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STRICT PROPORTIONAL POWER AND FAIR VOTING RULES IN COMMITTEES

(Extended abstract)

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Simple weighted committee is a pair [N, **w**], where N be a finite set of n committee members $i = 1,2,...,n$, and **w** = (w_1, w_2, …, w_n) be a nonnegative vector of committee members' voting weights (e.g. votes or shares). By 2^N we denote power set of N (set of all subsets of N). By voting configuration we mean an element S ∈ 2^N, subset of committee members voting uniformly (YES or NO), and $w(S) = \sum_{i \in S} w_i$ denotes voting weight of configuration S. Voting rule is defined by quota q, satisfying $0 < q \le w(N)$, where q represents minimal total weight necessary to approve the proposal. Triple [N, q, **w**] we call a *simple quota weighted committee*. Voting configuration S in committee [N, q, **w**] is called a winning one if $w(S) \ge q$ and a losing one in the opposite case. Winning voting configuration S is called critical if there exists at least one member k ∈ S such that w(S\k) < q (we say that k is critical in S). Winning voting configuration S is called minimal if any of its members is critical in S.

A priori voting power analysis seeks an answer to the following question: Given a simple quota weighted committee $[N, q, \mathbf{w}]$, what is an influence of its members over the outcome of voting? Absolute voting power of a member i is defined as a probability $\Pi_i[N, q, \mathbf{w}]$ that i will be decisive in the sense that such situation appears in which she would be able to decide the outcome of voting by her vote (Nurmi (1997)), and a relative voting power as

$$\pi_i[N, q, w] = \frac{\Pi_i[N, q, w]}{\sum_{k \in N} \Pi_k[N, q, w]}$$

Two most frequently used measures of a priori voting power are Shapley-Shubik power index (based on concept of pivot and Penrose-Banzhaf power index (based on concept of swing)

Concept of fairness is being discussed related to distribution of voting power among different actors of voting. This problem was clearly formulated by Nurmi (1982): "*If one aims at designing collective decision making bodies which are democratic in the sense of reflecting the popular support in terms of the voting power, we need indices of the latter which enable us to calculate for any given distribution of support and for any decision rule the distribution of seats that is 'just'. Alternatively, we may want to design decision rules that – given the distribution of seats and support – lead to a distribution of voting power which is identical with the distribution of support.*"

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Voting power is not directly observable: as a proxy for it voting weights are used (number of seats, number of votes, shares of population, square roots of shares of population, membership fees to some multilateral organization etc.). Therefore, fairness is usually defined in terms of voting weights (e.g. voting weights proportional to results of election).

Assuming, that a principle of fairness is selected for a distribution of voting weights, we are addressing the question how to achieve equality of voting power (at least approximately) to fair voting weights. The concepts of strict proportional power and randomized decision rule introduced by Holler (1985) and analyzed in Berg and Holler (1986), of optimal quota of Słomczyński and Życzkowski (2007), and of intervals of stable power (Turnovec (2008)) are used to find, given voting weights, a voting rule minimizing a distance between actors' voting weights and their voting power.

In the first section of the paper basic definitions are introduced and the applied power indices methodology shortly resumed. The second section introduces concepts of quota intervals of stable power and optimal quota. It is shown that in a simple weighted committees with finite number n of members, fixed weights and changing quota, there exists a finite number r of different quota intervals of stable power ($r \leq 2^n-1$) generating finite number of power indices vectors. If voting power is equal to blocking power, then number of different power indices vectors corresponding to majority quotas is equal to at most int (r/2) + 1. If the fair distribution of voting weights is defined, then fair distribution of voting power means to find a quota that minimizes distance between relative voting weights and relative voting power (optimal quota). Index of fairness is introduced as a function of quota. The problem of optimal quota has an exact solution via finite number of majority marginal quotas. While the framework of analysis of fairness is usually restricted to Penrose-Banzhaf concept of power, we are treating it in a more general setting and our results are relevant for any power index based on pivots or swings and for any concept of fairness.

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