# Coordination Incentives in Cross-Border Macroprudential Regulation

Alexis Derviz

Jakub Seidler

# Coordination Incentives in Cross-Border Macroprudential Regulation

Alexis Derviz[a]

Jakub Seidler[b]

[a] Czech National Bank, Monetary and Statistics Dept.,
Na Příkopě 28, CZ-115 03 Praha 1, Czech Republic,
e-mail: Alexis.Derviz@cnb.cz ,
corresponding author

[b] Czech National Bank and Institute of Economic Studies, Charles University

**Abstract:**

We discuss (dis)incentives for fair cooperation related to delegating macroprudential policy decisions to a supranational body, as well as their welfare implications. The question is studied by means of a signaling game of imperfect information between two national regulators. The model concentrates on informational frictions in an environment with otherwise fully aligned preferences. We show that even in the absence of evident conflicting goals, the non-transferrable nature of some regulatory information creates misreporting incentives. However, the major problem is not the reporting accuracy but the institutional arrangement focused on maximal multilateral satisfaction to the detriment of credible enforcement of rules. The main application is meant to be systemic risk management by the relevant EU institutions.

# 1. Introduction

Each financial crisis in the last decades brought about an extensive debate regarding the proper functioning of the current regulatory framework and led to questioning the international financial architecture (IFA). As a result, a number of possible amendments and changes of the IFA framework was introduced (see Goldstein, 2003, and references therein). One particular question accompanying the liberalization of international financial markets has been gaining an increased attention: whether efficient regulation and prevention of future crises can be guaranteed without existence of a supranational regulation organization (cf. Eatwell and Taylor, 1998, or Eichengreen, 2010).

A similar debate arose after the outbreak of the current global financial crisis, bringing about not only a wide range of suggestions for the future, but also many measures already implemented, including changes in the EU supervision architecture, establishing the European Systemic Risk Board (ESRB), and the new set of banking regulation guidelines known as Basel III. In line with those changes, new regulation optic is focused intensively on the so called macroprudential policies, which should prevent emergence of systemic risks. However, to what extent should these changes be entrusted to supranational bodies is still an open question. Moreover, little is known about functioning of macroprudential supervision in the case of small open economies, which gives to the question of the supranational regulator an additional twist. This is the motivation to address the named group of problems with particular relevance to the small open economies in our paper, which introduces a theoretical model dealing with delegation of macroprudential policy decisions to a supranational body.

One argument against international coordination of macroprudential policies in the financial services is suggested by the generally suspected policy inefficiency in an open economy. This inefficiency is related to the regulatory arbitrage and hence easy circumvention of country-level measures. Accordingly, a policy mandated by an authority independent of national regulators should be doubly suboptimal in such an economy: first due to openness and second due to domestic regulation costs insufficiently taken into account by a supranational regulator.

The opposite argument in favor of a supranational regulator is that only the latter is in the position to tackle the openness problem as, by expanding the validity of the policy outside national jurisdictions, it is able to eliminate regulatory arbitrage. As to the reflection of national regulation costs, the problem is usually downplayed by referring to the consensual nature of international regulatory bodies with equitable representation. This is, roughly, the conventional justification of tightening regulatory unification of the financial sector within the EU, richly fueled by the events of 2007-9. However, this "eurocentric" argument does not deal with the possibility of regulatory arbitrage making use of non-EU jurisdictions.[1]

Supporters of internationally centralized macroprudential regulation often refer to the issue of cross-border spillovers of financial shocks and the related demand for transnational coordination of global systemic risk containment measures. It is argued that the sheer complexity of information processing needed to efficiently counteract the threat of global financial instability calls for a centralized authority able to collect disaggregated data from individual countries and take actions based on centrally conducted analysis. On the other

---

[1] In this regard, one should remember that all efforts to spread a particular regulation globally through the G20-platform have failed so far. International consensus is especially difficult when it comes to the costs issue, as the example of the envisaged Systematically Important Financial Institutions (SIFI) regulation, the burden-sharing side of which remains unresolved.

hand, proponents of decentralized regulation point at the fact that whatever the source of financial instability spilling across borders, it always has a "home". It means that the particular instability invariably originated in a specific country in which the responsible authorities failed to act even if they had access to the relevant information. Further, they argue that a large portion of this information is of an intangible, human expert-dependent nature preventing it to be quantified and shared in a timely manner necessary for useful trans-national synergies.

A considerable part of the outlined discussion overlaps with a similar debate concerning "micro-prudential" regulation of financial institutions (cf. Pistor, 2010, as a strong proponent of decentralized regulation, as opposed to, e.g. Cerutti et al., 2010, whose punch line is, essentially, a fatwa on ring-fencing practices). The systemic risk focus of macroprudential regulation adds more complexity due to interactions with, i.a., monetary policy and international competitiveness issues. That is, the macro-level provides more, and stronger, sources of international spillovers as well as more entrenched vested interests in each involved country.

The described contradicting views often transform into deep disagreement between policymakers on the desirable institutional arrangement. On the other hand, the same conflicting opinions have so far received little attention from a formal economic analysis perspective. The present paper contributes to the task by stating the gist of opposing views on regulatory design in possibly abstract terms. In other words, we propose a model which strips the politically heavily laden problem to the bare essentials and gets down to the decision-theoretical bedrock of the interplay between two implications of (de)centralized regulation: dispersed information and regulatory costs. We believe that a first pass on understanding the problem formally can be made by abstracting from nearly all of the macroeconomic and financial intermediation specifics that occupy a prominent place in the current "post-crisis" literature on macroprudential regulation (cf. Angelini et al, 2010, de Walque et al., 2010, or Covas and Fujita, 2010).

Indeed, the essence of the clashing policy views outlined above can, in our opinion, be stylized in a fairly primitive microeconomic setting of two (or more) imperfectly informed strategically behaving national regulators in, or without, the presence of an overarching supranational authority. All one has to consider is the impact of one-dimensional national controls (summary statistics of local regulatory measures) on the common "supranational" fundamental variable (a summary statistic of global systemic risk factors) which co-determines the values of the two national regulatory loss functions.[2] This approach allows one to obtain a quantification of the aforementioned opposing opinions expressed by the centralists' and the autonomists' side of the macroprudential harmonization debate, and state basic hypotheses as to whose arguments receive stronger support under what circumstances. One also obtains some conjectures about contractual mechanisms that bring out the (dis)advantages of autonomous decision-making.

---

[2] At a first glance, due to its lack of specifics on the controlled fundamentals and policymakers' objectives, such a set-up could serve an analysis of policy coordination in any area of human activity. However, we have chosen it particularly for the discussion of macroprudential policies because formal representation of the latter in the mentioned parsimonious language is much easier than other problems of political economy. Most other controversial issues in areas of legal and institutional integration and public choice are, essentially, multidimensional. On the contrary, macroprudential regulation, thanks to its conceptual links with monetary policy, stands out as an isolated example of feasible quantification in a few elementary terms.

In the outlined environment, some results correct the conventional intuition about the workings of institutional arrangements normally associated with integration and decentralization. In particular, it turns out that information exchange does not have to be fully honest to maximize aggregate welfare, so that integration policies with too much stress on fair communication may end up in a suboptimal equilibrium. Moreover, some forms of integration determined to take care of the members' social welfare equitably, simply waste resources on an outcome indistinguishable from full decentralization. Conversely, certain institutional arrangements that seemingly neglect a part of the information content of the cooperating parties' communication, achieve a higher social welfare thanks to their ability to shift individual responsibility back to national regulators. Still, these arrangements lack many features one normally associates with politically viable integration constructs. So, instead of a "fair representation" variety of integration authority that offers its members all necessary space for strategic interaction, a near-mechanically functioning processor of member actions with pre-defined elementary rules and an exit option might be socially preferable.

Similar dilemmas associated with strategic interaction among national policy makers have been covered by the theory of national tax competition (e.g. Wilson, 1999, and references therein). Other aspects of interaction between national policymakers with imperfectly aligned incentives were extensively discussed in the time of the EMU establishment. The debate was naturally focused primarily on the common monetary policy, namely, on the question about how effectively can the main objective of the ECB be fulfilled in the environment of different countries following their own objectives (cf. Dixit, 2001). Implications of conflicting national goals were later considered for other kinds of policies, such as the borrower of the last resort function in a mutliregulator environment (Kahn and Santos, 2001). However, theoretical concepts of mutlinational interaction of financial regulators from a macroprudential angle have not yet been discussed intensively, although the current literature recognizes the importance of this topic, see Gaspar and Schinasi (2010). The present paper fills this gap by considering multi-national regulatory interaction in the context of a general macroprudential policy framework. We believe such an analysis to be especially topical in the present environment of massive regulatory changes both in the EU and worldwide.

The rest of the paper is organized as follows. In section 2 we give a verbal synopsis of the story to be later formalized as an imperfect information game between national regulators, and give an overview of the needed technical assumptions. Section 3 contains a technical description of the named communication game and states the necessary results about its equilibria in a number of alternative institutional settings. Section 4 offers an interpretation of the policy implications of the obtained formal results. Section 5 concludes. Proofs of technical propositions from Section 3 are collected in the Appendix.

## 2. Global risks, local costs and non-transferrable knowledge
## 2.1 Three depth levels of pro- vs. contra-integration debate

We consider financially integrated economies in which market disruptions spread across national borders quickly and financial frictions are, essentially, common. In such an environment, national authorities generally agree about what adverse developments are to be acted against. Still, they need to agree on burden sharing by the implementation of the necessary policies, and conflicts between self-interests make such an agreement complicated and its outcome ambiguous. The dispute between the regulatory pro- and contra-centralization sides can be then looked upon as a disagreement as to which of the two arrangements produces a more desirable outcome of the burden-sharing negotiations. The corresponding

debate takes place on at least three levels of analytical sophistication. If one abstracts from specifics of the numerous individual contributions available to date, the distinction between levels can be summarized as follows.

On the first level, one finds arguments that operate with simple cost and benefit parameters. What appears to be a beneficial policy measure from one country perspective, the autonomists say, may ignore costs incurred in other jurisdictions. A central regulatory power would always forward the interests of big members with a lot of political clout to the detriment of smaller ones, because no one will think of internalizing the preferences of the latter, they proceed. The unionists object that it is exactly in an integrated agency with an appropriate representation of all members that a fair regard to smaller participants can be safeguarded, whereas an uncontrolled world of independent regulators would mean exactly the harm to the weak the autonomists are campaigning against.[3]

The second-level debate is concerned with dispersed information, spillovers and synergies from a common regulatory "brain". Centralizers claim that informational synergies of an integrated regulator will be so strong that they are bound to dominate every gain from local competence. Autonomists object that the nature of regulatory information is imminently local and partially non-transferrable and that any central authority trying to take into account the totality of national economies and markets will inevitably end up as a bureaucratic slow-wit entirely dependent on country-level informational inputs. Therefore, instead of synergies, one shall mostly expect informational losses. To support this conjecture, they offer both theoretical and empirical knowledge on functioning of large hierarchical organizations.[4]

The third level goes even deeper into the domain covered by game theory and mechanism design, by asking whether a central regulator can prevent inefficiencies stemming from strategic non-cooperative behavior of the members and overcome welfare losses caused by asymmetric information.[5]

It is quite possible that for most practical purposes, the first two levels (or even the 1st level only) of discussion are quite sufficient for a viable judgment either for or against integrated macroprudential policies. All the experience available so far on the functioning of integrated agencies (be it within or without the EU) provides plentiful evidence for inadequate representation of smaller members and bespoke policies designed by, and in the interest of, biggest and most influential country representations. On the contrary, an example of an underdog being protected by "common" policies is quite hard to find. Nevertheless, in order to expand the discussion beyond the limits set by current political practice, we proceed by

---

[3] A formal treatment of this cost-benefit dilemma, including cross-jurisdictional spillovers, can be found in Seabright (1996). More specifically, voting decisions by EU members under partially conflicting preferences are modelled in Dixit (2001).

[4] The ultimate answer on whose arguments are more valid can only be an empirical one with formalized data at hand. Although a lot of anecdotal evidence on the functioning of supranational organizations supports the autonomists' concern, it remains to be seen whether negative experiences are, indeed, more numerous than positive ones or just more conspicuous.

[5] The theory of mechanism design, among other things, teaches us that not every objective set by a social planner (social choice rule) can be implemented in a game form under incentive-compatibility condition. Even among those objectives that can, many require a mechanism inconsistent with truthful revelation of private information (see Dasgupta et al., 1979, for an overview). Examples are the free rider problem in the provision of public goods and "strategic voting" games. Accordingly, one should not expect the problem of international regulatory cooperation considered here, to possess a clear-cut social welfare-maximizing "cheatproof" solution, either (this will also be the case in our setting, see particularly Subsections 3.2.3 and 3.2.4). Unfortunately, this circumstance remains largely ignored by the political manifestos underlying the current EU financial regulation reform.

allowing for a reasonably equitable representation (as a result of, e.g., efficient coalitions among small members). Accordingly, for the sake of the argument, we will assume that the autonomists' reservations on the first two of the mentioned debate levels have somehow been taken care of, and deal with the third level only. Specifically, to abstract from the level one-obstacles to integration, we restrict attention to a supranational regulatory body able to harmonize, at least, the "symmetric-information equivalents" of its members' preferences. That is, we allow for the existence of a central authority with the objective function which is optimized by the very same national macroprudential policies that each national regulator would choose given the policies of others, provided all uncertainties are common symmetrically observed random variables. (In other words, the corresponding social welfare function implements the unique Nash equilibrium of a hypothetical symmetric-information game between the two regulators.) In Section 3, we will work with an example of such. Also, in order to circumvent the level two-obstacles, let us allow the national regulators to act on that part of local information they are unable to share, and give the central authority the capacity to process and translate into policies all information that can be shared. (Formal examples of Section 3 take this feature on board.) In this setting, we will look at possible inefficiencies of non-cooperative behavior of national regulators when it comes to sharing local knowledge and the severity of welfare losses under different cooperation regimes.

## 2.2 National regulators and their information

We consider two regulators, A and B, who exercise partial control over a common "global" fundamental risk factor they strive to minimize in the presence of quadratic control costs. Their preferences (loss functions) over the common fundamental are proportional. Specifically, the relative size of the losses incurred by the residual risk surviving the implementation of both national policies derives from the relative size of the respective economies themselves, and from nothing else. Among other things, this means that when the two economies' sizes are unequal, the bigger economy (and its regulator) has a stronger impact on the loss of the smaller one than the smaller one on the loss of the bigger one. This makes the set-up formally applicable to the small open economy case, such as the Czech Republic vis-à-vis the EU.

Only a part of information in the hands of local regulators can be credibly communicated to other parties. Several factors may cause this.

First, there may be elements of "soft" knowledge accessible to lower-rank supervisors only (e.g. confidential information on individual institutions), which the decision-making body of a supranational regulatory agency would have had to process at a prohibitively high cost. This can be illustrated by the example of international systemically important financial institution (ISIFI) regulation: operation of a national affiliate of an ISIFI is often nearly impossible to follow in real time from across the border. This is so even when the involved countries are quite close and their financial regulators have a long-standing tradition of cooperation, as the case of Fortis resolution in Benelux in September 2008 has demonstrated.

Second, data take time to collect and the collection period coincides with the period of regulatory cycle itself, so that the outcome is worthless for outsiders because it is only available after the local regulatory decision has already been made. Consider, for example, a situation in which a credit bubble is forming more quickly in one country than in another. The regulator in the former country would need to have the loan-to-value (LTV) ratio limit for new loans lowered as soon as the data start to signal a bubble reliably. However, since bubble

detection is generically a slow process, the latter country regulators may be still collecting evidence that, on their side, the LTV restrictions are warranted as well. Under centralized regulation, the necessary decision may have to wait until information from both national sources has been processed, and come too late as a result. This could easily happen even if, under a hypothetical (although counterfactual) symmetric and timely data processing, the socially desirable LTV caps in both countries were the same.

Third, and this is what microeconomic literature is most familiar with, information may be impossibly or prohibitively costly to verify for anyone else but the regulator in question. Then, analogously to what has been long ago taken for granted in contract theory and microeconomics of financial intermediation, the regulator may not have the right incentives to report accurately to outsiders (in fact, may have an incentive to misreport), for strategic reasons. We will follow both the genesis of this misreporting incentive and the corresponding adjustment of the credible policy attributes, in the model.

The approach is necessarily highly stylized. This is not so much a limitation as a means to highlight the essentials of the problem. Various generalizations are possible, among them such that would allow one to vary the relative effect of the global risk factor on regulators' utility in the two national economies, keeping the utility contribution of national regulatory costs themselves fixed. The principal message would not change under this modification, although the latter may be useful for quantifying the roles of relative size and relative exposure to systemic risk (e.g. as a result of differing financial depth) separately.[6]

## 3. Model
## 3.1 Environment

Let $r$ be the fundamental risk factor introduced in the previous section, $a$ and $b$ – its national constituent parts and $u$, $v$ – the controls exercised by the two national regulators. Formally,

$$r = \alpha(a-u) + \beta(b-v). \tag{1}$$

Coefficients $\alpha$ and $\beta$ reflect relative strength of the contributions to $r$-containment by regulators A and B. These coefficients also reflect the weights with which the common risk factor $r$ enters the corresponding regulator loss function: the loss of A (B) is assumed to be $\tau(r/\alpha)^2/2$ ($\tau(r/\beta)^2/2$), with $\tau$ being a positive constant, for simplicity assumed the same for both. This definition means that each regulator accounts for the loss from its own residual fundamental ($a-u$ for A and $b-v$ for B) one-to-one, whereas the spillover from the other country residual enters the loss function with a rescaling factor depending on the relative sizes of the two economies. For instance, if economy B is bigger than A, regulator A cares about spillovers from B more than B cares about spillovers from A.

Further, let controls entail (tangible and intangible) quadratic costs so that they enter a regulator's loss function with common coefficient $\gamma$. So, when A (B) chooses control $u$ ($v$), it incurs costs $\gamma u^2/2$ ($\gamma v^2/2$). Accordingly, the overall loss functions of A and B are

$$f = -\frac{\tau}{2}\left(\frac{r}{\alpha}\right)^2 - \frac{\gamma u^2}{2}, \; g = -\frac{\tau}{2}\left(\frac{r}{\beta}\right)^2 - \frac{\gamma v^2}{2} \tag{2}$$

---

[6] See the proof of Proposition 3 in the Appendix for an example of quantification in this sense.

and depend both on regulatory policies ($u,v$) and realization of random risk factor $r$. The latter is stochastic since there is uncertainty in the realizations of its national drivers $a$ and $b$. Each regulator strives to minimize expected loss given both public and his private information.

There are exogenous noises affecting both national components of the aggregate risk factor to be controlled. So, we set $a=p+\varepsilon^a$, $b=q+\varepsilon^b$ with $\varepsilon^a$, $\varepsilon^b$ being mutually independent random disturbances with zero means and standard deviations $\sigma_a, \sigma_b$.[7] We can think of a part of these disturbances as originating in the outside world shocks, as it may be important to take account of a wider world outside the two economies in question. The contribution of outside world shocks is bigger the more important are global influences on the domestic financial system. For example, in the absence of other sources feeding $\varepsilon^a$, $\varepsilon^b$, a small open EU member economy is likely to be characterized by a higher value of $\sigma_a$ whereas a bigger semi-closed Union member – by a lower value of $\sigma_b$.

The part of the noise affecting $a$ and $b$ that reflects external shocks is likely to remain uncertain both to the national regulators and a possible overarching authority. Fortunately, in the chosen quadratic loss set-up chosen here, this feature can be accommodated without affecting the solution. Therefore, to make the exposition simpler we do not introduce such shocks explicitly.

No less important are factors contributing to random noises $\varepsilon^a$, $\varepsilon^b$ for purely "technological" reasons inside the respective regulatory systems. It is well-recognized that financial oversight utilizes a lot of "soft" information which is hard to either formalize or transfer to parties exterior to the oversight process. Apparently, such soft information would be even harder to communicate to partner regulators outside the country (cf. the discussion in Section 2.2). The phenomenon has been mainly documented for micro supervision of banks, but it is reasonable to assume that, when it comes to macroprudential policy, i.e. a synergy of micro-supervisory information with inferences on aggregate systemic risks, the problem of insider knowledge communication to outsiders becomes even more, not less, severe.

The next step in the argument is to recognize that ex ante estimates of a parameter that cannot be either measured or communicated exactly, are much more likely to be manipulated than a parameter known at least to one party with certainty. In the practice of communication between national regulators, distinguishing between honest measurement error and deliberate misreporting is as good as impossible. That is why country-specific noises existing at the time of signal exchange is a natural element of the cross-border communication model we create.

For the aforementioned reasons, the presence of random disturbances to the controlled fundamental, although it does not add any analytical sophistication worth a mention, to the formal derivation of our theoretical statements, is important for conceptual reasons. Computations themselves will be made slightly simpler by assuming that each national regulator eventually resolves its own uncertainty component after the exchange of signals but before own policy decision, i.e. A (B) knows $a(b)$ exactly at the time of setting $u(v)$. Generalizing to a non-zero residual uncertainty would be straightforward but is unimportant for our purposes.

---

[7] If the noises were not independent, i.e. there were a correlation cause by a common source of uncertainty, the latter could be, thanks to the chosen linear-quadratic set-up, easily factored out and separated in all analytical solutions analogous to those that we provide in the sequel. Qualitatively, the results would not be affected. This is why, for space economy reasons, we do not include any common noise terms.

*Stages of the game*
With the above discussion in mind, we define the timeline of the model with three distinct points.

At the initial moment, the two regulators learn the parameters of the game, including the distributions of random factors. Also, regulator A learns the value of $p$ and B – the value of $q$. Each of them also receives a prior information about the other party's average national risk factor, i.e. A learns value $q_0$ and the signal precision (distribution of the noise $b-q_0$), whereas B learns value $p_0$ and the distribution of the noise $a-p_0$.

At the second moment, regulators send out reports about the true value of their average fundamental: A sends a signal $m$ about $p$ and B – a signal $n$ about $q$.

At the third moment, A and B learn their national fundamental values exactly and choose their controls to minimize expected losses given their private information, i.e. they solve optimization programs

$$F = \max_u E^a[f], \; G = \max_v E^b[g]. \tag{3}$$

Symbols $E^a$ and $E^b$ stand for expectations with respect to information available to A and B, respectively.

*Non-transferrable local information and harmonization of preferences*
In accordance with the above definitions, we can identify the transferrable information with value $p$ (for A) and $q$ (for B) and the non-transferrable – with realizations, at the third stage, of shock $\varepsilon^a$ (observable only by A) and shock $\varepsilon^b$ (observable only by B).

At this junction, we are able to define separate criteria for autonomous decision-making (each regulator independently and non-cooperatively) as opposed to integrated, or harmonized, regulatory decisions by a joint authority.[8] Since, in accordance with the objectives spelled out in 2.2, we are only allowing the integrator to decide prior to the private shock realizations, its loss function must be defined in terms of transferrable information (and the parameters of the model). In other words, the integrated regulator has to set controls for both A and B based on their reported values of $p$ and $q$ and the known distributions of $\varepsilon^a$ and $\varepsilon^b$. Formally, the objective can be to solve the following optimization problem:

$$H^i = \max_{u,v} \left\{ -E^i \left[ \frac{r^2}{2} \right] - \mu_a \frac{\gamma u^2}{2} - \mu_b \frac{\gamma v^2}{2} \right\}, \tag{4}$$

with $E^i$ being the expectation with respect to the integrated regulator's information about uncertain values of $\varepsilon^a$ and $\varepsilon^b$, national regulators' signals given. Coefficients $\mu_a$ and $\mu_b$ can be selected so that the integrated control choice is identical with what non-cooperative regulators would have done independently if they had the same information. Indeed, if one sets $\mu_a = \alpha^2/\tau$, $\mu_b = \beta^2/\tau$, the joint regulatory authority that solves (4) would select the same controls as A and B solving

---

[8] The difference between harmonization and centralization is important, see discussion in Section 4.

$$F^i = \max_u E^i[f], \; G = \max_v E^i[g]. \tag{5}$$

Solution of (5) is what we will call truthtelling non-cooperative equilibrium in 3.2.2 below. Note that this is not a realistic Nash equilibrium notion since, in general, as is discussed in 3.2.3, deviations from truthtelling are possible and profitable. In general, the joint authority information set would contain the values of national regulators' signals, ($m$,$n$), instead of true values ($p$,$q$).

Given that $\varepsilon^a$, $\varepsilon^b$ are private non-transferrable information, (4) and, equivalently, (5), is actually, the closest the integrated regulator can get to replicating the individually optimal choices of controls implied by (3). Therefore, any discussion of advantages and disadvantages of international harmonization must take this "residual welfare discrepancy" into account. Its sign is model-specific and unobservable and, therefore, the verdict about (dis)advantages of harmonization can be only inspired, but not determined, by the presented theoretical framework. Still, the "local expertise" factor which we have formally associated with shocks $\varepsilon^a$ and $\varepsilon^b$, clearly belongs to the "level two-debate", as was outlined in 2.1 above, whereas our main interest in this paper are the "level three-factors" related to strategic information exchange.

## 3.2 Equilibria with accurate and distorted signals
### 3.2.1 Benchmark: complete symmetric knowledge of the game parameters

Purely hypothetically, and in disregard of the informational imperfections defined in 3.1, assume that A and B have perfect knowledge of both $a$ and $b$, from the outset. This counterfactual example is given for further reference. We then obtain the following characterization of equilibrium policies.

**Proposition 1** *The Nash equilibrium of a simultaneous-move game of perfect information between regulators A and B is characterized by regulatory controls*

$$u^0 = \frac{\tau}{\gamma + 2\tau}\left(a + \frac{\beta}{\alpha}b\right), \; v^0 = \frac{\tau}{\gamma + 2\tau}\left(\frac{\alpha}{\beta}a + b\right). \tag{6}$$

*The regulator loss function values in this equilibrium are given by*

$$f^0 = -\frac{\gamma\tau(\gamma + \tau)}{2(\gamma + 2\tau)^2}\left(a + \frac{\beta}{\alpha}b\right)^2, \; g^0 = -\frac{\gamma\tau(\gamma + \tau)}{2(\gamma + 2\tau)^2}\left(\frac{\alpha}{\beta}a + b\right)^2 \; \blacklozenge \tag{7}$$

### 3.2.2 Truthtelling controls under imperfect information

Another counterfactual special case arises when, as defined at the end of 3.1, the non-transferrable information (shocks $\varepsilon^a$ and $\varepsilon^b$) is observed privately, whereas the transferrable information is signaled perfectly truthfully at the first stage. In the notation introduced in 3.1, this means that $p_0=p$, $q_0=q$. We will call this (counterfactual if taken at face value, but with a chance to be implemented approximately, cf. Corollary 1 in 3.2.3) outcome *T-regime*, T standing for "truthful reporting".

9

**Proposition 2** *If truthful reporting of average national risk factors by national regulators (i.e. p by A and q by B) can be enforced, then the Nash equilibrium of the simultaneous-move game between A and B is described by strategies*

$$\hat{u} = \frac{\tau}{\gamma + \tau} a - \frac{\tau^2}{(\gamma + \tau)(\gamma + 2\tau)} p + \frac{\beta}{\alpha} \frac{\tau}{\gamma + 2\tau} q \,, \tag{8a}$$

$$\hat{v} = \frac{\tau}{\gamma + \tau} b + \frac{\alpha}{\beta} \frac{\tau}{\gamma + 2\tau} p - \frac{\tau^2}{(\gamma + \tau)(\gamma + 2\tau)} q \,. \tag{8b}$$

*In terms of signals and noises, these strategies can be expressed as*

$$\hat{u} = \frac{\tau}{\gamma + 2\tau}\left( p + \frac{\beta}{\alpha} q \right) + \frac{\tau}{\gamma + \tau}\varepsilon^a \,, \quad \hat{v} = \frac{\tau}{\gamma + 2\tau}\left( \frac{\alpha}{\beta} p + q \right) + \frac{\tau}{\gamma + \tau}\varepsilon^b \,. \tag{9}$$

*At the first stage of the game (i.e. before A learns a and B learns b), the ex ante loss function values are given by*

$$\hat{f} = -\frac{\gamma\tau(\gamma + \tau)}{2(\gamma + 2\tau)^2}\left( p + \frac{\beta}{\alpha} q \right)^2 - \frac{\gamma\tau}{2(\gamma + \tau)}\left\{ \sigma_a^2 + \frac{\beta^2}{\alpha^2}\frac{\gamma}{\gamma + \tau}\sigma_b^2 \right\}, \tag{10a}$$

$$\hat{g} = -\frac{\gamma\tau(\gamma + \tau)}{2(\gamma + 2\tau)^2}\left( \frac{\alpha}{\beta} p + q \right)^2 - \frac{\gamma\tau}{2(\gamma + \tau)}\left\{ \frac{\alpha^2}{\beta^2}\frac{\gamma}{\gamma + \tau}\sigma_a^2 + \sigma_b^2 \right\} \blacklozenge \tag{10b}$$

Observe that strategies in the above game are in terms of the true national risk factor value known to the corresponding regulator and the two signals (cf. (6) of the complete information game; expressions (10) can be derived directly from (6) by looking at (9)).

In our setting, joint social welfare is hard to define uniquely due to the existence of private regulatory information in each country. Most natural seems to compare the ex ante loss functions (10) of the non-cooperative equilibrium with the losses incurred under harmonized regulation following (4) and (5), which we hereinafter call the *I-regime* (I for integration). Note that, thanks to the quadratic nature of the loss functions, one can all but separate the welfare consequences of $\varepsilon$-uncertainty from the rest of the parties' problems. Indeed, (4) implies optimal controls of the form

$$u^i = \frac{\tau}{\gamma + 2\tau}\left( p + \frac{\beta}{\alpha} q \right), \quad v^i = \frac{\tau}{\gamma + 2\tau}\left( \frac{\alpha}{\beta} p + q \right). \tag{11}$$

The expected loss functions (under expectation operator $E^i$) are

$$f^i = -\frac{\gamma\tau(\gamma + \tau)}{2(\gamma + 2\tau)^2}\left( p + \frac{\beta}{\alpha} q \right)^2 - \frac{\tau}{2}\left\{ \sigma_a^2 + \frac{\beta^2}{\alpha^2}\sigma_b^2 \right\}, \tag{12a}$$

$$g^i = -\frac{\gamma\tau(\gamma + \tau)}{2(\gamma + 2\tau)^2}\left( \frac{\alpha}{\beta} p + q \right)^2 - \frac{\tau}{2}\left\{ \frac{\alpha^2}{\beta^2}\sigma_a^2 + \sigma_b^2 \right\}. \tag{12b}$$

Apparently, the difference from the losses under uncoordinated Nash equilibrium given by (10) is in the noise terms. Not only does the national regulator know that by opting for decentralized policy he reduces the loss ex post. Also ex ante, the variance terms enter (10) with coefficients strictly lower than those in (12), so that it is always true that

$$f^i < \hat{f}, \; g^i < \hat{g}. \tag{13}$$

The formal reason is, obviously, the possibility to take full advantage of the private information disclosed at the second stage, under decentralized policy, same as the awareness of this possibility open to the other regulator, finding its reflection in the strategic choices.

The result of (13) could be reversed if one assumed that the central authority were somehow able to improve each regulator's knowledge about the other regulator. Specifically, suppose that the knowledge of the other country fundamental is distorted by a stronger noise under decentralized than under centralized policies. This would mean a different set of values of $p$, $q$, $\sigma_a$ and $\sigma_b$ in equilibria described by (8) on the one hand and (11) on the other. (Since the same true fundamentals, $a$ and $b$, are now measured with different precision, both the mean values and the noise dispersions would have to undergo a change.) This would make comparison of loss functions (10) and (12) impossible without further quantitative assumptions. The (pretty bold and speculative) assumption able to reverse the result of (13) would, obviously, have to be a superior information-collection and processing ability of the central authority.

Realistically, the ability of multinational bodies to improve informational transparency of its constituent members should be made an empirical question which exceeds the scope of the present simple theoretical exercise. Here, we prefer to leave aside the role of informational precision under (hypothetically enforced) unbiased signals, and concentrate attention on the case, at least as important from the pragmatic point of view, of signals with a deliberate bias.

### 3.2.3 Equilibria with misreporting

In addition to the information precision problem just discussed, a more serious one arises because, once one has to give up the unnatural assumption of enforced truthtelling in the I-regime, the equilibrium solution of Proposition 2 immediately falls apart. To see this, it is enough to allow, say, regulator A to report a different than true value of $p$ while forcing B to both take this report as truthful and comply with the accurate equilibrium reporting strategy (8b). It is then easy to demonstrate that A's loss will be less than (10a) under a continuum of non-zero deviations from truthful signaling.

More generally, we consider below a set-up in which untruthful signals (in the notation of 3.1, $m \neq p$ and $n \neq q$), of pre-defined sizes, are allowed with known ex ante probabilities.[9] We then construct an equilibrium in which A and B take full advantage of the offered opportunity

---

[9] This randomization of signals has been examined in game theory as an attribute of "cheap talk" (see below) communication which is sometimes able to facilitate coordination on a mutually preferable equilibrium outcome. One of the easiest examples is the cheap talk signaling extension of the classical "Battle of the Sexes" game, as discussed in the review by Farrell and Rabin (1996). There is also a separate game theoretic literature on stochastic signals, see e.g. Hellas and Liu (1996). Alternatively, randomized signals emerge as a formal consequence of unobservable player attributes that determine the signal value, as covered by, e.g. Feldman (2004) and Feldman and Winer (2004).

to misreport. In other words, if regulators are provided with a well-specified option to lie, they will exercise it.

Our result goes through under a fairly general representation of misreporting opportunities. Formally, we assume that A operates for its signaling a randomization device according to which the difference of $p$ and $m$ is a stochastic variable distributed with density $\varphi$, and an analogous randomization with density $\psi$ is employed by B:

$$p = m + x \text{ with probability } \varphi(x), \quad q = n + y \text{ with probability } \psi(y). \tag{14}$$

Densities $\varphi$ and $\psi$ are supposed to be common knowledge.

After the randomized signals have been generated and disclosed, the game proceeds as was defined before in 3.1, i.e. the regulators wait until the exact value of their respective national fundamental becomes known with certainty and then choose their controls. One difference compared to 3.2.2 is that the value of the other country fundamental is now uncertain for two reasons: the first is the same private component $\varepsilon^a$ ($\varepsilon^b$) as was introduced in 3.1 and the second is the signal distortion $x$ ($y$), so that the expectations must be formed differently. This seemingly minor adjustment has far-reaching consequences for equilibrium behavior. The signal randomization (misreporting, or "lying") rules are known to both players and feed into their optimal strategies, which is why one cannot avoid mixing truth and lies in the proportion pinned down by (14) even in favor of the truth if one wants to remain credible and support the equilibrium outcome.

Analogous situations, in which signals of game participants have to contain uncertainty in order to be believable, are known from microeconomic literature on the so-called "cheap talk" (cf. Crawford and Sobel, 1982, or Stein, 1989). The latter term means costless messages by players in a non-cooperative game that help them to coordinate on the extraction of common benefit without being suspected of lying to secure an undue advantage in the zero-sum component of the payoff. In the present model, for reasons of analytical simplicity, we have chosen to represent message uncertainty by means of signal randomization. A set-up similar to that of Stein, 1989, based on fuzzy signals in the form of intervals, is equally possible.

With the defined timing, regulators' equilibrium strategies must be functions of both signals, exact values of known variables and distributions of uncertain ones. In particular, since the control is chosen after the resolution of both uncertainties on the regulator's own side ($\varepsilon^a$ and $x$ for A, $\varepsilon^b$ and $y$ for B), it must depend on the *signal* about own fundamental and not its true mean value ($m$ and not $p$ for A, $n$ and not $q$ for B), because otherwise, knowledge of the model would allow for extraction of the true value by the counterparty. The exact result is as follows.

**Proposition 3** *Equilibrium strategies of the signaling game with misreporting rules* (14) *are given by*

$$u^l = \frac{\tau a}{\gamma + \tau} - \frac{\tau^2 (m + \bar{x})}{(\gamma + \tau)(\gamma + 2\tau)} + \frac{\beta}{\alpha} \frac{\tau(n + \bar{y})}{\gamma + 2\tau}, \quad v^l = \frac{\tau b}{\gamma + \tau} + \frac{\alpha}{\beta} \frac{\tau(m + \bar{x})}{\gamma + 2\tau} - \frac{\tau^2 (n + \bar{y})}{(\gamma + \tau)(\gamma + 2\tau)}, \tag{15}$$

*with* $\bar{x} = \int x\varphi(x)dx$, $\bar{y} = \int y\psi(y)dy$ *being the average signal distortion (lie) values* ♦

Rule (14) was chosen for its relative simplicity in terms of representing signal distributions. However, a better interpretation can be obtained by parameterizing the space of misreporting events separately from the space of misreporting sizes. Namely, let $\Omega^a$, $\Omega^b$ be two random event spaces by means of which A and B form their signals (independently on each other), and let us fix probability measures $P^a$, $P^b$ on those spaces. Then, let a lie of size $x(\omega^a)$ ($y(\omega^b)$) take place with probability $\varphi(\omega^a)$ ($\psi(\omega^b)$) for any $\omega^a \in \Omega^a$ ($\omega^b \in \Omega^b$). Formula (15) is not affected by this event space change.

The above result characterizes equilibrium behavior on the condition that the misreporting size distributions (functions $x$ and $y$ defined on event spaces $\Omega^a$ and $\Omega^b$, respectively) are fixed. As a special case, when both $x$ and $y$ are identically zero, we obtain the truthtelling solution of Proposition 2. However, the main message of Proposition 3 is that, whatever the available distribution of reporting "mistakes", there exists an equilibrium in which these mistakes are committed. As regards the welfare comparison of equilibria under different choices of functions $x$ and $y$, there can be no general conclusion since the loss value is significantly dependent on the distributional characteristics of misreporting rules. However, one can conduct a simple comparison of the welfare consequences of changes in one lie magnitude under a given misreporting event (i.e. by fixing a random event $\omega^a \in \Omega^a$ or $\omega^b \in \Omega^b$ as well as well as all $x$ and $y$ realizations except for one by one regulator, and varying the latter lie size).

Since, when investigating welfare consequences of varying misreporting magnitudes, it is more natural to compare ex ante expected losses (before random signal selection) than ex post realizations of losses after the regulators have learnt all available private information, we look at expectations with respect to information available to A immediately after it learns $p$ and of B immediately after it learns $q$. We use operator $E^l$ in both cases (it should not cause confusion) and denote by $F^l(x)$ the $E^l$-expectation of A's losses considered a functional defined on the space of misreporting size functions $x$ (analogously for B). Let us call the outcome *L-regime*, or L($x$) if one needs to refer to the dependence on the lie size. Then we have the following general characterization in functional analysis terms.

**Proposition 4** *If the values of average national fundamentals observed by regulators A and B are equal to, respectively, p and q, the variation of the ex ante regulator A-loss functional $F^l$ at any initial point x is given by the linear functional*

$$h \mapsto -\frac{\gamma\tau^3}{(\gamma+\tau)(\gamma+2\tau)^2} \int x(\omega)h(\omega)dP^a(\omega), \tag{16}$$

*and an analogous result holds for functional $G^l$ evaluated by regulator B* ♦

The abstract form of Proposition 4 was chosen to stress the generality of result. To develop the necessary intuition about its meaning, we simplify the situation in the following

**Corollary 1** *Let the signal randomization space $\Omega^a$ of regulator A be final and contain N elements. Then feasible signal distortion sizes for A are simply vectors of dimension N (with one component equal to zero since we want to allow for truthful reporting as one possibility), whereas misreporting probabilities are N-vectors with positive components $\pi_j$ (j=1,…,N) summing up to 1. Functional $F^l$ becomes simply a function of N variables $x_1,…,x_N$. In that case, (16) is equivalent to the collection of N equalities*

$$\frac{\partial F^l}{\partial x_j} = -\pi_j \frac{\gamma \tau^3}{(\gamma + \tau)(\gamma + 2\tau)^2} x_j, \, j=1,\ldots,N. \tag{17}$$

In the situation defined by the above elementary event spaces, Proposition 4 simply tells us that, when the signal distortion is negative (positive), A's welfare can be increased by moving it up (down), in both cases closer to zero. The lowest losses for A are attained under truthful reporting. If one combines this finding with the intuitive fact that A's welfare benefits from an increase in B's truthfulness, we arrive at the natural conclusion that, in terms of social welfare, the truthtelling equilibrium of Proposition 2 dominates all other equilibria described by (15) with non-zero misreporting: $f^l < \hat{f}$, $g^l < \hat{g}$. This is an example of the general phenomenon of inefficiency caused by imperfect credibility in signaling games.

The above result should not be confused with the simplified (and false) claim that regulators would choose to signal truthfully in a given game. Remember that lie sizes are not choice variables in it. Strategies (15) are individually rational for *any pre-defined distribution of lies*, whilst equations (16) and (17) offer comparison of welfare *across different distributions of lies*. So, Propositions 3 and 4 can be equally well interpreted in such a way that, although offered a clearly welfare-superior game of truthful reporting, regulators always run a risk of relapsing into a welfare-inferior game with lies, the supply of which is unlimited.

A more cautious interpretation of the optimistic message provided by (17) is, in our view, more appropriate. One could say that, if, for some extraneous reason, the game between regulators according to the rules of this subsection could be repeated under varying magnitudes of misreporting, both participants would tend to choose every subsequent game with lie sizes below the levels of the previous one, until, eventually, the signal distortions become negligible. Note, however, that it will still be the game with the formal distinction between signals and privately observed values, and not the game of mandatory truthfulness from 3.2.2, which has a different strategy space. Recall that a transition to the behavior which ex ante excludes misreporting by one player would immediately provide a non-negligible misreporting incentive to the other player. Therefore, one can, at most, conclude that evolution of lie size rules in the present misreporting game is likely to result in a near-truthtelling game. At the end of such a development, one would see a (near-) maximization of welfare in the class of misreporting games defined by (14).

Comparing this fact, formally expressed by (16) or (17), with inequality (13), one sees that the equilibrium of the near-truthtelling signaling game between national regulators is superior in welfare terms to both centralized regulation (with truthtelling) and equilibria of any signaling game with non-negligible misreporting. However, this does not mean that the welfare of decentralized signaling with potential minor strategic misreporting cannot be improved upon. Next, we discuss one possibility to reduce losses by overcoming the limitations of strategic behavior. To do this, we will slightly change the rules of interaction between national regulators, at the same time avoiding unrealistic assumptions about information-extraction potential of transnational authorities.

### 3.2.4 Full responsibility for misreporting

Strategically sophisticated behavior finding its expression in misreporting equilibrium (15) has its welfare limits not so much due to inefficiencies stemming from distorted signals (those

are likely to recede with time, as we have argued in the previous subsection) as due to excessive weight put in the decision of one regulator on the signal statistics of the other. Put simply, this is an inefficiency caused by over-sophistication. Let us now again assume an overarching authority which has no incentive to dwell in the fineries of the players' strategic misreporting. We endow this authority with just one power: to collect signals from both regulators and implement national controls as their agent, but treating both signals as if they were fully truthful. Regulators are allowed to misreport according to the same scheme as in (14), but their own actions will be always formulated by the coordinating power on the premise that the other regulator does not lie. The equilibrium of such a signaling game with delegation is described below.

**Proposition 5** *If the national regulators endow the coordinating authority with the power to set controls based on their signals* (*m,n*) (*generated according to* (14)) *about mean values of national fundamentals, on condition that those signals are treated as truthful, the equilibrium controls are* (*superscript d stands for "delegation"*)

$$u^d = \frac{\tau}{\gamma + 2\tau}\left(m + \frac{\beta}{\alpha}n\right), \quad v^d = \frac{\tau}{\gamma + 2\tau}\left(\frac{\alpha}{\beta}m + n\right). \tag{18}$$

*By denoting the "true" average fundamental risk $\alpha p + \beta q$ by R and introducing the auxiliary "joint misreporting" variable $z = \alpha x + \beta y$, the loss function value for regulator A under* (18) *can be written as*

$$f^d(x, y) = -\frac{\gamma\tau(\gamma + \tau)}{2(\gamma + 2\tau)^2}\left(\frac{R}{\alpha}\right)^2 - \frac{\tau^2}{2(\gamma + 2\tau)^2}\frac{z}{\alpha}\left(2\gamma\frac{R}{\alpha} + (\gamma + 4\tau)\frac{z}{\alpha}\right) - \frac{\tau}{2}\left\{\sigma_a^2 + \frac{\beta^2}{\alpha^2}\sigma_b^2\right\} \tag{19}$$

*when A and B choose misreporting values x and y, and an analogous expression is valid for regulator B ♦*

In the above proposition, lie sizes and probabilities are fixed, as they were in the signaling equilibrium of Proposition 3. And, in the same way as was done in 3.2.3, we can ask how do regulators' loss functions in equilibrium (18) depend on the level of lies. For the sake of transparency, we give an answer for the simple example of just two possible signal values for each regulator. Namely, let A (B) report truthfully with probability $\pi^a$ ($\pi^b$) and give false signal $m = p - l^a$ ($n = q - l^b$) with the remaining probability. (More general settings yield analogous results, but their statement would only require more cumbersome notation without additional insight.[10]) It turns out that, as opposed to the (maximum sophistication) equilibria of Subsection 3.2.3, optimal misreporting size in the present case of a "credulous" central authority with delegation is non-zero.

**Corollary 2** *In the equilibrium of Proposition 5 with a single deviation from truth allowed to each regulator, the regulators' losses are minimized when the lie sizes satisfy*

---

[10] In fact, there is virtually no loss of generality compared to the situation of Corollary 1 with *N>2*, i.e. with more than one allowed level of signal distortion. As we show in the proof of Corollary 2 in the Appendix, optimal lie sizes must be equal, so that, if the regulator gets to choose misreporting magnitudes to minimize losses, multiple misreporting events are clamped down to just one.

$$l^a = -\frac{\pi^b \gamma \left( p + \frac{\beta}{\alpha} q \right)}{\pi^a + \pi^b - \pi^a \pi^b}, \quad l^b = -\frac{\pi^a \gamma \left( \frac{\alpha}{\beta} p + q \right)}{\pi^a + \pi^b - \pi^a \pi^b}. \tag{20}$$

*Under these values of (possible) signal distortions, losses of each regulator are strictly lower than losses of the same regulator in the centralized regulation case with perfectly truthful reporting, as given by* (12) ♦

The intuition behind the above result can be found in the elementary properties of socially optimal macroprudential controls in the settings in which they can be defined unambiguously, e.g. when information is symmetric. Aggregate welfare can be then expressed by means of a (weighted) average of national loss functions. It is then easy to check that, if optimal controls constitute an internal solution to the global loss function minimization, the partial derivative of *F* w.r.t. *u* (*G* w.r.t. *v*) must be negative, i.e. the socially optimal control should, typically, lie at a higher level than the non-cooperative game control (naturally, under the truthful reporting in equilibrium, since symmetric information excludes misreporting). In other words, hypothetical globally optimal national regulators usually exercise a higher effort than regulators in a non-cooperative setting. The myopic impartial mediator providing for our "optimal misreporting" mechanism assures, that, with their lies, the counterparties induce each other to move exactly in the desirable direction of higher effort.

In the remaining sections of the paper, we will denote the internationally coordinated regulatory regime in which the central authority is empowered to interpret the national regulator signals as truthful, by *D-regime* (D for delegation), and the specific case in which optimal misreporting is described by (20) in Corollary 2, by *D\*-regime*.

### 3.2.5 Non-transferrable information and exit option

Observe that, in Corollary 2, we have compared regulators' welfare with the centralized solution of 3.2.2 and not with the decentralized equilibria of either Proposition 2 or Proposition 3. The reason is that, as was assumed in 3.1, information on *a-p*, *b-q*, associated with private regulatory expertise, cannot be transferred to the central authority. Therefore, controls in a D-regime cannot make use of terms $\varepsilon^a$, $\varepsilon^b$, and the ex ante welfare measures of A and B in equilibrium (18) include, respectively, variance terms

$$-\frac{\tau}{2}\left\{\sigma_a{}^2 + \frac{\beta^2}{\alpha^2}\sigma_b{}^2\right\}, \quad -\frac{\tau}{2}\left\{\frac{\alpha^2}{\beta^2}\sigma_a{}^2 + \sigma_b{}^2\right\} \tag{21}$$

in the same way as in (12). On the contrary, losses in any decentralized equilibrium from Proposition 3 contain terms

$$-\frac{\gamma\tau}{2(\gamma+\tau)}\left\{\sigma_a{}^2 + \frac{\beta^2}{\alpha^2}\frac{\gamma}{\gamma+\tau}\sigma_b{}^2\right\}, \quad -\frac{\gamma\tau}{2(\gamma+\tau)}\left\{\frac{\alpha^2}{\beta^2}\frac{\gamma}{\gamma+\tau}\sigma_a{}^2 + \sigma_b{}^2\right\} \tag{22}$$

in which variances $\sigma_a{}^2$ and $\sigma_b{}^2$ enter with smaller coefficients than in (21). This is the welfare benefit from private regulatory expertise already discussed at the end of 3.2.2. Thus, although we clearly have the inequalities

$$f^i < f^d \,,\ g^i < g^d \tag{23}$$

analogous to (13), as stated in Corollary 2 ($f^d$ and $g^d$ are loss function values when controls are given by $u^d$, $v^d$), we cannot directly compare ($f^d, g^d$) with ($\hat{f}, \hat{g}$) without further assumptions. Remember that, as stated in Proposition 4, losses ($\hat{f}, \hat{g}$) under truthtelling provide the strict upper bound of loss functions attainable by various misreporting equilibria (15). The relation of this upper bound with ($f^d, g^d$) is ambiguous as two opposite forces are at work here. The difference between (21) and (22) gives welfare advantage to a decentralized control solution, whilst the effect stated in Corollary 2 speaks in favor of a central power able to limit excessively sophisticated strategic behavior. Apparently, the "sophistication inefficiency" is more important when the private information significance is low, and vice versa.

In any event, by giving the regulators an "exit option" from the delegation regime, one can make sure that the losses from unexploited private expertise under harmonized regulation do not get out of control. The exit option means that each regulator is free to choose, before the game start, between staying in the regime of 3.2.4 (central authority who interprets every signal as truthful) and reverting to the most elementary available version of decentralized regulation. Namely, instead of the signaling game defined in 3.2.3, one can choose a regime in which regulators act in mutual isolation (no signals), relying only on prior information. To make this last option more specific, one can assume that prior information, although very noisy, is unbiased, i.e. errors $a-p_0$ and $b-q_0$ faced by, respectively, B and A, have zero means.[11] Then, we obtain a backstop in the form of loss from high noise in the prior information about the other regulator. For instance, for A, it means deciding with the prior knowledge of $q_0$ (instead of $q$ entailing error $\varepsilon^b$ as was assumed in 3.2.2) and a variance of $b-q_0$ which may be higher than $\sigma_b^2$ and drive the loss term (22) further down away from zero. Evidently, the actual inequality sign between $f^d$ under low-variance noises and $F^l$ with high-variance noises will depend on the parameter values of the model.

More generally, an outside option in the form of reversion to non-cooperative regulatory autonomy would be useful in any environment in which the benefits of centralization are sensitive to exogenous parameters and rule-abidance by partners. The exposition in this section points, among others, at the following three deviations from rational behavior it can put a limit to:

1. the central authority reneging on the D*-regime under pressure from an influential national regulator, the latter forcing an evolution of the cross-border regulatory coordination towards some form of strategic misreporting game similar to the one described by Proposition 3, but with welfare additionally reduced by the loss of private regulatory information
2. one of the national regulators getting stuck in a misreporting behavior with a high misreporting level (in a decentralized regime)
3. the central authority putting excessive stress on, and demanding additional resources for, the enforcement of truthful reporting (i.e. striving after the I-regime described by (11)-(12) in 3.2.2), which, due to the loss of private regulatory information, would be inferior to the decentralized regulation under a sufficiently low strategic misreporting magnitude.

---

[11] By assuming a higher noise after exit compared to the one that prevails under policy coordination, we would additionally tighten the testing conditions for the exit option. Without this tightening, one would have a comparison of $\hat{f}$ with $f^d$, as discussed in the previous paragraph.

## 4. Interpretation and discussion

Different varieties of the model considered in the previous section provide us with the following outcomes. There are two possible regulatory organizations: centralized and decentralized (when each regulator decides autonomously). We have discussed individually rational decisions of national regulators under two regime types, I and D, for the centralized, and two further ones, T, and L, for the autonomous regulation organizations. The summary of findings concerning welfare implications is in the following table (for space economy reasons, only loss functions of the first regulator, i.e. $f^I$, $f^P$, etc. are spelled out):

**Regulatory coordination regimes and loss levels**

| Regime | Decentralized | | | Centralized | | |
|---|---|---|---|---|---|---|
| | **L($X$)** | **L($x$), $x<X$** | **T** | **I** | **D\*** | **D** |
| | Random misreporting with one possible lie size $X$ | Random misreporting with one possible lie size $x$ | Truthful reporting, voluntary | Truthful reporting, enforced | Truthful interpretation of submitted signals, optimal lie sizes | Truthful interpretation of submitted signals |
| **Welfare levels (national regulator A)** | $f^{LX}$ | $(f^{LX}<)f^{Lx}$ | $(f^{Lx}<)f^T$ | $(f^T>)f^I$ | $(f^I<)f^{D*}$ | $(f^{D*}>)f^D$ |

The centralized organization is associated with the loss of non-transferrable private information held by national regulators. Decentralization gives ex post gains to each of them conditioned on maximal feasible mutual truthtelling. The latter is not deviation-proof (at least a minor misreporting deviation always pays), i.e. remains hypothetical without an implementation device. If such device is proposed in the form of a central authority (integrated macroprudential regulation), then (a) the previous problem of non-transferrable private information returns to reduce aggregate welfare, and (b) one still needs to explain how to achieve truthful reporting. Since it is hard to justify why misreporting by integrated regulators shall be prevented by the mere fact of central authority existence (whose only feasible role can be to collect and disseminate data from participants), it shall be realistically counted upon rational misreporting under both decentralized and centralized regulatory regimes.

However, the second part of our story seems to turn upside-down the intuition of the first part. We take one step further from the misreporting equilibrium under a fixed distribution of possible signal distortions and ask whether, given any fixed distribution of possible lie sizes by the counterpart, the national regulator will be better off under a bigger or a smaller own size of misreporting. And one finds out that smaller (in absolute value) own lies entail higher welfare. The outcome follows from the strategic behavior of both parties, as given by (15). According to Proposition 3, each regulator conditions own actions on both misreporting choices of the other side *and the rational reaction of the other side on own misreporting* ("I know that you know, etc., … that I am lying to you," – the so called infinite regress of knowledge and beliefs explored by abstract game theory, cf. Townsend, 1983). This means that, given the equilibrium signaling policy of the other regulator, any regulator in this setting

would, prior to the start of the actual reporting game, seek external circumstances in favor of reduced misreporting. So, if he has enough alternatives to seek among, same as the other side, they should eventually coordinate on a nearly-truthtelling outcome.

The story also has the third part. The near-truthtelling equilibrium that, as we have seen, comes about as the evolutionary stable outcome of the search for optimal misreporting, is not identical with the socially optimal outcome. The latter would require more intensive controls than the non-cooperative truthtelling Nash equilibrium. The mechanism of impartial signal processing with obligatory truthful interpretation, i.e. the D*-regime that we have discussed last, moves regulatory actions in exactly that direction.

The theoretical distinction made in the previous section between decentralized decision-making by national regulators and delegation of their powers to a central authority shall by no means be confused with the actual international interaction patterns of regulatory agencies. At a closer look, all signaling and control strategies considered so far are perfectly imaginable under both centralized and decentralized cross-border policy regime. So, the question is not which regime generates which reporting behavior but rather, under which one does a given behavioral pattern have the best conditions to deploy. Accordingly, we shall now think about the chances of different variants of our signaling game to find a counterpart in the behavior of real-life macroprudential regulators. One needs to attribute the abstract outcomes of Section 3 to institutional realities of regulatory cooperation.

The featured variants of the discussed strategic game with private information are all too abstract to make policy-relevant interpretation immediate. Finding an equilibrium strategy requires both rationality and sufficient cognitive abilities on the part of each policymaker. Not least, the selection of optimal reporting/signal values, as it requires knowledge of parameters whose unobservability is at the very bottom of the problems we consider, cannot be much more than a trial-and-error exercise (supposing one designed a corresponding experiment economy to test the viability of equilibrium outcomes). In this regard, the most important part seems to be to identify the most appropriate game rules and then look for an institution best able to implement them.

To that effect, we argue that an "empathy-free" arrangement between sovereigns with formal application of pre-agreed procedures may be welfare-improving compared to arrangements trying to replicate national "micro-concerns" in exchange for delegated sovereignty (as the current EU practice often does to avoid criticisms of national interest neglect). This is exactly the difference we have visualized in Subsection 3.2.4 between controls set by a mediator with a clear limit to sophistication, and maximally sophisticated controls based on infinite-regress belief processing. Having an international coordinator that allows members to deploy the full range of their second-guessing abilities in the name of fairness, i.e. acting as a *regulation harmonizer*, may be detrimental to the overall welfare. Conversely, having a brainless automaton taking every member by its word, i.e. a very elementary *regulation centralizer*, may be welfare-improving. In addition, if cross-border coordination of this type is augmented with an exit option (the right to revert to decentralized strategic interaction), one receives a mechanism in which the downside risk losses from ignoring private non-transferrable regulatory information in the name of harmonization, is cut off.

At the same time, we stress that what the model has rendered as a welfare-superior mechanism is not a supranational regulatory autocracy one knows, e.g., from the fresh experience of ESRB. As such, it does not have to function as an administration enforcing

compliance but should instead act as an impartial mediator. It grants the parties a freedom of choice upon which it assumes full power while acting on submitted choices. Importantly, as opposed to the declared ideal of the EU bodies, an institution that would implement the said mechanism does not need to have preferences aligned with those of the member regulators.

## 5. Conclusion

In view of the lessons from our model, the main weakness of the current mechanism of EU-wide cooperation in macroprudential area can be summarized as too much strategic consideration in the quest for perfect balance of national interests, but not enough space for national responsibility for one's actions. One is too anxious to integrate claims and (often conflicting, in any event rarely aligned) interests across member states and almost entirely unconcerned with issues of finding appropriate contingency rules (put simply, the "crime and punishment" aspect).

Our reading of the model results is such that the main evil of integrated regulation in the actual EU-version does not seem to be an insufficient respect to national rights but rather, a complete lack of choices within the selected regime and no chance for the members to either make such choices or bear consequences. That is, regulators forced by a disinterested arbiter to face full consequences of their inaccurate signals jointly achieve higher social welfare than regulators integrated in an "empathic" supranational body. Unfortunately, this reading of the much-invoked subsidiarity principle has been entirely overlooked in the process of creating EU-wide institutions responsible for systemic risk oversight. On the other hand, the "empathy-free" arbiter arrangement we favor instead can be implemented between fully sovereign states on the basis of conventional international law. Naturally, it shall be much easier to transform the extant EU agencies for the same purpose than create new ones from scratch.

## References

1. Angelini, P., S. Neri, and F. Panetta (2010) *Grafting macroprudential policies in a macroeconomic framework, choice of optimal instruments and interaction with monetary policy*. Mimeo, Bank of Italy.
2. Cerutti, E., A. Ilyina, Y. Makarova, and C. Schmieder (2010) *Banks Without Borders? Implications of Ring-Fencing for European Cross-Border Banks*. IMF Working Paper 10/247 (November).
3. Covas, F., and S. Fujita (2010) Procyclicality of capital requirements in a general equilibrium model of liquidity dependence. *International Journal of Central Banking* 6, No. 4, 137–173.
4. Crawford, V., and J. Sobel (1982) Strategic Information Transmission. *Econometrica* 50 (November), 1431–51.
5. Dasgupta, P., P. Hammond, and E. Maskin (1979) The Implementation of Social Choice Rules: Some General Results on Incentive Compatibility. *Review of Economic Studies* 46, 185–216.
6. Dixit, A. (2001) Games of monetary and fiscal interactions in the EMU. *European Economic Review* 45 (May), 589–613.
7. Eatwell, J., and L. Taylor (1998)) International Capital Markets and the Future of Economic Policy. *CEPA Working Paper Series,* WP No. 9, New School for Social Research.

8. Eichengreen, B. (2010) Out-Of-The-Box Thoughts About The International Financial Architecture. *Journal of International Commerce, Economics and Polic*y (JICEP), World Scientific Publishing Co. Pte. Ltd., vol. 1 (01), 1–20.

9. Farrell, J., and M. Rabin (1996) Cheap Talk. *Journal of Economic Perspectives* 10 (3), 103–118.

10. Feldman, D. (2004) Separating Signaling Equilibria Under Random Relations Between Costs and Attributes: Discrete Attributes. *Mathematical Social Sciences* 48, 93-101.

11. Feldman, D., and R.S. Winer (2004) Separating Signaling Equilibria Under Random Relations Between Costs and Attributes: Continuum of Attributes. *Mathematical Social Sciences* 48, 81-91.

12. Gaspar V., and G. Schinasi (2010) *Financial Stability and Policy Cooperation*. Working Paper o201001, Banco de Portugal, Economics and Research Department.

13. Goldstein, M. (2003) *An Evaluation of Proposals to Reform the International Financial Architecture*, *NBER Chapters,* in: Managing Currency Crises in Emerging Markets, pp. 225–272, National Bureau of Economic Research.

14. Hellas, J., and M. Liu, (1996) Repeated Games with Partial Monitoring: the Stochastic Signaling Case. *Game Theory and Information* 9605001, EconWPA.

15. Kahn Ch., and J. Santos (2001) *Allocating bank regulatory powers: lender of last resort, deposit insurance and supervision*. BIS Working Paper 102, Bank for International Settlements.

16. Pistor, K. (2010) *Host's Dilemma: Rethinking EU Banking Regulation in the Light of the Global Crisis*. Columbia Law and Economics Working Paper No.378.

17. Seabright, P. (1996) Accountability and decentralisation in government: An incomplete contracts model. *European Economic Review* 40, 61–89.

18. Stein, J. (1989) Cheap Talk and the Fed: A Theory of Imprecise Policy Announcements. *American Economic Review* 79, No.1 (March), 32–42.

19. Townsend, R. M. (1983), Forecasting the Forecasts of Others. *Journal of Political Economy* 91, 546–588.

20. de Walque, G., O. Pierrard, and A. Rouabah (2010) Financial (In)stability, Supervision and Liquidity Injections: A Dynamic General Equilibrium Approach, *Economic Journal* 120 (December), 1234–1261.

21. Wilson, J.D. (1999) Theories of tax competition. *National Tax Journal* 52, 269-304.

**Appendix: Proofs**

**Proof of Proposition 1**

Although the statement is pretty straightforward, we reproduce the key steps mainly for the purposes of subsequent reference.

When regulator A takes the control exercised by B, i.e. $v$, as given, the loss function $f$ has the partial $u$-derivative equal to

$$\frac{\partial f}{\partial u} = \tau\left[a + \frac{\beta}{\alpha}(b - v)\right] - (\gamma + \tau)u . \tag{A1}$$

This implies the reaction function of A on B and, by symmetry, the reaction function of B on A in the form

$$u = \frac{\tau\left(a + \frac{\beta}{\alpha}b\right)}{\gamma + \tau} - \frac{\beta}{\alpha}\frac{\tau v}{\gamma + \tau}, \quad v = \frac{\tau\left(\frac{\alpha}{\beta}a + b\right)}{\gamma + \tau} - \frac{\alpha}{\beta}\frac{\tau u}{\gamma + \tau} . \tag{A2}$$

The result (6) of proposition 1 then follows directly from solving (A2) for $u$ and $v$, whereas (7) – from substituting (6) into the expressions for the loss function ●

**Proof of Proposition 2**

Take regulator A first. A minimizes expectation $E^a[]$ of loss function $f$, i.e. he knows $a$ exactly but, regarding $b$, only has knowledge of value $q$ which is its unbiased estimate. Accordingly, his knowledge of $v$ is an estimate as well. Instead of (A1), the first order condition of optimality for A is

$$\frac{\partial F}{\partial u} = \tau\left\{a + \frac{\beta}{\alpha}\big(q - E^a[v]\big)\right\} - (\gamma + \tau)u = 0 . \tag{A3a}$$

By symmetry, a similar expression is valid for B:

$$\frac{\partial G}{\partial u} = \tau\left\{\frac{\alpha}{\beta}\big(p - E^b[u]\big) + b\right\} - (\gamma + \tau)v = 0 . \tag{A3b}$$

Our educated guess, inspired by (A2) and (6), is that equation system (A3) must be solved by control $\hat{u}$ which is a linear function of three variables: $a$, $p$ and $q$, whereas control $\hat{v}$ must be a linear function of $b$, $p$ and $q$. So, we write

$$\hat{u} = c_a a + c_p p + c_q q, \quad \hat{v} = d_b b + d_p p + d_q q , \tag{A4}$$

then substitute (A4) into (A3) and look for coefficient values $c_a$, $c_p$, $c_q$, $d_b$, $d_p$, $d_q$ that satisfy (A3) and (A4) identically for all values of $a$, $b$, $p$ and $q$. The result is exactly as stated in (8) or, if one replaces $a$ by $p+\varepsilon^a$ and $b$ by $q+\varepsilon^b$, (9). Given that, before exact

values of *a* and *b* are revealed to the respective regulators, $\varepsilon^a$ and $\varepsilon^b$ are random noises to both, expected loss function expressions (10) can be obtained directly by substituting (9) into (2), taking expectations and simplifying ●

**Proof of Proposition 3**

Since it does not constitute any significant increase in complexity, we shall consider the case of slightly more general regulator loss functions than in (2), namely

$$f = -\frac{\rho_a}{2}r^2 - \frac{\gamma u^2}{2}, \ g = -\frac{\rho_b}{2}r^2 - \frac{\gamma v^2}{2}. \tag{A5}$$

Formulation (A5) allows one to visualize the role of differentiated significance of global systemic risk across countries. (Preferences (2) are a special case of (A5) with $\rho_a = \tau/\alpha^2$, $\rho_b = \tau/\beta^2$.)

As was mentioned immediately prior to the statement of Proposition 3, optimal regulatory controls are formulated after both the national fundamental and own signal uncertainty has been resolved, i.e. they shall depend on the exact value of own national fundamental, own realized signal, observed other regulator signal and the totality of other possible signals, by both regulators. In other respects, the search for equilibrium strategies can proceed as in the proof of Proposition 2 above. Our educated guess analogous to (A4) will be

$$u = c_a a + c_p m + \int c_m(x)\varphi(x)dx + c_q n + \int c_n(y)\psi(y)dy, \tag{A6a}$$

$$v = d_b b + d_p m + \int d_m(x)\varphi(x)dx + d_q n + \int d_n(y)\psi(y)dy, \tag{A6b}$$

and our task is to find coefficients $c_a, c_p, c_m, c_q, c_n, d_b, d_p, d_m, d_q, d_n$ such that the first order conditions analogous to (A3) are satisfied identically for all *a*, *b*, *x* and *y*. (Note that *x* and *y* are functions on the supports of densities $\varphi$ and $\psi$, so that one pair of realizations corresponds to the observed signals *m* and *n*.)

It is a matter of simple checking that the coefficients solving this problem are unique and render equilibrium strategies

$$u^I = \frac{\alpha^2 \rho_a a}{\gamma + \alpha^2 \rho_a} - \frac{\alpha^2 \beta^2 \rho_a \rho_b (m + \bar{x})}{(\gamma + \alpha^2 \rho_a)(\gamma + \alpha^2 \rho_a + \beta^2 \rho_b)} + \frac{\alpha\beta\rho_a(n + \bar{y})}{\gamma + \alpha^2 \rho_a + \beta^2 \rho_b}, \tag{A7a}$$

$$v^I = \frac{\beta^2 \rho_b b}{\gamma + \beta^2 \rho_b} + \frac{\alpha\beta\rho_b(m + \bar{x})}{\gamma + \alpha^2 \rho_a + \beta^2 \rho_b} - \frac{\alpha^2 \beta^2 \rho_a \rho_b (n + \bar{y})}{(\gamma + \beta^2 \rho_b)(\gamma + \alpha^2 \rho_a + \beta^2 \rho_b)}. \tag{A7b}$$

Expressions (A7) reduce to (15) when one sets $\rho_a = \tau/\alpha^2$, $\rho_b = \tau/\beta^2$. It is interesting to observe that, whenever one country becomes more sensitive to global systemic risk than the other (the corresponding coefficient $\rho$ grows *ceteris paribus*), the relative importance of strategic interaction concerns for its regulatory policy, expressed by the last two terms in (A7a) or (A7b), declines ●

**Proof of Proposition 4**

By definition, variation (also called differential) of functional $F^l$, evaluated at a fixed random function $x$: $\Omega^a \to \mathbf{R}$ giving the sizes of lies in all possible misreporting events, is a random (i.e. dependent on $\omega \in \Omega^a$) linear functional $\delta_x F^l$ for which

$$F^l(x+h) - F^l(x) = \int_{\Omega^a} \delta_x F^l(\omega) h(\omega) dP^a(\omega) + h.o.t.(h) \qquad (A8)$$

for every $h$: $\Omega^a \to \mathbf{R}$ from an appropriately chosen normed linear functional space. Acronym $h.o.t.(h)$ denotes all the terms converging to zero quicker than the corresponding norm of $h$ in the said functional space, as that norm goes to zero. Let the space in question be the (real) Hilbert one with scalar product defined as

$$\langle h_1, h_2 \rangle = \int_{\Omega^a} h_1(\xi) h_2(\xi) dP^a(\xi).$$

In this case, variation $\delta_x F^l$ is also an element of this Hilbert space (this is the so called linear functional Representation Theorem in Hilbert spaces). We will now consider optimal controls $u^l$ and $v^l$ from (15) as (affine) functionals on the same space. Then, $F^l$ is a (quadratic) function of $u^l$ and $v^l$ and the well-known facts from elementary calculus imply that the scalar product of $\delta_x F^l$ with any given trial function $h$ can be written as

$$\langle \delta_x F^l, h \rangle = \int_{\Omega^a} E^{\varepsilon,\omega} \left[ \frac{\partial f}{\partial u}(a,b,u^l,v^l) \delta_x u^l(\xi) + \frac{\partial f}{\partial v}(a,b,u^l,v^l) \delta_x v^l(\xi) \right] h(\xi) dP^a(\xi). \quad (A9)$$

In (A9), $E^{\varepsilon,\omega}$ (which can also be written in more detail as $E^{\varepsilon^a,\varepsilon^b,\omega^a,\omega^b}$) is the expectation over the values of all four uncertain variables of the model. Next, observe that, by definition of equilibrium strategy, the first order condition of optimality of $u=u^l$ is tantamount to the equality

$$E^{\varepsilon^b,\omega^b} \left[ \frac{\partial f}{\partial u} | a, m \right] = 0 \qquad (A10)$$

for any realization of $a$ and $m$. At the same time, from (15) follows immediately that the functional $u^l$ is affine in $x$ and $\delta_x u^l$ is independent of $b$ and $n$. Therefore, (A10) implies that

$$E^{\varepsilon,\omega} \left[ \frac{\partial f}{\partial u}(a,b,u^l,v^l) \delta_x u^l(\xi) \right] = 0$$

for every $\xi \in \Omega^a$ (the dependence of the partial derivative of $f$ on $\xi$ has been dropped for notational simplicity). In view of this "Envelope Theorem", we only have to calculate the second term in (A9). To do this, we write the risk factor $r$ as a function $r^l(\omega)$ of random parameter $\omega \in \Omega^a$ and observe that

$$\frac{\partial f^l}{\partial v}(\omega) = \frac{\beta}{\alpha^2} \tau r^l(\omega). \qquad (A11)$$

In (A11), $f^l$ denotes A's loss function calculated at $u=u^l$, $v=v^l$. At the same time, by looking at the second term of the second equation in (15), we obtain

$$\left\langle \frac{\partial f^l}{\partial v} \delta_x v^l, h \right\rangle = \frac{\alpha}{\beta} \frac{\tau}{\gamma+2\tau} \int_{\Omega^a} \frac{\beta}{\alpha^2} \tau E^{\varepsilon,\omega^b} \left[ r^l(\xi) \right] \left( \overline{h} - h(\xi) \right) dP^a(\xi)$$

$$= \frac{\tau^2}{\gamma+2\tau} \int_{\Omega^a} E^{\varepsilon,\omega^b} \left[ \frac{r^l(\xi)}{\alpha} \right] \left( \overline{h} - h(\xi) \right) dP^a(\xi). \tag{A12}$$

Here, as usual, $\overline{h}$ means the integral of $h$ w.r.t. $P^a$.

By looking again at (15), substituting $m$ with $p$-$x$ and $n$ with $q$-$y$, and after some algebra, one arrives at

$$E^{\varepsilon,\omega^b} \left[ \frac{r^l(\xi)}{\alpha} \right] = \frac{\gamma \left( p + \frac{\beta}{\alpha} q \right)}{\gamma+2\tau} + \frac{\gamma\tau(x(\xi) - \overline{x})}{(\gamma+\tau)(\gamma+2\tau)} \tag{A13}$$

for any $\xi \in \Omega^a$. Except for the term $\gamma\tau x(\xi)/(\gamma+\tau)(\gamma+2\tau)$, the rest of the right hand side of (A13) is independent of $\xi$ and can be taken outside of the integral in (A12). But, $\overline{h} - h$ has the mean value zero, so that all that remains of (A12) is

$$\left\langle \delta_x F^l, h \right\rangle = - \frac{\gamma\tau^3}{(\gamma+\tau)(\gamma+2\tau)^2} \int_{\Omega^a} x(\xi) h(\xi) dP^a(\xi).$$

This is exactly the statement of (16) and, in the context of Corollary 1, of (17) as well ●

**Proof of Proposition 5**

Expression (18) is an immediate consequence of the rules by which the integrated authority is instructed to function: if it has to treat signals $m$ and $n$ as if they were equal to values $p$ and $q$ actually observed by the national regulators, then, subjectively, it faces the situation of the centralized regulator in the truthful world from 3.2.2. Accordingly, controls must be set as in (11) with signaled values substituted for the actual ones.

To prove (19), rewrite the expressions for $u^d$, $v^d$ from (18) as

$$u^d = \frac{\tau}{\gamma+2\tau} \frac{R-z}{\alpha}, \ v^d = \frac{\tau}{\gamma+2\tau} \frac{R-z}{\beta}$$

and substitute this into (1). We then see that the controlled fundamental in this notation equals

$$r(x,y) = \frac{\gamma R + 2\tau z}{\gamma+2\tau} + \alpha\varepsilon^a + \beta\varepsilon^b.$$

Using the definition (2) of the loss function and taking expectations over $\varepsilon^a$, $\varepsilon^b$ (recall that they are independent and have zero means), we arrive at (19) •

**Proof of Corollary 2**

The proof of (20) amounts to finding under which value of A's lie the second term in the expectation over distortions $x,y$ of the loss function expression (19) attains its maximum and whether this maximum value is positive, as claimed. Since the only term in (19) dependent on the lie values is the second one, and the latter is linear-quadratic with negative coefficient by $z^2$, we know that, when A reports $p$ distortion $x$, his loss is minimized in expectation if and only if

$$\gamma R + (\gamma + 4\tau)(\alpha x + \beta \overline{y}) = 0. \tag{A14}$$

This implies immediately that all lie sizes are equal under optimal behavior (also applies to B by symmetry). Therefore, it is, indeed, sufficient to consider only binary distributions of $x$ and $y$, as was announced in footnote 4.

Considering this simplified case, we take regulator A who knows that $y$ is zero with probability $\pi^b$ and $l^b \neq 0$ with probability 1- $\pi^b$. The expected lie $\overline{y}$ by B is simply (1- $\pi^b$)$l^b$. Analogously, his own randomized reporting generates the truth with probability $\pi^a$ and a lie $l^a \neq 0$ with probability 1- $\pi^a$. Optimal choice of $l^a$ by A, according to (A14), implies the following reaction function:

$$l^a = -\frac{\beta}{\alpha}\left(1 - \pi^b\right)l^b - \frac{\gamma}{\gamma + 4\tau}\frac{R}{\alpha}.$$

By symmetry, an analogous reaction function is valid for B. Solving these two equations for $l^a$ and $l^b$, we arrive at (20).

Now observe that the loss function of A under centralized regulation with truthful signals, (12a), is equal to the sum of the first and the third terms on the right hand side of (19). Evidently, the linear-quadratic second term there is strictly positive at and around the maximum, which proves that A's (and B's) losses are lower when they misreport according to (20) than when they tell the truth •

# IES Working Paper Series

All papers can be downloaded at: http://ies.fsv.cuni.cz