

## **MATHEMATICS OF POLITICS**

### **ECONOMICS METHODOLOGY IN POLITICAL SCIENCE**

František Turnovec

Charles University in Prague, Faculty of Social Sciences, Institute of Economic Studies

**Abstract:** In this paper the concept of politometrics is proposed, defined in a similar way as Ragnar Frisch in 1932 had defined the econometrics: as the discipline summarizing application of mathematical and statistical techniques to political science problems and theories. Different possible topics of politometrics are discussed, such as models of voting, measures of influence in committee systems and regression models of socio-demographic determinants of voters' behavior. Relevant research agenda is briefly outlined.

**Keywords:** economics, mathematics, politics, power, voting

#### **1. Economics, political science and mathematics**

The use of mathematics in the social sciences is expanding both in breadth and depth at an increasing rate. It has made its way from economics into the other social sciences, often accompanied by the same controversy that raged in economics in the 1950's. The reasons for this expansion are several: "First, mathematics makes communication between researchers succinct and precise. Second, it helps make assumptions and models clear; this bypasses arguments in the field that are a result of different implicit assumptions. Third, proofs are rigorous, so mathematics helps avoid mistakes in the literature. Fourth, its use often provides more insights into the models. And finally, the models can be applied to different contexts without repeating the analysis, simply by renaming the symbols."<sup>1</sup>

In this paper we follow the Duncan Black's idea that "Economics and Political Science are the same in kind: that when we do eventually obtain a 'satisfactory' Political Science, it will have the same distinguishing marks as Walras' Elements or Pareto's Manuel, or perhaps Marshall's Principles, with the admixture of the rigorously formal and the descriptive treatment – rather than those of the existing texts in Politics. And the core of the treatment, we hold, will consist of a set of formal or mathematical propositions"<sup>2</sup>.

---

<sup>1</sup> Schofield, N. (2004), *Mathematical Methods in Economics and Social Sciences*, Springer, Berlin, Heidelberg, New York.

<sup>2</sup> Quoted from Black D., *The Theory of Committees and Elections*, revised second edition, edited by Iain McLean, Alistair McMillan and Burt L. Monroe, Kluwer Academic Publishers, Boston, Dordrecht, London, 1998, 353-361.

First Nobel Prize Laureate for economics (1969), Norwegian economist Ragnar Frisch, introduced in 1932 concept of econometrics as “the application of mathematical and statistical techniques to economic problems and theories”. During last 70 years econometrics evolved to be one of the fundamental instruments of economic analyses. In a broader sense (as was originally interpreted by Ragnar Frisch) econometrics formulate mathematical models of economic processes and using observable data discovers directly unobservable properties, verifies propositions or conclusions derived from abstract model analysis. In a narrow sense econometrics is an instrument for empirical testing of hypotheses of economic theory.

While the focus of economics is on human behavior on the market place, characterized by data such as GDP, inflation, unemployment, income, consumption, investments, savings, trade, etc., in political science we study human behavior in the public arena, outside of the market, characterized by electoral preferences, political behavior in representative bodies, behavior of central, territorial and local governments etc. In both cases we have some observable data, time series, provided by statistical service, sample studies, electoral statistics etc. It was economics which started to understand that traditional dichotomy between “homo oeconomicus” and “homo politicus” is counterproductive and that it is not possible to understand and explain economic phenomena without study of political behavior: in a society where 40% of GDP is redistributed by different decision making bodies, where provision of public goods is regulated by various councils, committees, parliaments, where positive and negative externalities lead to market failures, the doctrine of “self-regulated market place” and “invisible hand of competition” does not provide satisfactory answers to appealing questions. Thus, approximately since 1948, within the framework of economic sciences new disciplines emerged, studying the problems of collective choice, bureaucracy behavior, rent seeking, voting behavior, institutions etc. In this respect we can speak about elements of application of economics methodology to political sciences. This approach extends power and deepness of economic analyses and provides new interesting theoretical and empirical results. It is interesting, that among Nobel Prize Laureates for economics it is possible to find outstanding scientists representing this orientation in economics: Kenneth J. Arrow (1972), James M. Buchanan (1986), John Nash, John Harsanyi and Richard Selten (1994), Amartya K. Sen (1998).

The question is: why political sciences, using similar data, should not use similar methodology? Concept of politometrics, proposed in this paper, can be defined in the same way as Ragnar Frisch defined econometrics: the application of mathematical and statistical techniques to political problems and theories.

In this paper we try to illustrate in a very simple way possible subjects of politometrics, such as modeling of political processes, analyzing electoral systems, measuring power in committees, explaining voters’ behavior. We also shortly outline some topics of the relevant research agenda.

First time the concept of politometrics was introduced in Turnovec (2003). In recent literature the comprehensive treatment of the mathematics applications in political science see e.g. in Brams (2008), Taylor and Pacelli (2008), Schofield (2004).

## **2. Voting as an aggregation of individual preferences**

By voting we mean the following pattern of collective choice: There is a set of alternatives and a group of individuals. Individual preferences over the alternatives are exogenously specified and are supposed to be orderings. The group is required to choose an alternative on the basis of stating and aggregating of all individual preferences, or to produce a ranking of alternatives from the most preferred to the least preferred.

To show that problems with voting are not as simple as one can expect, let us start with almost trivial example.

**Example 1:** Consider 3 candidates,  $A = \{x, y, z\}$ , and 9 voters,  $N = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$  with a preference profile given in the Table 1.

Table 1

1	2	3	4	5	6	7	8	9
z	z	z	z	x	x	x	y	y
x	y	y	y	y	y	y	x	x
x	x	x	x	z	z	z	z	z

We can produce a pair-wise comparisons matrix with entries indicating how many voters prefer a "row" candidate to the "column candidate" (Table 2).

Table 2

	x	y	z
x	-	3	5
y	6	-	5
z	4	4	-

Let us apply several standard voting procedures to this situation:

**THE CONDORCET' VOTING PROCEDURE:** The candidate is chosen if he is not defeated by a strict majority by any other candidate. In our case the candidate y is selected and we receive the ordering  $y P x P z$  (by  $a P b$  we denote that a candidate  $a$  is collectively, by voting, preferred to candidate  $b$ ).

**PLURALITY VOTING PROCEDURE:** The candidate who is preferred by most of the voters to all other candidates is selected. In our case this voting procedure selects candidate z and generates the ordering  $z P x P y$ .

**PLURALITY WITH RUN-OFF:** If no alternative receives a majority of votes on the first ballot, the top two vote getters are considered and the candidate who receives majority in the run-off is selected. In our case the candidate x is selected and we receive the ordering  $x P z P y$ .

We can see that using three different voting procedures we can get three completely different results of voting based on the same individual preferences.

The problem of voting has two aspects: individual decision of the voter (his choice among the alternatives) and the counting the votes, evaluation of results. Social choice theory investigates two aspects of voting rules and procedures: Individual voting behavior expresses individual preferences of the voter. The method of counting should guarantee an aggregation of individual preferences into something called social preferences. The key question is: how to aggregate individual preferences by some intuitively acceptable and not contradictory way into social preferences?

Selected problems:

- a) Democratic legitimacy, existence of not-contradictory algorithms of aggregating individual preferences (Arrow's impossibility theorem puzzle – collective choice is either not transitive or dictatorial).

- b) Manipulation by strategic voting: is it possible for an individual or a group of individuals to benefit by misrepresentation of their preferences? (Gibbard-Satterthwaite theorem saying that any voting procedure is either manipulable or dictatorial). Information complexity of manipulation. Rational voters (voting by sincere preferences), irrational voters (voting randomly) and sophisticated voters (voting strategically) and models of their behavior.
- c) Agenda manipulation and voting rules manipulation, how to influence result of voting by its institutional framework? Political districting in majority electoral systems and fair seats allocation in proportional electoral systems.

### 3. Calculus of influence – voting power in committees

Having a committee elected it makes sense to try to analyze *of distribution of power* among its members, quantitative evaluation of an influence and voting power of different members of the committee. At this introductory level we shall again illustrate the problem of power by a simple example.

**Example 2:** Distribution of votes among the parties in a committee is not a sufficient characteristic of power or influence distribution. This can be clearly seen by a simple example of the committee with 3 parties and 100 seats (see Table 3).

Table 3

parties	seats
1	49
2	2
3	49

With respect to the simple majority rule all three parties have the same position in the voting process (any two-parties coalition is a winning one, no single party can win). In fact, under certain circumstances (if the two large parties 1 and 3 are on the opposite sides of the political spectrum) the role of the party 2 could be essential. Quite a different situation can be observed for a qualified majority, say, 60%. In this case the party 2 has no influence on the outcomes of voting and a co-operation of parties 1 and 3 is needed for approving any bill.

It is known that a distribution of votes among the groups in a committee is not a sufficient characteristic of their voting power or an influence distribution. So called power indices are used to estimate an influence of the members of a committee as a function of a voting rule and of a structure of representation in a committee.<sup>3</sup>

The majority of proposed power indices are based on the game theoretical model of simple games in characteristic function form and on different concepts of "decisiveness" of members of a committee with respect to winning coalitions. They usually express probability of members of the body to be "decisive" in a given sense.

---

<sup>3</sup> In 1954 Lloyd Shapley and Martin Shubik published a short paper in the American Political Science Review, proposing that the Shapley value for cooperative characteristic function form games could serve as a measure of voting power in committees. In 1965 John Banzhaf proposed a new index of voting power. Since that more than twenty new definitions (with more or less satisfactory theoretical justification) of so called power indices have been published.

For illustration let us consider one of the most frequently used power indices proposed by John Banzhaf, the so called Banzhaf power index. All possible winning coalitions are considered. Each of the winning coalition is analyzed and the so called "swing" voters are identified: i.e. those who by changing their vote from "yes" to "no" could change the coalition from winning to losing. The relative "voting power" of individual members is then measured by a ratio of the number of member swings to the total number of swings in the committee. Let us apply Banzhaf measure in our example (assuming always simple majority rule). Winning coalitions and swings:  $\{A^*, B^*\}$ ,  $\{A^*, C^*\}$ ,  $\{B^*, C^*\}$ ,  $\{A, B, C\}$ . Each party has 2 swings out of 6, i.e. the relative power of each of them is  $1/3$ , i.e. the vector of relative power indices equals to  $(1/3, 1/3, 1/3)$ .

In fact, under certain circumstances (if the two large parties A and C are on the opposite sides of the political spectrum) the role of the party B can be essential. Let us suppose that A and C are strictly opposed blocs (they never vote together). Here we have the following winning coalitions and swings:  $\{A^*, B^*\}$ ,  $\{B^*, C^*\}$ . Hence, in this case B has two swings out of four swings, while A and C has only one swing and relative voting power of the parties can be evaluated as  $(1/4, 1/2, 1/4)$ .

We can introduce another assumption: let us assume that A, B, C is an ordering of the parties over some political dimension (say, left and right), and that only "ideologically connected" coalitions can be created (for example in forming a government coalition). Then we shall have the following swings:  $\{A^*, B^*\}$ ,  $\{B^*, C^*\}$ ,  $\{A, B^*, C\}$ . Each of the parties A and C has one swings, while party B has three swings out of six, therefore the evaluation of relative voting power will be  $(1/5, 3/5, 1/5)$ . We can see that even simple measure of power is flexible enough to reflect different assumptions about parties' behavior.

Selected problems:

- d) Binary (YES-NO) voting and abstention, existing models do not consider abstention as a strategic factor, while in some cases abstention could mean NO and in some cases abstention could mean YES.
- e) Fairness in voting: voting rules that guarantee distribution of power proportional to distribution of votes?
- f) Coalition formation. Waiving assumption about equal probability of different voting coalitions: how to incorporate different propensity of committee members to cooperate into the model?

#### 4. Explaining voters' behavior

Parliamentary elections provide political scientists with valuable data. Statistics and econometrics propose efficient instruments for analysis of electoral results. One of the possibilities is to look for socio-economic determinants of voters' behavior. Let us demonstrate this opportunity on a simple linear model.

Let us denote by

- n number of electoral districts in the country ( $j = 1, 2, \dots, n$ ),
- m number of political parties participating in election ( $i = 1, 2, \dots, m$ ),
- r number of socio-demographic factors (explanatory variables), such as inflation rate, average income, age structure of population, income structure, educational structure, professional structure, rate of rural population (urbanization) etc. ( $k = 1, 2, \dots, r$ ),

$p_i$  percentage of votes submitted for party  $i$  in the country,  
 $p_{ij}$  percentage of votes submitted for party  $i$  in district  $j$ ,  
 $x_k$  observed value of socio-demographic factor  $k$  in the country,  
 $x_{kj}$  observed value of socio-demographic factor  $k$  in district  $j$ ,  
 $\beta_{ik}$  rate of influence of socio-demographic factor  $k$  on voters' decision to vote for party  $i$  (how much the percentage of votes for party  $i$  will change if the value of factor  $k$  changes by a unit).

Assuming, that the percentage of votes for a given party is a linear function of explanatory variables, we want to identify  $m$  functions

$$p_i(\mathbf{x}, \boldsymbol{\beta}_i) = \sum_{k=1}^r x_k \beta_{ik}$$

expressing electoral results of party  $i$  as a linear function of socio-demographic factors.

Based on observable data  $p_{ij}$ ,  $x_{kj}$ , for each  $i$  we can estimate parameters  $\beta_{ik}$  of the function  $p_i(\mathbf{x}, \boldsymbol{\beta}_i)$  as values minimizing the sum of square deviations of the function from observed values taken by electoral districts:

$$\sum_{j=1}^n (p_{ij} - \sum_{k=1}^r x_{kj} \beta_{ik})^2$$

If the number of electoral districts  $n$  is significantly greater than the number of explanatory variables  $r$ , we can use standard econometric techniques to estimate parameters  $\beta_{ik}$  and to test how significant estimated parameters are. Careful selection of explanatory variables and cautious interpretation of results (estimated rates of influence of socio-demographic factors on voters' electoral choice) open a broad space for qualitative analysis explaining different aspects of voters' decision making and political parties concerns.

Selected problems:

- g) How to estimate (in proportional electoral systems) switches at the two consecutive elections, what fraction of voters who voted for party  $i$  in the first election switched to party  $k$  in second election (ecological regression)?
- h) Financing of political parties and efficiency of electoral campaign expenditures; does power in democracy depend on money investments?
- i) Political landscape: multi-dimensionality of ideological space. Are voters voting on ideological basis?

## 5. Roots

Politometrics can find its roots in several economic disciplines, such as econometrics, public choice and social choice, constitutional economics, welfare economics. Generally one credits James Buchanan, the Nobel Prize Laureate, and Gordon Tullock as the intellectual fathers of economic studies of politics. Their book from 1962 *Calculus of Consent* remains a classic in the relevant literature. But some of the ground-stones of the public choice were laid before James Buchanan and Gordon Tullock introduced the whole area as a separate field of economic theory.

Another Nobel Prize Laureate, Kenneth Arrow, formulated the basic problem of discovering of social preferences in his work from 1951 *Social Choice and Individual Values*. In 1958 Duncan Black in his book *The Theory of Committees and Elections* and in 1957 Anthony Downs in *Economic Theory of Democracy* extended concepts of economic competition to political competition. Indian economist and 1998 Nobel Prize Laureate Amartya K. Sen contributed to the economic theory of justice (*Collective Choice and Social Welfare*, 1970). Game theory, developed in the 40's by John von Neumann and Oskar Morgenstern provided theorists with adequate methodology. In 1954 Lloyd Shapley and Martin Shubik started the branch of research focused on power analysis.

We also should not forget contribution of outstanding scientists of the 18th and 19th centuries, who were forgotten for many years and rediscovered only in the second half of 20th century, in voting theory French mathematicians Marquis de Condorcet (1743 - 1794), who was also an important political figure shortly after the Great French Revolution, and Jean Charles de Borda (1733-1799), contributed to intellectual background of democratic ideas and originated the mathematical theory of voting. In 19th century, British mathematician Charles Dodgson (1832-1898), better known as Lewis Carroll, the author of Alice in Wonderland, extended the theory of voting.

### **Acknowledgements**

This research was supported by the Grant Agency of the Czech Republic, project No. 402/09/1066 "Political economy of voting behavior, rational voters' theory and models of strategic voting".

### **References:**

- Black D. (1998), *The Theory of Committees and Elections*, revised second edition, edited by Iain McLean, Alistair McMillan and Burt L. Monroe, Kluwer Academic Publishers, Boston, Dordrecht, London.
- Brams J. (2008), *Mathematics and Democracy*, Princeton University Press, Princeton.
- Schofield N. (2004), *Mathematical Methods in Economics and Social Choice*, Springer, Berlin, Heidelberg, New York.
- Taylor A.D. - Pacelli A.M. (2008), *Mathematics and Politics, Strategy, Voting, Power and Proof* (second edition), Springer, Berlin, Heidelberg, New York.
- Turnovec F. (2003), *Quantitative Methods in Political Sciences*. In: Dvořáková V. and A. Heroutová eds., 2. kongres českých politologů, Česká společnost pro politické vědy, Praha, 153-160.

Prof. RNDr. Ing. František Turnovec, PhD, Charles University in Prague, Faculty of Social Sciences, Institute of Economic Studies, e-mail: [turnovec@fsv.cuni.cz](mailto:turnovec@fsv.cuni.cz).