therefore, also the interbank rate as given. Consequently, all banks face exactly the same decision problem given by (11). The optimal individual borrowing from the refinancing operations RO_i^{opt} as well as the optimal individual lending to the non-banking sector L_i^{opt} are identical for all banks and are, therefore, equal to the respective aggregate values RO and L.

5.2 Optimal Borrowing from the Refinancing Operations

Determining a bank's optimal behavior at the first stage, we can restrict our attention to the case $i^{IBM} \in \left[i^{RO} - \gamma, i^{RO} + \gamma\right]$. Suppose $i^{IBM} < i^{RO} - \gamma$. Then, no bank has an incentive to borrow from the central bank's refinancing operations at the first stage since borrowing from the interbank market at the second stage is strictly cheaper. However, refusing to borrow from the central bank's refinancing operations at the first stage implies that there is no liquidity to be traded on an interbank market so that the interbank market is inexistent. Therefore, $i^{IBM} < i^{RO} - \gamma$ constitutes no possible equilibrium. If $i^{IBM} > i^{RO} + \gamma$ each bank would be incentivized to borrow unlimitedly from the central bank's refinancing operations to place its liquidity in the interbank market. Apparently, this cannot be an equilibrium either. Considering this and solving the optimization problem (11) we obtain

Lemma 1: *Suppose that $i^{IBM*} \in \left[i^{RO} - \gamma, i^{RO} + \gamma\right]$. If $\bar{t}_i^{opt} \geq \tilde{t} := -\frac{c}{(1-c)\chi}$, then at the first stage, bank i will borrow from the central bank's refinancing operations according to:*

$$i^{RO} = \max\left\{i^{IBM*} - \gamma, i^{DF}\right\} G\left(\bar{t}_i^{opt}\right) + \min\left\{i^{IBM*} + \gamma, i^{LF}\right\}\left[1 - G\left(\bar{t}_i^{opt}\right)\right]. \quad (13)$$

Proof: See appendix.

Optimal borrowing from the central bank's refinancing operations requires marginal costs of this borrowing to be equal to expected marginal revenues. Marginal costs are equal to the interest rate on these operations given by the left hand side of (13). The right hand side of (13) reflects expected marginal revenues. With probability $G(\bar{t}_i^{opt})$, bank i will face a liquidity surplus at the second stage, i.e. $N_i < 0$. In this case, the bank will either lend its excess liquidity in the interbank market or place it in the deposit facility, depending on which alternative yields the higher marginal revenues. With probability $1 - G(\bar{t}_i^{opt})$, the bank will face a liquidity deficit, i.e. $N_i > 0$, so that it will borrow from the interbank market or close its liquidity gap by borrowing from the central bank's lending facility. In this case borrowing from the central bank's refinancing operations implies marginal revenues in the form of avoided illiquidity costs.

The adjustment process in the case marginal costs differ from expected marginal revenues plays a crucial role in our analysis. This process can be described as follows. Assume that marginal costs are higher than expected marginal revenues. Then, the bank has an incentive to reduce its borrowing from the central bank's refinancing operations: If RO_i declines, the critical value $\bar{t}_i$ will decrease as formally revealed by equation (12). A decreasing $\bar{t}_i$ means that the probability of facing a liquidity deficit at the second stage $G(\bar{t}_i)$ increases. As marginal revenues in the case of a liquidity deficit given by $\min\left\{i^{IBM} + \gamma, i^{LF}\right\}$ are strictly larger than those in the case of a liquidity surplus given by $\max\left\{i^{IBM} - \gamma, i^{DF}\right\}$, expected marginal revenues will increase if the bank reduces its borrowing from the refinancing operations. Consequently, as long as marginal costs are higher than marginal revenues, the bank wishes to reduce RO_i until expected marginal revenues equal marginal costs.

However, if the non-negativity constraint on RO_i becomes binding, the bank cannot reduce RO_i any further. This will be the case if $\bar{t}_i^{opt} < \tilde{t}$. As a result, $\bar{t}_i^{opt}$ can not be realized by bank i and $RO_i^{opt} = 0$. Accordingly, marginal costs of borrowing from the refinancing operations remain higher than expected marginal revenues.

5.3 Optimal Lending to the Non-Banking Sector

Solving the optimization problem (11) with respect to the optimal lending L_i^{opt} to the non-banking sector, we obtain

Lemma 2: *Bank i will supply loans at the first stage according to the following first order condition:*

$$i^L = \lambda L_i^{opt} + ci^{RO} + (1 - c)\chi\phi, \tag{14}$$

with

$$\phi = \max\left\{i^{IBM} - \gamma, i^{DF}\right\} \int_{t^{min}}^{\max\left\{\bar{t}_i^{opt}, \tilde{t}\right\}} t_i g(t_i)\, dt_i$$
$$+ \min\left\{i^{IBM} + \gamma, i^{LF}\right\} \int_{\max\left\{\bar{t}_i^{opt}, \tilde{t}\right\}}^{t^{max}} t_i g(t_i)\, dt_i, \tag{15}$$

where $\tilde{t}$ is defined in Lemma 1 and where $\bar{t}_i^{opt}$ is implicitly defined by (13)*.*

Proof: See appendix.

Optimal lending L_i^{opt} to the non-banking sector requires balancing marginal revenues with expected marginal costs of granting loans. Marginal revenues are equal to the interest rate i^L. Expected marginal costs consist of marginal management costs λL_i^{opt} and expected marginal funding costs $ci^{RO} + (1-c)\chi\phi$. The latter can be divided into two parts. The first part ci^{RO} refers to the certain funding costs due to borrowing from the central bank's refinancing operations at the first stage. The second part $(1-c)\chi\phi$ corresponds to the expected second stage marginal funding costs.

If bank i faces a liquidity surplus at the beginning of the second stage as $t_i \leq \max\{\bar{t}_i^{opt}, \tilde{t}\}$, it will either lend its liquidity surplus in the interbank market at $i^{IBM} - \gamma$ or place it in the deposit facility at i^{DF}. Hence, the first term on the right hand side of (15) determines the expected (negative) marginal funding costs in the case of a liquidity surplus. The second term determines the expected marginal funding costs in the case of a liquidity deficit. A liquidity deficit will occur if $t_i > \max\{\bar{t}_i^{opt}, \tilde{t}\}$. In this case, bank i will borrow either from the interbank market at $i^{IBM} + \gamma$ or from the lending facility at i^{LF}.

Expected second stage marginal funding costs $(1-c)\chi\phi$ can be rewritten to

$$(1-c)\chi \int_{\max\left\{\bar{t}_i^{opt}, \tilde{t}\right\}}^{t^{max}} t_i g(t_i)\, dt_i 2\gamma \qquad \text{if } \gamma \leq \bar{\bar{\gamma}}, \tag{16}$$
$$(1-c)\chi \int_{\max\left\{\bar{t}_i^{opt}, \tilde{t}\right\}}^{t^{max}} t_i g(t_i)\, dt_i \left(i^{LF} - i^{DF}\right) \qquad \text{if } \gamma > \bar{\bar{\gamma}}. \tag{17}$$

Equation (16) and (17) reveal that expected second stage marginal funding costs consist of the expected share per unit of loans for which funding costs are expected to accrue and the relevant funding costs. If $\gamma \leq \bar{\bar{\gamma}}$ and if $RO = cL$, the expected share per unit of loans will be $2(1-c)\chi \int_{\max\{\bar{t}_i^{opt}, \tilde{t}\}}^{t^{max}} t_i g(t_i)\, dt_i$. If $\gamma > \bar{\bar{\gamma}}$ or if $RO \neq cL$, the expected share per unit of loans will be $(1-c)\chi \int_{\max\{\bar{t}_i^{opt}, \tilde{t}\}}^{t^{max}} t_i g(t_i)\, dt_i$.[11]

[11] For details with respect to a bank's expected second stage marginal funding costs we refer the reader to Appendix A.3.

6 Equilibrium

After having clarified the optimal behavior of an individual bank, we are now in a position to determine the equilibrium of our model. We have a continuum of ex-ante identical banks of unit mass. Consequently, the bank-individual optimal values L_i^{opt} and RO_i^{opt} correspond to the respective equilibrium aggregate levels L^* and RO^*.

In the euro area, aggregate borrowing from the ECB's main refinancing operations has been systematically equal to or higher than the ECB's benchmark allotment.[12] As in our model aggregate cash withdrawals cL^* corresponds to the benchmark allotment, we focus in our analysis on equilibria in which $RO^* \geq cL^*$. These equilibria will emerge if $G(0) < 0.5$ and if $i^{LF} - i^{RO} \geq i^{RO} - i^{DF}$. The latter means that the corridor, which the rates on the central bank's facilities form around the main policy rate, is symmetric or asymmetric in the sense that the lower difference is smaller than the upper difference. These two cases have been relevant in the euro area.[13] The former implies a left-skewed distribution of T so that for each single bank the probability of facing a net deposit outflow is larger than the probability of a net deposit inflow. Due to $E[T] = 0$, each individual bank thus expects small outflows with a high probability and large inflows with low probability. To understand this pattern, it is useful to distinguish between two types of bank customers. First, each bank possesses a huge number of bank customers predominantly generating relatively small deposit outflows, such as households who make payments for consumption purposes. Second, each bank possesses a small number of bank customers predominantly receiving relatively large payments, such as firms as suppliers of consumption goods. With a small probability these firms may benefit from spikes in demand for their goods, e.g. caused by a major innovation. In consequence, their respective bank would face a large net deposit inflow. Moreover, a large net deposit inflow may also occur if outflows decline to a large extent. This is the case, if households experience a massive shock that significantly reduces their spending, e.g. if in a region cash card payments are not feasible. As the probability of such a shock is also small, banks expect a large net deposit inflow with a small probability and small net deposit outflows with a large probability.

[12]See footnote 5 for a description of the benchmark allotment. Note that in our model there are no reserve requirements and autonomous factors consists of currency holdings.

[13]From April 1999 until November 2013 the rates on the Eurosystem's facilities generally formed a symmetric corridor around the rate on the main refinancing operations. However, due to the zero lower bound, the rate on the deposit facility was not decreased in November 2013 contrary to the rates on the lending facility and on the main refinancing operations. Consequently, there has been an asymmetric interest rate corridor since then.

Combining the results of Proposition 1, Lemma 1 and Lemma 2, we obtain

Proposition 2: *Assume that $G(0) < 0.5$ and that $i^{LF} - i^{RO} \geq i^{RO} - i^{DF}$. Then, depending on γ, we have to distinguish between three equilibria*

$$\text{I}: \quad RO^* = cL^*, \quad DF^* = 0, \; LF^* = 0, \quad i^{IBM*} = i^{RO} - \gamma\,[1 - 2G\,(0)] \qquad \text{if } \; \gamma \leq \bar{\gamma}, \tag{18}$$

$$\text{II}: \quad RO^* > cL^*, \quad DF^* > 0, \; LF^* = 0, \quad i^{IBM*} = i^{DF} + \gamma \qquad \text{if } \; \gamma \in (\bar{\gamma}, \bar{\bar{\gamma}}], \tag{19}$$

$$\text{III}: \quad RO^* > cL^*, \quad DF^* > LF^* > 0 \qquad \text{if } \; \gamma > \bar{\bar{\gamma}}, \tag{20}$$

with

$$\bar{\gamma} := \frac{i^{RO} - i^{DF}}{2[1 - G(0)]}, \tag{21}$$

and $\bar{\bar{\gamma}}$ being defined in (9).

Proof: See appendix.

In Equilibrium I, interbank market transaction costs are that low that each bank borrows exactly an amount equal to its cash withdrawals from the central bank's refinancing operations and balances its liquidity needs resulting from the deposit transfers of its customers solely by using the interbank market. None of the facilities is used. Only this behavior implies that the optimality condition given by (13) is fulfilled. To see this, suppose as a starting point that $\gamma = 0$. If banks borrowed an amount larger than their cash withdrawals from the refinancing operations, there would be an aggregate surplus at the second stage bringing the interbank market to its lower bound $i^{DF} + \gamma = i^{DF}$. However, this cannot be an equilibrium, as then marginal costs of borrowing from the refinancing operations given by i^{RO} would exceed expected marginal revenues which in this case are equal to i^{DF}.[14] Consequently, banks will have an incentive to reduce their borrowing from the refinancing operations to balance marginal costs and expected marginal revenues (see the adjustment process described in Subsection 5.2). Analogously, if banks borrowed an amount lower than their cash withdrawals, there would be an aggregate liquidity deficit bringing the interbank rate to its upper bound $i^{LF} - \gamma = i^{LF}$. This is no equilibrium either, as for this interbank rate expected marginal revenues of borrowing from the refinancing operations, which are equal to i^{LF}, exceed marginal costs given by i^{RO}, so that banks wish to increase their borrowing. Consequently, for $\gamma = 0$ an equilibrium will be reached if each bank borrows an amount equal to its cash withdrawals from the refinancing

[14] One obtains expected marginal revenues by inserting the equilibrium interbank market rate into the right hand side of (13).

operations. This implies that the interbank market rate i^{IBM*} equals the central bank's policy rate i^{RO}.

Let us assume next that γ becomes positive. Then, expected marginal revenues of borrowing from the central bank's refinancing operations increase. If a bank faces a liquidity surplus at the second stage, its marginal revenues from placing liquidity in the interbank market will decrease but its avoided marginal illiquidity costs in case of a liquidity deficit will increase (see Lemma 1). Due to the left-skewed distribution of T, the probability of facing a liquidity deficit at the second stage will be higher than of facing a liquidity surplus as long as each bank borrows an amount equal to its cash withdrawals from the central bank's refinancing operations, i.e. as long as $RO_i = cL_i$. Consequently, a bank's expected marginal revenues of borrowing from the refinancing operations will increase if γ becomes positive. To balance marginal costs and expected marginal revenues again, each bank will have an incentive to increase its borrowing from the refinancing operations above cL_i (see the adjustment process described in Subsection 5.2). However, such an aggregate borrowing behavior will result in excess aggregate liquidity so that the interbank rate will decline. This decline will reduce expected marginal revenues of borrowing from the refinancing operations. Accordingly, the incentive to borrow more than cL_i becomes weaker. No bank will borrow more than an amount equal to its cash withdrawals from the refinancing operations if the interbank rate decreases to $i^{IBM} = i^{RO} - \gamma \left[1 - 2G(0)\right]$. As long as the interbank rate is unrestricted, an increase in expected marginal revenues due to higher interbank market transaction costs will thus be offset by a decrease of the interbank rate. The price mechanism works. This mechanism ensures that banks have no incentive to borrow more than an amount equal to their cash withdrawals from the central bank's refinancing operations. Equilibrium I as described by equation (18) will be realized.

If transaction costs exceed the critical level $\bar{\gamma}$, further decreases of the interbank rate will not be possible to balance marginal costs and expected marginal revenues as the interbank rate has reached its lower bound $i^{DF} + \gamma$. Consequently, a further increase in γ implies that banks actually start to increase their borrowing from the refinancing operations. This will reduce the probability of facing a liquidity deficit and, therefore, expected marginal revenues. As the price mechanism does not function, marginal costs and expected marginal revenues are balanced via a quantity effect. As this behavior implies that banks increase their borrowing from the refinancing operations above the cash withdrawals, an aggregate liquidity surplus will materialize, $RO^* > cL^*$, while the interbank rate will remain at its lower limit. The excess liquidity will then be placed in the deposit facility. Hence, for sufficiently high transaction costs, Equilibrium II given in equation (19) will be realized. In this equilibrium, all banks with a liquidity deficit still cover their liquidity needs by using the interbank market. Some surplus banks have to use the deposit facility due to the aggregate excess liquidity.

If transaction costs reach the critical level $\bar{\bar{\gamma}}$, the deficit banks are no longer willing to borrow their liquidity from the interbank market but prefer to use the central bank's lending facility instead. The interbank market breaks down. Both, the deficit banks as well as the surplus banks, exclusively use the facilities to balance their liquidity needs at the second stage. As there is aggregate excess liquidity, it follows that $DF^* > LF^*$. Equilibrium III as given in equation (20) will be realized.

In Proposition 2 we assume that $G(0) < 0.5$ and that $i^{LF} - i^{RO} \geq i^{RO} - i^{DF}$. These assumptions imply that the equilibrium is characterized by $RO^* \geq cL^*$, which is the situation observed in the euro area. However, for the sake of completeness, we will briefly comment on the possible equilibrium $RO^* < cL^*$. This equilibrium will emerge if the distribution of T becomes sufficiently right-skewed or if the interest rate corridor becomes sufficiently asymmetric with $i^{LF} - i^{RO} < i^{RO} - i^{DF}$. We start with the importance of the distribution of T. Let us assume that T is distributed symmetrically around $t_i = 0$ and that the interest rates on the facilities form a symmetric corridor around the main policy rate. Then, the probability of facing a net deposit outflow equals the probability of facing a net deposit inflow due to customers' deposit transfers. In this case, only $RO_i = cL_i$ implies that (13) is fulfilled. Transaction costs play no role as they increase marginal revenues of borrowing from the refinancing operations in the case of a liquidity deficit and decrease them in the case of a liquidity surplus, and for $RO_i = cL_i$ both scenarios occur with the same probability given the symmetric distribution of T.

Now let us assume that the distribution of T becomes right-skewed. For $RO_i = cL_i$, the probability of facing a liquidity surplus increases. In this case, transaction costs imply that expected marginal revenues of borrowing from the refinancing operations decrease. Accordingly, banks are incentivized to borrow less from the refinancing operations. Analogously to the case of a left-skewed distribution of T this results in an increase in the interbank market rate to balance marginal costs and expected marginal revenues of borrowing from the refinancing operations. However, if the interbank rate reaches its upper bound so that it cannot increase further, banks start to borrow less from the refinancing operations and $RO^* < cL^*$.

In order to highlight the importance of the asymmetry of the interest rate corridor let us assume that net deposit transfers T are distributed symmetrically around zero. If then the asymmetry of the interest rate corridor is, contrary to our assumption in Proposition 2, given by $i^{LF} - i^{RO} < \gamma < i^{RO} - i^{DF}$, the optimal behavior of bank i will be $RO_i^{opt} < cL_i^{opt}$: Facing relatively high interbank market transaction costs ($i^{LF} - i^{RO} < \gamma$), banks with a liquidity deficit will only accept an interbank rate below i^{RO}. Otherwise, they would prefer to use the lending facility. However, an interbank rate below i^{RO} implies for $RO_i = cL_i$ expected marginal revenues of borrowing from the refinancing operations to be lower than marginal costs.[15] Therefore, banks are incentivized to borrow less from the refinancing operations. Generally, this would lead to an increase in the interbank rate balancing expected marginal revenues and marginal costs again. However, such an adjustment is not possible as in case of a liquidity deficit a bank would not accept a higher interbank rate. Therefore, all banks actually start to borrow less from the refinancing operations to balance expected marginal revenues so that marginal costs and $RO^* < cL^*$.

7 Monetary Policy and Bank Loan Supply

This section analyzes the impact of monetary policy on bank loan supply. As the strength of this effect depends on the extent of uncertainty about the net deposit transfer and interbank market transactions costs, we will start with a brief look on their direct impact on bank loan supply.

7.1 Uncertainty and Interbank Market Frictions

Uncertainty about the net deposit transfer and interbank market frictions have a negative impact on bank loan supply.[16] Both, uncertainty and interbank market transaction costs, influence expected second stage marginal funding costs as Lemma 2 reveals. An increase in uncertainty about the net deposit transfer leads to an increase in both, the expected liquidity surplus and the expected liquidity deficit per unit of loans as revealed by the equations (16) and (17). As this surplus or deficit has to be costly balanced either in the interbank market or via the central bank's facilities, an increase in uncertainty implies

[15] For $RO_i^{opt} = cL_i^{opt}$ we get that $\bar{t}_i = 0$ as equation (12) shows. This means for a symmetric distribution of T that $G(\bar{t}_i = 0) = 0.5$, so that expected marginal revenues of borrowing from the refinancing operations are equal to the interbank market rate as revealed by the right hand side of equation (13). As marginal costs of borrowing from the refinancing operations are given by i^{RO}, $i^{IBM} < i^{RO}$ implies expected marginal revenues to be lower than marginal costs.

[16] We provide the respective formal analysis in Appendix A.4.

higher expected marginal funding costs of granting loans. Consequently, banks start to reduce their loan supply.

Obviously, a change in interbank market transaction costs will only have an impact on bank loan supply if these costs are still low enough to ensure an active interbank market, i.e. if $\gamma \leq \bar{\bar{\gamma}}$. In this case, it follows from Lemma 2 that an increase in marginal transaction costs γ results in an increase in expected marginal funding costs. Accordingly, banks will reduce their loan supply.

7.2 Main Monetary Policy Rate

The central bank has several alternatives to impose an effect on bank loan supply. Starting with the possibility of changing its policy rate i^{RO}, we obtain

Proposition 3: *A change in the policy rate implies for Equilibrium j*

$$\frac{\partial L^{j*}}{\partial i^{RO}} = -\frac{1}{\lambda}\left[c + (1-c)\chi\bar{t}^{*}\right] < 0 \quad \forall \quad j \quad \textit{with}$$
$$\frac{\partial L^{\mathrm{III}*}}{\partial i^{RO}} < \frac{\partial L^{\mathrm{II}*}}{\partial i^{RO}} < \frac{\partial L^{\mathrm{I}*}}{\partial i^{RO}} < 0.$$

Furthermore, we get

$$\frac{\partial^2 L^{\mathrm{III}*}}{\partial i^{RO}\partial\chi} < \frac{\partial^2 L^{\mathrm{II}*}}{\partial i^{RO}\partial\chi} < \frac{\partial^2 L^{\mathrm{I}*}}{\partial i^{RO}\partial\chi} = 0,$$
$$\frac{\partial^2 L^{\mathrm{III}*}}{\partial i^{RO}\partial\gamma} = 0, \quad \frac{\partial^2 L^{\mathrm{II}*}}{\partial i^{RO}\partial\gamma} < 0, \quad \frac{\partial^2 L^{\mathrm{I}*}}{\partial i^{RO}\partial\gamma} = 0.$$

Proof: See appendix.

This proposition reveals that, independently of potential frictions in the interbank market and the extent of uncertainty about the net deposit transfer, by changing its policy rate i^{RO} the central bank affects banks' marginal funding costs and, therefore, their loan supply in all equilibria. However, frictions in the interbank market and uncertainty about the net deposit transfer may reinforce the impact of this monetary policy impulse on bank loan supply.

In Equilibrium I, in which $\bar{t}^{*} = 0$, the effect of monetary policy on aggregate loan supply depends only on the management cost parameter λ and the currency ratio c. The impact of monetary policy on bank loan supply is not influenced by interbank market frictions or by uncertainty about the net deposit transfer. An increase in i^{RO} implies marginal costs of borrowing from the main refinancing operations to become higher than expected marginal revenues, leading to the adjustment process described in Subsection 5.2. The interbank rate increases which again balances marginal costs and expected marginal revenues. In this case, only the price effect prevails. A quantity effect to balance marginal costs and expected marginal revenues does not occur so that $\bar{t}^*$ remains unchanged. Therefore, we can conclude from Lemma 2 that in Equilibrium I only first stage marginal costs of granting loans will change if the monetary policy rate is changed. Frictions in the interbank market and uncertainty about the net deposit transfer, which both refer to the second stage, play no role for the effectiveness of monetary policy.

In contrast, in Equilibrium II and III a change in the policy rate i^{RO} does not only have an impact on first stage marginal funding costs but also on expected second stage marginal funding costs, i.e. also on the third term given on the right hand side of (14). The reason is that in both equilibria, a change in the interbank market rate to balance marginal costs and expected marginal revenues of borrowing from the central bank's refinancing operations is not feasible. Either the interbank market rate is at its lower bound or an interbank market does not exist due to high transaction costs.[17] Consequently, banks start to adjust their borrowing from the refinancing operations to balance marginal costs and expected marginal revenues. According to Lemma 2, this behavior implies a change in expected second stage marginal funding costs. Suppose the central bank decreases i^{RO}. Then, marginal costs of borrowing from the refinancing operations fall below expected marginal revenues inducing banks to increase their borrowing from the refinancing operations. This borrowing behavior implies an increase in $\bar{t}_i^{opt}$. The expected liquidity deficit per unit of loans, which has to be covered costly by borrowing from the interbank market (Equilibrium II) or from the central bank's lending facility (Equilibrium III), decreases, and therefore, also banks' expected second marginal funding costs (see equations (16) and (17)). Consequently, banks are willing to supply more loans. This means that contrary to Equilibrium I, in Equilibrium II and III not only first stage marginal funding costs decrease but in addition, there is a reduction of expected second stage marginal funding costs. Therefore, the impact of monetary policy on bank loan supply is stronger than in Equilibrium I.

In Equilibrium II, the reinforcing effect increases in interbank market transaction costs. Crucial is the decrease of the expected liquidity deficit per unit of loans due to the increased

[17] Note that in Equilibrium II, the interbank rate has reached its lower bound so that an increase in the interbank rate is generally possible. This would however imply a switch from Equilibrium II to Equilibrium I. In this section, we thus only consider changes in i^{RO} which ensure that $\gamma > \frac{i^{RO}-i^{DF}}{2[1-G(0)]}$, i.e. we stay in Equilibrium II and the interbank rate is fixed at its lower bound.

borrowing from the refinancing operations, formally reflected by an increase in $\bar{t}_i^{opt}$. As the deficit will be costly refinanced in the interbank market, the impact of the decreased expected liquidity deficit on expected marginal funding costs is the stronger, the larger the interbank market transaction costs are. Furthermore, in both equilibria, II and III, the reinforcing effect is the stronger the higher the extent of uncertainty about a bank's net deposit transfer χ is. The reason is that the expected liquidity deficit per unit of loans is determined by this uncertainty (see equations (16) and (17)). Therefore, a large χ gives a higher weight to the decrease of the expected deficit due to the increased borrowing from the refinancing operations.

Finally, Proposition 3 shows that the reinforcing effect is stronger in Equilibrium III than in Equilibrium II. The intuition behind this result is that covering a liquidity deficit by borrowing from the lending facility in Equilibrium III is more expensive than of using the interbank market in Equilibrium II.[18]

7.3 Main Policy Rate and the Rates on the Facilities

Alternatively, the central bank might change all its interest rates instead of solely changing its policy rate. In this case we obtain

Proposition 4: *A likewise change in all central bank's interest rates, i.e.* $di^{RO} = di^{DF} = di^{LF}$, *implies for Equilibrium j*

$$\frac{\partial L^{j*}}{\partial i^{RO}} = -\frac{c}{\lambda} < 0 \quad and \quad \frac{\partial^2 L^{j*}}{\partial i^{RO} \partial \chi} = \frac{\partial^2 L^{j*}}{\partial i^{RO} \partial \gamma} = 0 \;\; \forall \;\; j.$$

Proof: See appendix.

If the central bank changes all interest rates to the same extent, a reinforcing effect will be avoided. Recall from Proposition 3 that the reinforcing effect will occur if the interbank rate has already reached its lower bound so that the interbank rate cannot adjust to changes in the policy rate anymore (Equilibrium II) or if due to high transaction costs an interbank market does not exist (Equilibrium III). Changing the rates of the facilities leads to a change of the lower bound which allows the interbank rate to adjust in Equilibrium II as well. The price mechanism works again so that the quantity effect which is responsible for the reinforcing effect does not occur, i.e. $\bar{t}_i^{opt} = \bar{t}^*$ does not change.

[18] Appendix A.3 reveals that relevant costs in Equilibrium III are given by $i^{LF} - i^{DF}$, in Equilibrium II they are 2γ. As in Equilibrium II $\gamma \leq \bar{\bar{\gamma}} = (i^{LF} - i^{DF})/2$, the relevant marginal funding costs in Equilibrium III are higher than in Equilibrium II.

In Equilibrium III, a likewise change in the rates on the facilities as in the main policy rate implies that expected marginal revenues of borrowing from the refinancing operations change to the same extent as marginal costs.[19] Consequently, banks do not change their borrowing behavior so that also in this equilibrium $\bar{t}_i^{opt} = \bar{t}^*$ does not change. A quantity effect, which is responsible for the reinforcing effect, does not occur. Therefore, in all equilibria a likewise change in all central bank's interest rates will only affect first stage marginal funding costs (see Lemma 2). This means that in all equilibria, this central bank behavior has the same impact on bank loan supply. A reinforcing effect does not occur.

However, these results point to the problem of a zero lower bound on interest rates. In Equilibrium II and III, the reinforcement of an expansionary monetary policy cannot be avoided by a likewise change in the rates on the facilities if the rate on the deposit facility i^{DF} is already equal to zero. As a result, the zero lower bound on the deposit facility rate may imply an unavoidable reinforcement effect of an expansionary monetary policy.

The findings of this subsection point to the importance of the rates on the facilities in the presence of uncertain liquidity needs and interbank market frictions. Also the next two subsections illustrate their high relevance. They show that in the presence of uncertain liquidity needs and sufficiently high interbank market frictions, the facilities represent an additional effective monetary policy instrument.

7.4 Changing the Width of the Interest Rate Corridor

If the central bank changes the width of the interest rate corridor around its policy rate, we will obtain

Proposition 5: *Suppose, we have a symmetric interest rate corridor, i.e.* $i^{RO} = \frac{i^{DF}+i^{LF}}{2}$. *A change in the width of the interest rate corridor, i.e.* $di^{LF} = -di^{DF}$ *and* $di^{RO} = 0$, *implies*

$$\frac{\partial L^{\text{I}*}}{\partial(i^{LF}-i^{DF})} = 0,$$
$$\frac{\partial L^{\text{II}*}}{\partial(i^{LF}-i^{DF})} = -\frac{(1-c)\chi}{\lambda}\bar{t}^* < 0,$$
$$\frac{\partial L^{\text{III}*}}{\partial(i^{LF}-i^{DF})} = -\frac{(1-c)\chi}{\lambda}\int_{\bar{t}^*}^{t^{max}} t_i g(t_i) dt_i < 0,$$

[19] Referring to the right hand side of (13), expected marginal revenues are given by $i^{DF}G(\bar{t}_i^{opt}) + i^{LF}[1 - G(\bar{t}_i^{opt})]$ in Equilibrium III.

with

$$\frac{\partial^2 L^{\mathrm{I}*}}{\partial(i^{LF}-i^{DF})\partial\chi}=0,\quad \frac{\partial^2 L^{\mathrm{II}*}}{\partial(i^{LF}-i^{DF})\partial\chi}<0,\quad \frac{\partial^2 L^{\mathrm{III}*}}{\partial(i^{LF}-i^{DF})\partial\chi}<0,$$
$$\frac{\partial^2 L^{\mathrm{I}*}}{\partial(i^{LF}-i^{DF})\partial\gamma}=0,\quad \frac{\partial^2 L^{\mathrm{II}*}}{\partial(i^{LF}-i^{DF})\partial\gamma}<0,\quad \frac{\partial^2 L^{\mathrm{III}*}}{\partial(i^{LF}-i^{DF})\partial\gamma}=0.$$

Proof: See appendix.

Proposition 5 shows that by changing the width of the interest rate corridor, the central bank may influence banks' loan supply without changing its policy rate i^{RO}. The central bank thus has an additional instrument at hand.

Obviously, changing the width of the corridor has no effect in Equilibrium I. In this equilibrium, banks never use the facilities to balance their liquidity needs so that the rates on the facilities are irrelevant for their loan supply decision.

In Equilibrium II and III a bank may use one of the facilities. Consequently, the rates on the facilities become relevant for a bank's expected marginal funding costs, and therefore, for its optimal loan supply. The facilities are used to balance uncertain liquidity needs. If a bank has to use the deposit facility, from the ex post perspective it would have been better to borrow less from the refinancing operations as the bank pays for the liquidity the rate i^{RO} and it receives only the rate i^{DF}. Analogously, if a bank has to use the lending facility, from the ex post perspective it would have been better to borrow more from the refinancing operations as then the bank would have paid for the liquidity only the rate i^{RO} instead of i^{LF}. Due to the uncertainty about actual liquidity needs additional funding costs may thus occur. These costs of uncertainty are the higher, the stronger the rates on the facilities deviate from the rate on the main refinancing operations. This means that an increase in the width of the corridor leads to an increase in expected marginal funding costs and corresponds therefore to a contractionary monetary policy. As in Equilibrium II expected marginal funding costs increase in γ and χ, the impact of this monetary policy impulse is the stronger the higher the frictions in the interbank market and the uncertainty about the net deposit transfer are. In Equilibrium III, in which the interbank market is not used, it is only the uncertainty about the net deposit transfer which reinforces the monetary policy impulse.

Note that if the zero lower bound on the deposit rate becomes binding, changing the width of the corridor will not be a feasible instrument for conducting contractionary monetary policy. However an expansionary monetary policy is still feasible.

7.5 Changing the Asymmetry of the Interest Rate Corridor

The central bank can also change the interest rate corridor in the sense that it becomes more or less asymmetric around the main policy rate. Then we obtain

Proposition 6: *A change in the interest rate corridor in the form of* $di^{LF} = di^{DF}$ *and* $di^{RO} = 0$*, implies*

$$\frac{\partial L^{\mathrm{I}*}}{\partial i^{DF}} = 0,$$
$$\frac{\partial L^{\mathrm{j}*}}{\partial i^{DF}} = \frac{(1-c)\chi}{2\lambda}\overline{t}^{*} > 0 \quad \textit{for} \quad j = \mathrm{II}, \mathrm{III},$$

with

$$\frac{\partial^2 L^{\mathrm{I}*}}{\partial i^{DF}\partial\chi} = 0 \quad \frac{\partial^2 L^{\mathrm{II}*}}{\partial i^{DF}\partial\chi} > 0, \quad \frac{\partial^2 L^{\mathrm{III}*}}{\partial i^{DF}\partial\chi} > 0,$$
$$\frac{\partial^2 L^{\mathrm{I}*}}{\partial i^{DF}\partial\gamma} = 0 \quad \frac{\partial^2 L^{\mathrm{II}*}}{\partial i^{DF}\partial\gamma} > 0, \quad \frac{\partial^2 L^{\mathrm{III}*}}{\partial i^{DF}\partial\gamma} = 0.$$

Proof: See appendix.

As in Proposition 5, in Equilibrium I a change in the rates on the facilities has no impact on bank loan supply as the facilities will not be used and are not expected to be used. However, in the Equilibrium II and III, the central bank has an additional effective instrument at its disposal by changing the asymmetry of the interest rate corridor. If, for example, the central bank increases both rates but leaves its main rate unchanged, its monetary policy will be expansionary. In Equilibrium II, only the deposit facility may be used. Therefore, the increase in i^{DF} implies a decrease in the costs of uncertainty (see Subsection 7.4) leading to decreasing expected marginal funding costs. The increase in i^{LF} plays no role. In Equilibrium III, a bank may use the deposit facility or the lending facility. The higher i^{DF} implies lower costs of uncertainty resulting in lower expected marginal funding costs. The higher i^{LF} leads to higher costs of uncertainty, and therefore also to higher expected marginal funding costs. However, the former effect outweighs the latter: The increase in both, i^{DF} and i^{LF}, result in higher marginal revenues of borrowing from the refinancing operation (see Lemma 1). Consequently, banks actually start to borrow more from the refinancing operations and the probability of facing a liquidity surplus at the second stage increases. This means that the decreasing effect on expected marginal funding costs due to the increase in i^{DF} outweighs the increasing effect due to the increase in i^{LF}. As a result, this monetary policy impulse has a positive impact on bank loan supply. Analogously to the situation described in Section 7.2, in Equilibrium II, this monetary policy impulse is the more effective, the higher the transaction costs in the interbank market are, and in Equilibria II and III the impact of this monetary policy impulse on bank loan supply increases in the extent of uncertainty about the net deposit transfer.

7.6 Collateralization and Minimum Reserves

In our analysis, we have not considered the collateralization of central bank credits and minimum reserve requirements. Both are elements of the Eurosystem's operational framework. However, introducing these elements would not change the qualitative results of our analysis.

If we considered the collateralization of central bank credits, banks would face opportunity costs of holding collateral. As a result, expected marginal funding costs would increase.[20] Bank loan supply would be lower but our model results would not change qualitatively.

Considering reserve requirements lead to a structural liquidity deficit of the banking sector. In our model, a structural liquidity deficit is already captured by considering cash withdrawals. Introducing reserve requirements would therefore simply increase the already existing structural deficit. A main feature of the Eurosystem's minimum reserve system is that banks can make use of averaging provision of required reserves during the reserve maintenance period. This allows banks to smooth out liquidity fluctuations. In our model, costs resulting from uncertain liquidity fluctuations, and thus banks' expected marginal funding costs would decrease. Although this would have a positive effect on bank loan supply, the qualitative results of our model would not change.

8 Summary

The interbank market is regarded to play a crucial role for the implementation of monetary policy as it serves as the starting point of the transmission mechanism. Based on a theoretical model, this paper analyzes in how far interbank market frictions in the form of transaction costs influence the effectiveness of monetary policy and draws conclusions for monetary policy implementation.

[20] For a respective analysis see, for example, Neyer and Wiemers (2004) and Berentsen and Monnet (2008).

We show that independently of interbank market frictions monetary policy is effective. The central bank is able to influence banks' funding costs of granting loans, and therefore their loan supply, just by changing its main policy rate. However, frictions in the interbank market may reinforce this effect.

Generally, the central bank can steer this reinforcing effect by changing the rates on its facilities. This indicates that for sufficiently high interbank market transaction costs, the standing facilities present an additional effective monetary policy instrument. By changing the width or the asymmetry of the interest rate corridor the central bank can influence banks' expected marginal funding costs and therefore, their loan supply. It should be noted that lowering the rate on the deposit facility taken for itself, corresponds to a contractionary monetary policy. This implies that the zero lower bound of the rate on the deposit facility is not a problem as long as the central bank wants to conduct a an expansionary monetary policy.

Appendix

A.1 Proof of Lemma 1

Recall from (11) that the first stage optimization problem of bank i reads:

$$\begin{aligned}\max_{L_i,RO_i} E\left[\pi_i\right] = i^L L_i - \tfrac{1}{2}\lambda L_i^2 - i^{RO} RO_i - \max\left\{i^{IBM}-\gamma, i^{DF}\right\} \int_{t^{min}}^{\bar{t}_i} N_i g(t_i)\, dt_i \\ - \min\left\{i^{IBM}+\gamma, i^{LF}\right\} \int_{\bar{t}_i}^{t^{max}} N_i g(t_i)\, dt_i,\end{aligned} \tag{22}$$

subject to (8) and (12). By applying the Leibniz rule and making use of the fact that $N_i = 0$ for $t_i = \bar{t}_i$, we obtain:

$$\begin{aligned}\tfrac{\partial E[\pi_i]}{\partial RO_i} = -i^{RO} - \max\left\{i^{IBM}-\gamma, i^{DF}\right\} \int_{t^{min}}^{\bar{t}_i} \tfrac{\partial N_i}{\partial RO_i} g(t_i) dt_i \\ - \min\left\{i^{IBM}+\gamma, i^{LF}\right\} \int_{\bar{t}_i}^{t^{max}} \tfrac{\partial N_i}{\partial RO_i} g(t_i) dt_i.\end{aligned} \tag{23}$$

We can infer from (8) that $\frac{\partial N_i}{\partial RO_i} = -1$. Insertion of this in (23) and rewriting terms yields

$$\tfrac{\partial E[\pi_i]}{\partial RO_i} = -i^{RO} + \max\left\{i^{IBM}-\gamma, i^{DF}\right\} G\left(\bar{t}_i\right) + \min\left\{i^{IBM}+\gamma, i^{LF}\right\} \left[1 - G\left(\bar{t}_i\right)\right]. \tag{24}$$

Note that $\frac{\partial E[\pi_i]}{\partial RO_i}$ decreases in $G\left(\bar{t}_i\right) \in [0,1]$, which in turn (weakly) increases in $\bar{t}_i$. Moreover, we know from (12) that

- $\bar{t}_i$ increases in RO_i, so that $\frac{\partial E[\pi_i]}{\partial RO_i}$ (weakly) decreases in RO_i,
- and from the restriction $RO_i \geq 0$ that $\bar{t}_i$ is restricted to $\bar{t}_i \geq -\frac{c}{(1-c)\chi} =: \tilde{t}$.

Denoting optima by the superscript *opt*, we can distinguish three cases:

1. If $i^{IBM} > i^{RO} + \gamma$, then $\frac{\partial E[\pi_i]}{\partial RO_i} > 0$ for all $G\left(\bar{t}_i\right)$. Therefore, we obtain $\bar{t}_i^{opt} = \infty$. In conjunction with (12), this yields $RO_i^{opt} = \infty$.
2. If $i^{IBM} \in \left[i^{RO}-\gamma, i^{RO}+\gamma\right]$, then $\frac{\partial E[\pi_i]}{\partial RO_i} = 0$ only if $\bar{t}_i = \bar{t}_i^{opt}$, where $\bar{t}_i^{opt}$ is implicitly defined by (13). In conjunction with (12) and the restriction $RO_i \geq 0$, this yields $RO_i^{opt} = \max\left\{0, \bar{t}_i^{opt}\left(1-c\right)\chi L_i^{opt} + cL_i^{opt}\right\}$, which brings us to two subcases.
   - If $\bar{t}_i^{opt} > \tilde{t}$ and thus $G\left(\bar{t}_i^{opt}\right) > G\left(\tilde{t}\right)$, then $RO_i^{opt} = \bar{t}_i^{opt}\left(1-c\right)\chi L_i^{opt} + cL_i^{opt} > 0$.
   - If $\bar{t}_i^{opt} \leq \tilde{t}$ and thus $G\left(\bar{t}_i^{opt}\right) \leq G\left(\tilde{t}\right)$, then $RO_i^{opt} = 0$.

3. If $i^{IBM} < i^{RO} - \gamma$, then $\frac{\partial E[\pi_i]}{\partial RO_i} < 0$ for all $G\left(\bar{t}_i\right)$. Therefore, we obtain $\bar{t}_i^{opt} = -\infty$. In conjunction with (12) and the restriction $RO_i \geq 0$, this yields $RO_i^{opt} = 0$.

Consequently, we have shown that only if $i^{IBM*} \in \left[i^{RO} - \gamma, i^{RO} + \gamma\right]$ a bank's optimal borrowing from the refinancing operation is described by the first order condition given by equation (13).

A.2 Proof of Lemma 2

From (12) and due to the restriction $RO_i \geq 0$ it follows that $\bar{t}_i$ is restricted to $\bar{t}_i \geq -\frac{c}{(1-c)\chi} =: \tilde{t}$. By applying the Leibniz rule on (22) and making use of the facts that $N_i = 0$ for $t_i = \bar{t}_i$ and that optimal borrowing in the refinancing operation implies $\bar{t}_i = \max\left\{\bar{t}_i^{opt}, \tilde{t}\right\}$, we obtain:

$$\begin{aligned}\frac{\partial E[\pi_i]}{\partial L_i} &= i^L - \lambda L_i - \max\left\{i^{IBM} - \gamma, i^{DF}\right\} \int_{t^{min}}^{\max\left\{\bar{t}_i^{opt}, \tilde{t}\right\}} \frac{\partial N_i}{\partial L_i} g(t_i) dt_i \\ &\quad - \min\left\{i^{IBM} + \gamma, i^{LF}\right\} \int_{\max\left\{\bar{t}_i^{opt}, \tilde{t}\right\}}^{t^{max}} \frac{\partial N_i}{\partial L_i} g(t_i) dt_i + \frac{\partial E[\pi_i]}{\partial RO_i^{opt}} \frac{\partial E\left[RO_i^{opt}\right]}{\partial L_i}.\end{aligned} \tag{25}$$

We can infer from (8) and the envelope theorem that $\frac{\partial N_i}{\partial L_i} = c + (1-c)\chi t_i$ and $\frac{\partial E[\pi_i]}{\partial RO_i^{opt}} \frac{\partial E\left[RO_i^{opt}\right]}{\partial L_i} = 0$. Insertion of this in (25) and rewriting terms yields

$$\begin{aligned}\frac{\partial E[\pi_i]}{\partial L_i} &= i^L - \lambda L_i - (1-c)\chi \max\left\{i^{IBM} - \gamma, i^{DF}\right\} \int_{t^{min}}^{\max\left\{\bar{t}_i^{opt}, \tilde{t}\right\}} t_i g(t_i) dt_i \\ &\quad - (1-c)\chi \min\left\{i^{IBM} + \gamma, i^{LF}\right\} \int_{\max\left\{\bar{t}_i^{opt}, \tilde{t}\right\}}^{t^{max}} t_i g(t_i) dt_i \\ &\quad - c \max\left\{i^{IBM} - \gamma, i^{DF}\right\} G\left(\max\left\{\bar{t}_i^{opt}, \tilde{t}\right\}\right) \\ &\quad - c \min\left\{i^{IBM} + \gamma, i^{LF}\right\} \left[1 - G\left(\max\left\{\bar{t}_i^{opt}, \tilde{t}\right\}\right)\right].\end{aligned} \tag{26}$$

This brings us to two cases.

- If $\bar{t}_i^{opt} > \tilde{t}$ and thus $G\left(\bar{t}_i^{opt}\right) > G\left(\tilde{t}\right)$, then insertion of (13) in (26) implies that $\frac{\partial E[\pi_i]}{\partial L_i} = 0$ only if (14) is met.
- If $\bar{t}_i^{opt} \leq \tilde{t}$ and thus $G\left(\bar{t}_i^{opt}\right) \leq G\left(\tilde{t}\right)$, then $\frac{\partial E[\pi_i]}{\partial L_i} = 0$ only if (14) is met.

A.3 A Bank's Expected Second Stage Marginal Funding Costs

Expected second-stage marginal funding costs in the case of a liquidity surplus are given by

$$(1-c)\chi\frac{\max\left\{i^{IBM}-\gamma,i^{DF}\right\}\int_{t^{min}}^{\max\left\{\bar{t}_i^{opt},\tilde{t}\right\}}t_i g(t_i)\,dt_i}{\max\left\{G\left(\bar{t}_i^{opt}\right),G\left(\tilde{t}\right)\right\}},$$

while expected second stage-marginal funding costs in the case of a liquidity deficit are

$$(1-c)\chi\frac{\min\left\{i^{IBM}+\gamma,i^{LF}\right\}\int_{\max\left\{\bar{t}_i^{opt},\tilde{t}\right\}}^{t^{max}}t_i g(t_i)\,dt_i}{1-\max\left\{G\left(\bar{t}_i^{opt}\right),G\left(\tilde{t}\right)\right\}}.$$

As the former occurs with the probability $\max\{G(\bar{t}_i^{opt}),G(\tilde{t})\}$ and the latter with the probability $(1-\max\{G(\bar{t}_i^{opt}),G(\tilde{t})\})$, expected second-stage marginal funding costs are given by $(1-c)\chi\phi$.

Equation (16) reveals that these costs are formally the same for all cases described in Proposition 1 in which the interbank market is active. For interpreting this term in more detail, it is useful to consider that, due to $E[T]=0$, the expected second stage liquidity deficit per unit of loans equals the negative value of the expected second stage liquidity surplus per unit of loans:

$$(1-c)\chi\int_{\max\left\{\bar{t}_i^{opt},\tilde{t}\right\}}^{t^{max}}t_i g(t_i)\,dt_i=-(1-c)\chi\int_{t^{min}}^{\max\left\{\bar{t}_i^{opt},\tilde{t}\right\}}t_i g(t_i)\,dt_i. \tag{27}$$

If $RO=cL$, banks will balance their liquidity needs solely via the interbank market and the facilities will not be used. Considering (14), expected second stage marginal funding costs are then

$$(1-c)\chi\int_{t^{min}}^{\max\left\{\bar{t}_i^{opt},\tilde{t}\right\}}t_i g(t_i)\,dt_i(i^{IBM*}-\gamma)+(1-c)\chi\int_{\max\left\{\bar{t}_i^{opt},\tilde{t}\right\}}^{t^{max}}t_i g(t_i)\,dt_i(i^{IBM*}+\gamma). \tag{28}$$

Obviously, the interbank rate constitutes negative marginal funding costs in the case of an individual liquidity surplus and positive marginal funding costs in the case of an individual liquidity deficit, while transaction costs have a positive impact on marginal funding costs in both cases. Expression (28) shows that the effects of the interbank rate on expected marginal funding costs compensate each other so that only interbank market transaction costs are relevant for a bank's expected second stage marginal funding costs. Considering

(27), the expected share per unit of loans for which funding costs are expected to accrue is $2(1-c)\chi \int_{\max\{\bar{t}_i^{opt},\tilde{t}\}}^{t^{max}} t_i g(t_i)\, dt_i$, the relevant funding costs are γ, so that (28) is equivalent to (16).

If $RO > cL$, banks will cover their individual liquidity deficit in the interbank market at marginal costs of $i^{IBM*} + \gamma = i^{DF} + 2\gamma$. In the case of an individual liquidity surplus, they place their excess liquidity in the interbank market or in the deposit facility so that marginal revenues are given by i^{DF}. Consequently, expected second stage marginal funding costs are

$$(1-c)\chi \int_{t^{min}}^{\max\{\bar{t}_i^{opt},\tilde{t}\}} t_i g(t_i)\, dt_i i^{DF} + (1-c)\chi \int_{\max\{\bar{t}_i^{opt},\tilde{t}\}}^{t^{max}} t_i g(t_i)\, dt_i (i^{DF} + 2\gamma), \tag{29}$$

which is again equivalent to (16). The expected marginal funding costs given by (29) show that the interest rate i^{DF} has a negative impact on these costs in the case of a liquidity surplus and a positive impact in the case of a liquidity deficit. As these effects compensate each other, the expected share per unit of loans for which those funding costs are expected to accrue is equal to the expected liquidity deficit per unit of loans given by $(1-c)\chi \int_{\max\{\bar{t}_i^{opt},\tilde{t}\}}^{t^{max}} t_i g(t_i)\, dt_i$, the relevant funding costs are 2γ. Transaction costs are relevant for two reasons. First, they will accrue if the bank borrows liquidity in the interbank market and, second, because they imply a higher interbank rate which is $i^{DF}+\gamma$.

Analogously, if $RO < cL$, the expected share per unit of loans for which funding costs are expected to accrue is $-(1-c)\chi \int_{t^{min}}^{\max\{\bar{t}_i^{opt},\tilde{t}\}} t_i g(t_i)\, dt_i = (1-c)\chi \int_{\max\{\bar{t}_i^{opt},\tilde{t}\}}^{t^{max}} t_i g(t_i)\, dt_i$, and the relevant funding costs are 2γ.

In case there is no interbank market as $\gamma > \bar{\bar{\gamma}}$, expected second stage marginal funding costs are

$$(1-c)\chi \int_{t^{min}}^{\max\{\bar{t}_i^{opt},\tilde{t}\}} t_i g(t_i)\, dt_i i^{DF} + (1-c)\chi \int_{\max\{\bar{t}_i^{opt},\tilde{t}\}}^{t^{max}} t_i g(t_i)\, dt_i i^{LF}. \tag{30}$$

Considering (27) we obtain that expected second stage marginal funding costs are equal to (17). The expected share per unit of loans for which funding costs are expected to accrue is $(1-c)\chi \int_{\max\{\bar{t}_i^{opt},\tilde{t}\}}^{t^{max}} t_i g(t_i)\, dt_i$, and the relevant funding costs are $i^{LF} - i^{DF}$.

A.4 Proof of Proposition 2

Following Proposition 1, we distinguish between an active and an inactive interbank market to determine the interbank rate and bank aggregate borrowing and lending in equilibrium.

Active Interbank Market ($\gamma \leq \bar{\bar{\gamma}}$)

It is useful to distinguish between the three cases described in Proposition 1:

1. Suppose that an equilibrium exists with $RO^* < cL^*$. Then, (10) implies $i^{IBM^*} = i^{LF} - \gamma$ while according to (12) $RO^* = \bar{t}^* (1-c)\chi L^* + cL^* \geq 0$ implies $\bar{t} < 0$ and thus
$$G(\bar{t}) < G(0).$$
Insertion of i^{IBM^*} in (13) yields
$$G(\bar{t}) = \tfrac{i^{LF} - i^{RO}}{2\gamma} < G(0).$$
For all $\gamma \in [0, \bar{\bar{\gamma}}]$ it follows due to $i^{LF} - i^{RO} \geq i^{RO} - i^{DF}$ that $\frac{i^{LF} - i^{RO}}{2\gamma} > 0.5$. As we assume that $G(0) < 0.5$, $RO^* < cL^*$ does not constitute an equilibrium.

2. Suppose that an equilibrium exists with $RO^* = cL^*$. Then, (10) implies
$$i^{IBM^*} \in \left[i^{DF} + \gamma, i^{LF} - \gamma\right],$$
while according to (12) $RO^* = \bar{t}^* (1-c)\chi L^* + cL^* > 0$ implies $\bar{t} = 0$ and thus $G(\bar{t}) = G(0)$. Insertion of $G(\bar{t})$ in (13) yields
$$i^{IBM^*} = i^{RO} - \gamma + 2\gamma G(0),$$
and thus $G(0) \in \left[\frac{i^{DF} + 2\gamma - i^{RO}}{2\gamma}, \frac{i^{LF} - i^{RO}}{2\gamma}\right]$. As $RO^* = cL^*$, there is no aggregate liquidity deficit at the second stage so that neither the lending nor the deposit facility is used.

3. Suppose that an equilibrium exists with $RO^* > cL^*$. Then, (10) implies $i^{IBM^*} = i^{DF} + \gamma$ while according to (12) $RO^* = \bar{t}^* (1-c)\chi L^* + cL^* > 0$ implies $\bar{t} > 0$ and thus
$$G(\bar{t}) > G(0).$$
Insertion of i^{IBM^*} in (13) yields
$$G(\bar{t}) = \tfrac{i^{DF} + 2\gamma - i^{RO}}{2\gamma} > G(0).$$

As $RO^* > cL^*$, banks have to place their aggregate liquidity surplus of the second stage in the deposit facility so that $DF^* > 0$.

Inactive Interbank Market ($\gamma > \bar{\bar{\gamma}}$)

It is useful to distinguish between the same three cases as for an active interbank market:

1. Suppose that an equilibrium exists with $RO^* < cL^*$. Then, we have $i^{IBM^*} \in \left[i^{LF} - \gamma, i^{DF} + \gamma\right]$ while $RO^* = \bar{t}^* (1-c)\,\chi L^* + cL^* > 0$ implies $\bar{t} < 0$ and thus

$$G\left(\bar{t}\right) < G\left(0\right).$$

   Insertion of i^{IBM^*} in (13) yields

$$G\left(\bar{t}\right) = \frac{i^{LF}-i^{RO}}{i^{LF}-i^{DF}} < G\left(0\right).$$

   It follows due to $i^{LF} - i^{RO} \geq i^{RO} - i^{DF}$ that $\frac{i^{LF}-i^{RO}}{i^{LF}-i^{DF}} > 0.5$. As we assume that $G(0) < 0.5$, $RO^* < cL^*$ does not constitute an equilibrium.

2. Suppose that an equilibrium exists with $RO^* = cL^*$. Then, we have $i^{IBM^*} \in \left[i^{LF} - \gamma, i^{DF} + \gamma\right]$ while $RO^* = \bar{t}^* (1-c)\,\chi L^* + cL^* > 0$ implies $\bar{t} = 0$ and thus

$$G\left(\bar{t}\right) = G\left(0\right).$$

   Insertion of i^{IBM^*} in (13) yields again

$$G\left(\bar{t}\right) = \frac{i^{LF}-i^{RO}}{i^{LF}-i^{DF}} = G\left(0\right).$$

   Due to $G(0) < 0.5$, $RO^* = cL^*$ does not constitute an equilibrium either.

3. Suppose that an equilibrium exists with $RO^* > cL^*$. Then, we have $i^{IBM^*} \in \left[i^{LF} - \gamma, i^{DF} + \gamma\right]$ while $RO^* = \bar{t}^* (1-c)\,\chi L^* + cL^* > 0$ implies $\bar{t} > 0$ and thus

$$G\left(\bar{t}\right) > G\left(0\right).$$

   Insertion of i^{IBM^*} in (13) yields again

$$G\left(\bar{t}\right) = \frac{i^{LF}-i^{RO}}{i^{LF}-i^{DF}} > G\left(0\right).$$

Banks with a liquidity deficit have to borrow from the lending facility, while banks with a liquidity surplus have to place their excess liquidity in the deposit facility. As $RO^* > cL^*$, it follows that $DF^* > LF^* > 0$.

A.5 Impact of Uncertainty and Transaction Costs on Loan Supply

In this subsection, we present the derivation of the argument presented in Section 7 that an increase in both uncertainty and frictions in the interbank market have a negative effect on bank loan supply.

It follows from (14) and Proposition 2 that

$$L^{j*} = \frac{1}{\lambda}\left[i^L - ci^{RO} - \left[(1-c)\chi \int_{\bar{t}^*}^{t^{max}} t_i g(t_i) dt_i\right]\xi\right] \tag{31}$$

with

$$\xi = \begin{cases} 2\gamma & \text{if} \quad \gamma \leq \bar{\bar{\gamma}}, \\ i^{LF} - i^{DF} & \text{if} \quad \gamma > \bar{\bar{\gamma}}. \end{cases} \tag{32}$$

Uncertainty

Applying the Leibniz rule on (31), the first derivative w.r.t. χ reads

$$\frac{\partial L^{j*}}{\partial \chi} = -\frac{(1-c)\xi}{\lambda}\left[\int_{\bar{t}^*}^{t^{max}} t_i g(t_i) dt_i - \chi \frac{\partial \bar{t}^*}{\partial \chi} \bar{t}^* g(\bar{t}^*)\right].$$

We derive from (13) the function

$$F := \max\left\{i^{IBM} - \gamma, i^{DF}\right\} G\left(\bar{t}^*\right) + \min\left\{i^{IBM} + \gamma, i^{LF}\right\}\left[1 - G\left(\bar{t}^*\right)\right] - i^{RO} = 0. \tag{33}$$

Applying the implicit function theorem on (33) thus yields

$$\frac{\partial \bar{t}^*}{\partial \chi} = -\frac{\frac{\partial F}{\partial \chi}}{\frac{\partial F}{\partial \bar{t}^*}} = 0 \;\; \forall \;\; \bar{t}^* \tag{34}$$

so that

$$\frac{\partial L^{j*}}{\partial \chi} = -\frac{(1-c)\xi}{\lambda} \int_{\bar{t}^*}^{t^{max}} t_i g(t_i) dt_i < 0 \;\; \forall \;\; j.$$

Transaction Costs

If $\gamma \leq \bar{\bar{\gamma}}$, it follows from (32) that $\xi = 2\gamma$. Applying the Leibniz rule on (31), the first derivative w.r.t. γ then reads

$$\frac{\partial L^{j*}}{\partial \gamma} = -\frac{(1-c)2\chi}{\lambda}\left[\int_{\bar{t}^*}^{t^{max}} t_i g(t_i) dt_i - \gamma \frac{\partial \bar{t}^*}{\partial \gamma} \bar{t}^* g(\bar{t}^*)\right].$$

Applying the implicit function theorem on (33) yields

$$\frac{\partial \bar{t}^*}{\partial \gamma} = -\frac{\frac{\partial F}{\partial \gamma}}{\frac{\partial F}{\partial \bar{t}^*}} = -\frac{G(\bar{t}^*)}{\gamma g(\bar{t}^*)} \qquad \text{if } \ j = \text{I}, \tag{35}$$

$$\frac{\partial \bar{t}^*}{\partial \gamma} = -\frac{\frac{\partial F}{\partial \gamma}}{\frac{\partial F}{\partial \bar{t}^*}} = \frac{1-G(\bar{t}^*)}{\gamma g(\bar{t}^*)} \qquad \text{if } \ j = \text{II}, \tag{36}$$

so that

$$\frac{\partial L^{\text{I}*}}{\partial \gamma} = \frac{2(1-c)\chi}{\lambda} G\left(\bar{t}^*\right)\left[E\left[t_i | t_i < \bar{t}^*\right] - \bar{t}^*\right] < 0,$$
$$\frac{\partial L^{\text{II}*}}{\partial \gamma} = -\frac{2(1-c)\chi}{\lambda}\left[1 - G(\bar{t}^*)\right]\left[E\left[t_i | t_i \geq \bar{t}^*\right] - \bar{t}^*\right] < 0.$$

If $\gamma > \bar{\bar{\gamma}}$, it follows from (32) that $\xi = i^{LF} - i^{DF}$. Applying the implicit function theorem on (33) yields

$$\frac{\partial \bar{t}^*}{\partial \gamma} = -\frac{\frac{\partial F}{\partial \gamma}}{\frac{\partial F}{\partial \bar{t}^*}} = 0 \tag{37}$$

so that $\frac{\partial L^{\text{III}*}}{\partial \gamma} = 0$.

Moreover, it follows from (34) that the mixed partial derivative with respect to χ reads

$$\frac{\partial^2 L^{\text{I}*}}{\partial \gamma \partial \chi} = \frac{2(1-c)}{\lambda} G\left(\bar{t}^*\right)\left[E\left[t_i | t_i < \bar{t}^*\right] - \bar{t}^*\right] < 0,$$
$$\frac{\partial^2 L^{\text{II}*}}{\partial \gamma \partial \chi} = -\frac{2(1-c)}{\lambda}\left[1 - G(\bar{t}^*)\right]\left[E\left[t_i | t_i \geq \bar{t}^*\right] - \bar{t}^*\right] < 0.$$

A.6 Proof of Proposition 3

We proof this proposition in two steps. First, we determine the derivative with respect to i^{RO} for each feasible equilibrium. Afterwards, we derive the respective mixed partial derivative with respect to χ and γ.

1. Applying the Leibniz rule on (31), the first derivative w.r.t. i^{RO} reads

$$\frac{\partial L^{j*}}{\partial i^{RO}} = -\frac{1}{\lambda}\left[c - (1-c)\chi \frac{\partial \bar{t}^*}{\partial i^{RO}} \bar{t}^* g(\bar{t}^*)\xi\right] \quad \forall \;\; j.$$

Applying the implicit function theorem on (33) yields

$$\frac{\partial \bar{t}^*}{\partial i^{RO}} = -\frac{\frac{\partial F}{\partial i^{RO}}}{\frac{\partial F}{\partial \bar{t}^*}} = 0 \qquad \text{if} \;\; j = \text{I},$$

$$\frac{\partial \bar{t}^*}{\partial i^{RO}} = -\frac{\frac{\partial F}{\partial i^{RO}}}{\frac{\partial F}{\partial \bar{t}^*}} = -\frac{1}{\xi g(\bar{t}^*)} \qquad \text{if} \;\; j = \{\text{II}, \text{III}\}.$$

As $\bar{t}^* = 0$ for $j = \text{I}$, it follows for all j

$$\frac{\partial L^{j*}}{\partial i^{RO}} = -\frac{1}{\lambda}\left[c + (1-c)\chi \bar{t}^*\right].$$

If $\gamma \leq \bar{\bar{\gamma}}$, it follows that $2\gamma < i^{LF} - i^{DF}$. In Equilibrium II expected marginal revenues of borrowing from the refinancing operations are given by

$$i^{DF} + 2\gamma\left[1 - G\left(\bar{t}_i^{opt}\right)\right], \tag{38}$$

while in Equilibrium III they read

$$i^{DF} G\left(\bar{t}_i^{opt}\right) + i^{LF}\left[1 - G\left(\bar{t}_i^{opt}\right)\right] \tag{39}$$

Comparing (38) and (39) shows that $G(\bar{t}_i^{opt})^{\text{II}} < G(\bar{t}_i^{opt})^{\text{III}}$ so that $\bar{t}_i^{opt\text{II}} < \bar{t}_i^{opt\text{III}}$. Due to $L_i^{opt} = L^*$ and $RO_i^{opt} = RO^*$, it follows that $\bar{t}^{*\text{II}} < \bar{t}^{*\text{III}}$ and thus $\frac{\partial L^{\text{III}*}}{\partial i^{RO}} \leq \frac{\partial L^{\text{II}*}}{\partial i^{RO}} < \frac{\partial L^{\text{I}*}}{\partial i^{RO}} < 0$.

2. (a) In order to determine the mixed partial derivative with respect to χ, we make use of the result obtained in (34). It thus follows that

$$\frac{\partial^2 L^{j*}}{\partial i^{RO} \partial \chi} = -\frac{1-c}{\lambda}\bar{t}^*,$$

so that $\frac{\partial^2 L^{\text{III}*}}{\partial i^{RO}\partial\chi} < \frac{\partial^2 L^{\text{II}*}}{\partial i^{RO}\partial\chi} < 0$ and $\frac{\partial^2 L^{\text{I}*}}{\partial i^{RO}\partial\chi} = 0$.

(b) In order to determine the mixed partial derivative with respect to γ, we make use of the results obtained in (35), (36) and (37). It thus follows that

$$\begin{aligned}
\frac{\partial^2 L^{\mathrm{I}*}}{\partial i^{RO}\partial\gamma} &= 0,\\
\frac{\partial^2 L^{\mathrm{II}*}}{\partial i^{RO}\partial\gamma} &= -\frac{(1-c)\,\chi}{\lambda}\frac{\partial\bar{t}^*}{\partial\gamma} = -\frac{(1-c)\,\chi\left[1-G(\bar{t}^*)\right]}{\lambda\gamma g(\bar{t}^*)} < 0,\\
\frac{\partial^2 L^{\mathrm{III}*}}{\partial i^{RO}\partial\gamma} &= 0.
\end{aligned}$$

A.7 Proof of Proposition 4

We proof this proposition in two steps. First, we apply the total derivative to determine the impact of a change in the overall interest rate level. Afterwards, we derive the respective mixed partial derivative with respect to χ and γ.

1. Given $di^{LF} = di^{RO} = di^{DF}$, applying the total derivative on (31) yields

$$dL^{j*} = \frac{1}{\lambda}\left[-c di^{RO} - \left[(1-c)\chi d\bar{t}^* \frac{\partial}{\partial \bar{t}^*}\int_{\bar{t}^*}^{t^{max}} t_i g(t_i) dt_i\right]\xi\right].$$

   Moreover, applying the total derivative on (33) yields

$$\begin{aligned}
\frac{\partial G\left(\bar{t}^*\right)}{\partial\bar{t}^*}d\bar{t}^* &= 0 \quad \text{if} \quad j = \mathrm{I},\\
di^{DF} - 2\gamma\frac{\partial G\left(\bar{t}^*\right)}{\partial\bar{t}^*}d\bar{t}^* - di^{RO} &= 0 \quad \text{if} \quad j = \mathrm{II},\\
di^{LF} - \left(i^{DF} - i^{LF}\right)\frac{\partial G\left(\bar{t}^*\right)}{\partial\bar{t}^*}d\bar{t}^* + (di^{DF} - di^{LF})G\left(\bar{t}^*\right) - di^{RO} &= 0 \quad \text{if} \quad j = \mathrm{III}.
\end{aligned}$$

   Due to $di^{RO} = di^{DF} = di^{LF}$ it follows for all j that $d\bar{t}^* = 0$ so that

$$\frac{dL^{j*}}{di^{RO}} = -\frac{c}{\lambda}. \tag{40}$$

2. It follows directly from (40) that $\frac{\partial^2 L^{j*}}{\partial i^{RO}\partial\chi} = \frac{\partial^2 L^{j*}}{\partial i^{RO}\partial\gamma} = 0$ for all j.

A.8 Proof of Proposition 5

We proof this proposition analogously to the proof of Proposition 4 in two steps. First, we apply the total derivative to determine the impact of a change in the rates of the facilities. Afterwards, we derive the respective mixed partial derivative with respect to χ and γ.

1. Given $di^{LF} = di^{DF}$ and $di^{RO} = 0$, applying the total derivative on (31) yields

$$
\begin{aligned}
dL^{j*} &= -\frac{2\gamma(1-c)}{\lambda}\chi d\bar{t}^* \frac{\partial}{\partial \bar{t}^*}\int_{\bar{t}^*}^{t^{max}} t_i g(t_i) dt_i && \text{if } j = \{\mathrm{I}, \mathrm{II}\}, \\
dL^{j*} &= -\frac{1-c}{\lambda}\chi\left[(i^{LF} - i^{DF})d\bar{t}^* + 2di^{LF}\right]\frac{\partial}{\partial \bar{t}^*}\int_{\bar{t}^*}^{t^{max}} t_i g(t_i) dt_i && \text{if } j = \mathrm{III}.
\end{aligned}
$$

Moreover, applying the total derivative on (33) yields

$$
\begin{aligned}
\frac{\partial G\left(\bar{t}^*\right)}{\partial \bar{t}^*} d\bar{t}^* &= 0 \quad \text{if } j = \mathrm{I}, \\
di^{DF} - 2\gamma\frac{\partial G\left(\bar{t}^*\right)}{\partial \bar{t}^*} d\bar{t}^* &= 0 \quad \text{if } j = \mathrm{II}.
\end{aligned}
$$

As long as the interest rate corridor is symmetric, it follows for $\gamma > \bar{\gamma}$ that $G\left(\bar{t}^*\right) = 0.5$ so that

$$
d\bar{t}^* = 0 \quad \text{if } j = \mathrm{III}.
$$

Considering $\frac{\partial i^{LF}}{\partial(i^{LF} - i^{DF})} = 0.5$ and $\frac{\partial i^{DF}}{\partial(i^{LF} - i^{DF})} = -0.5$ it follows

$$
\frac{\partial L^{\mathrm{I}*}}{\partial(i^{LF} - i^{DF})} = \frac{dL^{\mathrm{I}*}}{di^{LF}}\frac{\partial i^{LF}}{\partial(i^{LF} - i^{DF})} = 0, \tag{41}
$$

$$
\frac{\partial L^{\mathrm{II}*}}{\partial(i^{LF} - i^{DF})} = \frac{dL^{\mathrm{II}*}}{di^{DF}}\frac{\partial i^{DF}}{\partial(i^{LF} - i^{DF})} = -\frac{(1-c)\chi}{\lambda}\bar{t}^* < 0, \tag{42}
$$

$$
\frac{\partial L^{\mathrm{III}*}}{\partial(i^{LF} - i^{DF})} = \frac{dL^{\mathrm{III}*}}{di^{LF}}\frac{\partial i^{LF}}{\partial(i^{LF} - i^{DF})} = -\frac{(1-c)\chi}{\lambda}\int_{\bar{t}^*}^{t^{max}} t_i g(t_i) dt_i < 0. \tag{43}
$$

2. Making use of the result obtained in (34), it follows directly from (41) to (43) that

$$
\begin{aligned}
\frac{\partial^2 L^{\mathrm{I}*}}{\partial(i^{LF} - i^{DF})\partial\chi} &= 0, \\
\frac{\partial^2 L^{\mathrm{II}*}}{\partial(i^{LF} - i^{DF})\partial\chi} &= -\frac{(1-c)}{\lambda}\bar{t}^* < 0, \\
\frac{\partial^2 L^{\mathrm{III}*}}{\partial(i^{LF} - i^{DF})\partial\chi} &= -\frac{(1-c)}{\lambda}\int_{\bar{t}^*}^{t^{max}} t_i g(t_i) dt_i < 0.
\end{aligned}
$$

Making use of the result obtained in (36) and (37), it follows that

$$
\begin{aligned}
\frac{\partial^2 L^{\mathrm{j}*}}{\partial(i^{LF} - i^{DF})\partial\gamma} &= 0 \quad \text{for } j = \mathrm{II},\, \mathrm{III}, \\
\frac{\partial^2 L^{\mathrm{II}*}}{\partial(i^{LF} - i^{DF})\partial\gamma} &= -\frac{(1-c)}{\lambda}\frac{1 - G(\bar{t}^*)}{\gamma g(\bar{t}^*)} < 0.
\end{aligned}
$$

A.9 Proof of Proposition 6

We proof this proposition in the same two steps as in the proof of Proposition 5.

1. Given $di^{LF} = -di^{DF}$ and $di^{RO} = 0$, applying the total derivative on (31) yields

$$dL^{j*} = -\frac{2\gamma(1-c)}{\lambda}\chi d\bar{t}^* \frac{\partial}{\partial \bar{t}^*}\int_{\bar{t}^*}^{t^{max}} t_i g(t_i) dt_i \qquad \text{if} \quad j = \{\text{I}, \text{II}\},$$
$$dL^{j*} = -\frac{1-c}{\lambda}\chi(i^{LF} - i^{DF})d\bar{t}^* \frac{\partial}{\partial \bar{t}^*}\int_{\bar{t}^*}^{t^{max}} t_i g(t_i) dt_i \qquad \text{if} \quad j = \text{III}.$$

   Moreover, applying the total derivative on (33) yields

$$\frac{\partial G\left(\bar{t}^*\right)}{\partial \bar{t}^*} d\bar{t}^* = 0 \quad \text{if} \quad j = \text{I},$$
$$di^{DF} - 2\gamma \frac{\partial G\left(\bar{t}^*\right)}{\partial \bar{t}^*} d\bar{t}^* = 0 \quad \text{if} \quad j = \text{II},$$
$$di^{LF} - (i^{LF} - i^{DF}) \frac{\partial G\left(\bar{t}^*\right)}{\partial \bar{t}^*} d\bar{t}^* = 0 \quad \text{if} \quad j = \text{III},$$

   so that

$$\frac{\partial L^{\text{I}*}}{\partial i^{DF}} = 0, \tag{44}$$
$$\frac{\partial L^{\text{j}*}}{\partial i^{DF}} = \frac{(1-c)\chi}{\lambda}\bar{t}^* > 0 \quad \text{for} \quad j = \text{II, III}. \tag{45}$$

2. Making use of the result obtained in (34), it follows directly from (44) to (45) that

$$\frac{\partial^2 L^{\text{I}*}}{\partial i^{DF} \partial \chi} = 0,$$
$$\frac{\partial^2 L^{\text{j}*}}{\partial i^{DF} \partial \chi} = \frac{(1-c)}{\lambda}\bar{t}^* > 0 \quad \text{for} \quad j = \text{II, III}.$$

   Making use of the result obtained in (36) and (37), it follows that

$$\frac{\partial^2 L^{\text{j}*}}{\partial i^{DF} \partial \gamma} = 0 \quad \text{for} \quad j = \text{I, III},$$
$$\frac{\partial^2 L^{\text{II}*}}{\partial i^{DF} \partial \gamma} = \frac{(1-c)}{\lambda}\frac{1 - G(\bar{t}^*)}{\gamma g(\bar{t}^*)} > 0.$$

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