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Bachelor Thesis

Centralized or Decentralized Provision of Local Public Goods with Spillovers: Which to Choose and Under what Circumstances?

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Abstract

This paper revises the classic trade-off between centralized and decentralized provision of local public goods. It studies the performance of centralization under different tax systems and compares them with decentralization. It argues that a conflict of interest between citizens in different regions caused by shared costs is significantly suppressed or completely eliminated when district specific head-tax is introduced. The outcome then depends on the degree of spillovers, heterogeneity in tastes for public spending and the mechanisms in which the centralized legislature composed of locally elected representatives works.

Abstract

[in Slovak] Táto práca sa zaoberá klasickým problémom centralizovanej alebo decentralizovanej produkcie lokálnych verejných statkov. Študuje výhody centralizácie v rôznych daňových systémoch a porovnáva ich s decentralizáciou. Tvrdí, že konflikt záujmov medzi občanmi v rôznych regiónoch vznikajúci z dôvodu spoločných nákladov je výrazne potlačený alebo úplne obmedzený v daňovom systéme, v ktorom je daň z hlavy pre každý región odlišná. Relatívna výhodnosť' každého zo systémov potom závisí na miere externalít, rôznorodosti v preferenciách voči verejným statkom a na mechanizme, na ktorom centralizovaná vláda pozostávajúca z lokálne zvolených reprezentantov funguje.

Declaration

I hereby declare that the whole bachelor thesis was elaborated on my own and that I used only the listed sources.

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Prague, May 23, 2005 Peter Tuchyňa

To Beatrix

Contents

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1 Introduction

There is now a classic problem in the theory of fiscal federalism that asks which tier of government should be responsible for particular taxing and spending decisions. This area of research is still an interesting field to study due to its importance in the debate over the future shape of the European Union. The widely emphasized principle of subsidiarity states that functions should be decentralized where possible. On the other hand, still more competencies are being assigned to European institutions such as e.g. the competition policy (the supervision of cartels, mergers and acquisitions as well as sectoral and regional subsidies of the member states) and the environmental policy (the establishment of common norms for the protection of the environment).

This paper builds on the recent model of Besley and Coate (2003). This framework enables us to study the main determinants in deciding the case for centralized or decentralized provision of local public goods, namely heterogeneity in public goods preferences and the degree of externalities. These are the key elements in comparing the various benefits and costs of centralized and decentralized systems.

Besley and Coate (2003) depart from the existing literature in emphasizing the *political processes* of decision making. If governments under centralized systems were allowed to allocate different levels of local public goods to different districts, they could respect the preferences of citizens in each district while optimally accounting for cross-border spillovers. This would make the centralized system preferable. If there is a case for a decentralized system then, it must follow from the political economy considerations.

Centralization has been typically modeled as a system in which public spending is financed by general taxation and all jurisdictions receive a uniform level of the local public good. In a decentralized system local public goods are financed by local taxation and each district chooses its own preferred level. This approach has been adopted by Oates (1972) who argued that the drawbacks of centralized and decentralized systems are uniformity in provision and absence of reflecting the benefits going to other regions, respectively. This logic relies crucially on the assumption that centralization provides uniform levels of public goods. Besley and Coate (2003) relax this assumption and then study the various forms of centralized decision making.

An innovation of the paper is to model public spending under centralization financed by district-specific head-tax which depends on the public good level provided to each region. Under this tax system, taxing is not shared across regions any more and hence does not create such a significant common-pool problem where citizens use the political process to exploit the budgetary externality that the centralization with uniform taxation produces.

The remainder of the paper is organized as follows. Section 2 outlines the framework for our analysis. Section 3 provides a brief review of the standard analysis. Section 4 presents a political economy analysis with two forms of taxation beginning with a centralized system based on minimum winning coalition. Section 5 continues in this direction considering a more cooperative legislature. Finally, Section 6 offers some concluding remarks.

2 The Model

There are 2 geographically distinct regions or districts indexed by $i \in \{1, 2\}$ each populated by a continuum of citizens with a mass of unity. The citizens are immobile between the regions. The economy contains 3 goods: a single private good, x, and two local public goods, g_1 and g_2, each one associated with a particular district.[1] Each citizen is endowed with some of the private good and throughout we will assume that the endowments are high enough for each citizen to meet their required tax obligations. To produce one unit of either of the public goods requires p units of the private good.[2]

Each citizen in district i is characterized by a public goods preference parameter λ.[3] The preferences of a type λ citizen in district i are

$$x + \lambda[(1 - \kappa) \ln g_i + \kappa \ln g_{-i}],^{4}$$

where parameter $\kappa \in \langle 0, 1/2\rangle$ indexes the degree of spillovers. When $\kappa = 0$, citizens consume only the public good in their own district, while for $\kappa = 1/2$ they equally consume public

[1] The parameters g_1 and g_2 can be thought of as being the same public good provided at different levels in each district, as further assumptions may indicate. But to make the model richer, we will assume two distinct local public goods such as e.g. airports and roads each associated with a particular region.

[2] That is, we define one unit of each of the public good to represent such a quantity that costs p units of the private good.

[3] The parameter λ can be interpreted as indicating the interest in public goods of both districts.

[4] See discussion papers of Besley and Coate for more universal specifications of public goods preferences.

goods in both districts. Uniform κ (across regions) means that citizens from both districts consume both public goods in the same proportion.[5]

The range of preference types is $\langle 0, \overline{\lambda} \rangle$ in each district. The respective median type in district i is denoted by m_i. We assume, without loss of generality, that the median citizen in district 1 is at least as pro-public spending as his counterpart in district 2, i.e. $m_1 \geq m_2$. We also assume that $2m_1 < \overline{\lambda}$. The latter condition will be needed in Section 5 to obtain interior solutions.

Under a *decentralized system,* the level of public good in each district is chosen by the government of that district and public goods are financed by a uniform head tax on local residents. Thus, if district i chooses a public good level g_i, each citizen in this district pays a tax of pg_i. Under a *centralized system,* the levels of public goods are determined by a government that represents both regions. Spending is being financed by two possible tax systems whose outcomes will be compared. The first one is a uniform head tax on all citizens; with public good levels (g_1, g_2), this tax is $\frac{p(g_1+g_2)}{2}$. The second one is a head tax which is non-uniform across districts, but uniform for all citizens within a given region. Citizen of each region pays a head tax proportional to his consumption of both public goods; thus, public good levels (g_1, g_2) and degree of spillovers κ result in a head tax of $pg_i(1-\kappa)+pg_{-i}\kappa$ in district i.[6, 7]

Our social welfare criterion for comparing the performance of centralized and decentralized provision of local public goods will be aggregate public goods surplus. With public good levels (g_1, g_2), it is defined as

$$S(g_1, g_2) = [m_1(1-\kappa) + m_2\kappa] \ln g_1 + [m_2(1-\kappa) + m_1\kappa] \ln g_2 - p(g_1 + g_2).$$

[5]However, it could be that citizens from district 1 cared equally about the public goods in both districts, whereas citizens from district 2 cared only about their own public good. That is to say, proportional public goods preferences κ could differ among regions. Moreover, differences among citizens from the same district could occur, i.e. a citizen could, for instance, derive benefit from both public goods, but his neighbor from the same district could be interested only in the public good provided by his own district. We will, however, assume these spillovers to be of a purely technical nature. Moreover, we will assume them to be the same for citizens from both districts as well as for the citizens in a given region just to capture the fact that the resolution between centralization and decentralization depends on the degree of spillovers and the extent of heterogeneity in preferences for public goods.

[6]Of course, this tax system is uniform in the case when the levels of public goods in both districts equal. Otherwise, it is non-uniform, which is the reason we have identified it as such.

[7]What is worth noting here regarding both tax systems is that uniform taxation implies shared financing of the public goods whereas non-uniform tax system means that each district pays proportionally to its consumption. If it consumes less, it pays less; if it consumes more, it pays more.

The surplus is in fact a sum of differences between utilities of median citizens in both districts or, more specifically, a median citizens' sum of marginal benefits from their consumption of public goods:

$$\begin{aligned}\Delta U_{m_1} + \Delta U_{m_2} &= x_1 - pg_1 + m_1[(1-\kappa)\ln g_1 + \kappa \ln g_2] - x_1 \\ &\quad + x_2 - pg_2 + m_2[(1-\kappa)\ln g_2 + \kappa \ln g_1] - x_2 \\ &= S(g_1, g_2).\end{aligned}$$

The surplus-maximizing public good levels are

$$(g_1, g_2) = \left(\frac{m_1(1-\kappa) + m_2\kappa}{p}, \frac{m_2(1-\kappa) + m_1\kappa}{p}\right).$$ [8]

This result reveals that the surplus-maximizing public good levels take account of the benefits received by citizens from both districts.

3 The Standard Analysis

The above outlined model allows a simple exposition of the traditional analysis according to Oates (1972), who influenced many public finance economists' views on the relative merits of centralization and decentralization. He supposed that, in a decentralized system, each district's government independently chooses the policy which maximizes public goods surplus in the region (which is ΔU_{m_i}, $i \in \{1,2\}$). A pair of expenditure levels (g_1^d, g_2^d) will form a Nash equilibrium, which requires that:

$$g_i^d = \arg\max_{g_i}\{m_i[(1-\kappa)\ln g_i + \kappa \ln g_{-i}^d] - pg_i\}, \quad i \in \{1,2\}.$$

Taking first order conditions yields:

$$(g_1^d, g_2^d) = \left(\frac{m_1(1-\kappa)}{p}, \frac{m_2(1-\kappa)}{p}\right).$$

Each region's government thus only takes into account the benefits received by his constituency and local public goods are surplus-maximizing only when there are no spillovers, regardless of heterogeneity in tastes. When spillovers occur, public goods production results

[8] The vector is an interior solution of a simple maximization of function $S(g_1, g_2)$. We obtain it by taking first-order conditions, i.e. by differentiating this function w.r.t. g_1 and g_2 and setting equal to zero.

in under-provision in both districts and this under-provision is increasing in the extent of spillovers.

Under a centralized system, Oates assumed that the government would be restricted to provide a uniform level of the public goods, denoted g^c. He further assumed that expenditures would be financed by a uniform head tax, which is in the case of uniform provision of public goods identical to our proposed non-uniform head tax that takes into account the proportional consumption of both goods by citizens from each region.[9] This common level of public goods satisfies

$$g^c = \arg\max_g \{[m_1 + m_2] \ln g - 2pg\},$$

resulting in

$$g^c = \frac{m_1 + m_2}{2p}.$$

The uniform level of public goods is independent of the level of spillovers and results in the surplus-maximizing level only in the case of identical districts.[10] However, when $m_1 > m_2$, centralization over-provides public goods to district 2 and under-provides them to district 1 except when spillovers are maximal, i.e. $\kappa = 1/2$. In this situation, citizens consume public goods in both districts equally which leads to uniform provision of public goods in both regions.

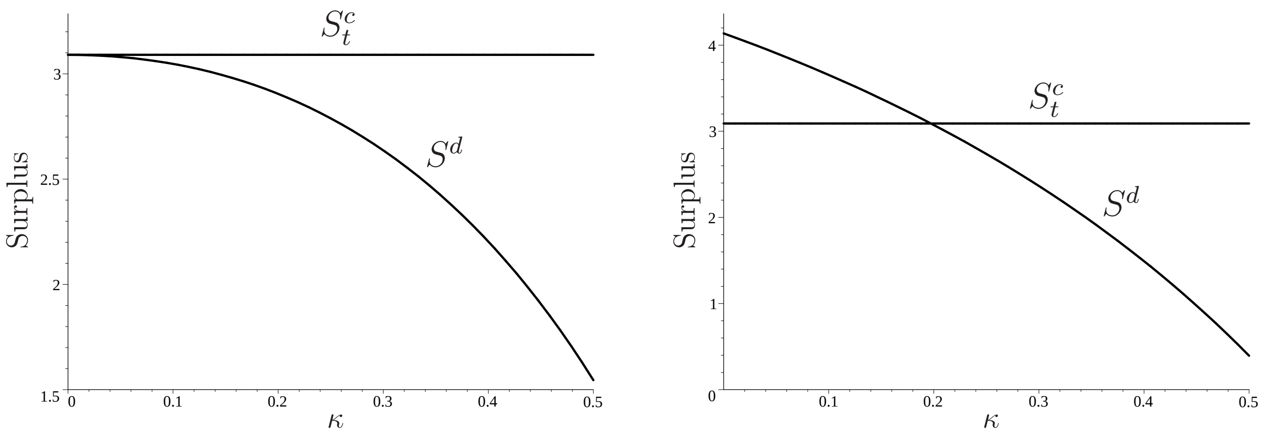
3.1 Comparative Statics

When regions are homogeneous, centralization produces surplus maximizing public goods levels and dominates decentralization when spillovers are present.[11] However, the two systems generate the same level of public goods surplus when spillovers do not occur (Figure 1a).

[9]This stems from the simple fact that, with uniform provision and identical prices of local public goods, Oates' head tax $\frac{p(g+g)}{2} = pg$ equals $pg(1-\kappa) + pg\kappa = pg$.

[10]Throughout the text, the phrases *identical* (*non-identical*) and *homogeneous* (*heterogeneous*) *districts* will indicate that the median citizens from each region *have* (*do not have*) the same public goods preferences.

[11]In this case, centralization has the conventional advantage of internalizing spillovers.



(a) Identical districts (b) Non-identical districts

Figure 1: Aggregate public goods surpluses under decentralization (S^d) and centralization in the standard analysis (S_t^c).

In the case of heterogeneous districts, decentralization produces surplus maximizing public goods levels and dominates centralization when there are no spillovers. On the other hand, when the spillovers are maximal, centralization produces surplus maximizing public goods levels and dominates decentralization. Finally, when the spillovers are somewhere in between these two polar cases, there exists a critical level of spillovers above which centralization dominates and under which decentralization is preferred (Figure 1b).

Proposition 1. *Suppose that the assumptions of the standard analysis are satisfied. Then*

(i) If the regions are homogeneous and spillovers are present ($\kappa > 0$), a centralized system produces a higher level of surplus than does decentralization. Absent spillovers ($\kappa = 0$), the two systems generate the same level of surplus.

(ii) If the districts are heterogeneous, there is a critical value of κ, greater than zero but less than $1/2$, such that a centralized system produces a higher level of surplus if and only if κ exceeds this critical level.[12]

According to Oates, without spillovers, a decentralized system is preferred. With spillovers and homogeneous districts, a centralized system is superior. With spillovers and heterogeneous regions, it is necessary to compare the extent of the two effects.

[12]The proof of this as well as the other results may be found in the Appendix.

It is often suggested that heterogeneity favors the case for decentralization. However, in our model, this does not follow. To establish such a proposition, it would be necessary to show that the critical level of spillovers is increasing in heterogeneity. But there is no guarantee that this is so.[13]

Modeling the trade-off in the standard analysis relies on the assumption of uniform expenditures in a centralized system. But this assumption is too strong and does not correspond with empirical evidence that expenditures vary across districts in many countries. We will now relax this assumption and model the decision making institutions which decide upon allocation of resources in both centralized and decentralized systems.

4 A Political Economy Analysis with Two Forms of Taxation

4.1 Policy Determination Under Decentralization

In a decentralized system, we assume that each region elects a single representative from that region to choose policy. Our model is based on the citizen-candidate approach to political decision making, which has two stages. First, elections determine which citizen from each district is selected to constitute the decision making government in that district (election stage). Second, policies are chosen simultaneously by the elected representatives in each district (policy-selection stage).

Using backward induction,[14] let us proceed as follows. First, we find what elected

[13]This may be analyzed by letting $S^d(\kappa,\alpha)$ and $S^c_t(\kappa,\alpha)$ denote surpluses under decentralization and centralization, respectively, when $(m_1,m_2)=(\alpha\omega,(1-\alpha)\omega)$, where $\alpha\in\langle 1/2,1\rangle$ measures the degree of heterogeneity between the two regions. Districts are identical when $\alpha=1/2$ and become more heterogeneous when α increases.
Then $S^c_t(\kappa,\alpha)=\omega\ln\dfrac{\omega}{2p}-\omega$, which is independent of both κ and α. Therefore we can write $S^c_t(\kappa,\alpha)=S^c_t$. The critical value of κ, denoted $\kappa^*(\alpha)$, is uniquely defined by the equation $S^d(\kappa^*,\alpha)=S^c_t$. To show that κ^* is an increasing function of α, it is necessary to show that for all α, $\partial S^d(\kappa^*,\alpha)/\partial\alpha>0$. Differentiating, we obtain

$$\frac{\partial S^d}{\partial\alpha}(\kappa,\alpha)=\omega(1-2\kappa)\ln\frac{\alpha}{1-\alpha}+\omega\kappa\frac{1-2\alpha}{\alpha(1-\alpha)}.$$

The first term is positive, while the second one is negative. As spillovers increase, the first term goes to zero. Thus, it is possible that $\partial S^d(\kappa,\alpha)/\partial\alpha<0$. (In our specification of public goods preferences, surplus under decentralization is always decreasing in heterogeneity for all $\kappa>1/4$. This finding makes it possible that the critical level of spillovers is decreasing in heterogeneity, i.e. the case for centralization could be strengthened as the regions become more diverse.)

[14]Backward induction is an iterative process for solving finite extensive form games. First, one determines

representatives select (stage 2 or policy-selection stage) and then we discuss whom citizens, considering outcomes which are subsequently selected by representatives, will appoint to an office (stage 1 or election stage). Beginning with stage 2, let the types of the representatives in district 1 and 2 be λ_1 and λ_2, respectively.[15] Then the policy outcome $(g_1(\lambda_1), g_2(\lambda_2))$ satisfies

$$g_i(\lambda_i) = \arg\max_{g_i}\{\lambda_i[(1-\kappa)\ln g_i + \kappa \ln g_{-i}(\lambda_{-i})] - pg_i\}, \quad \text{for } i \in \{1, 2\}.$$

Solving this with first-order conditions yields

$$(g_1(\lambda_1), g_2(\lambda_2)) = \left(\frac{\lambda_1(1-\kappa)}{p}, \frac{\lambda_2(1-\kappa)}{p}\right).$$

The level of each district's public goods spending is higher the stronger is the public good preference of its representative and lower the higher the level of spillovers.

Now let us move to stage 1. With the representatives λ_1 and λ_2 in region 1 and 2, respectively, a citizen of type λ in district i will enjoy a public goods surplus

$$\Delta U_{\lambda,i} = \lambda\left[(1-\kappa)\ln\frac{\lambda_i(1-\kappa)}{p} + \kappa\ln\frac{\lambda_{-i}(1-\kappa)}{p}\right] - \lambda_i(1-\kappa).$$

These preferences over types determine citizens' voting decisions. A pair of representative types $(\lambda_1^*, \lambda_2^*)$ is majority preferred under decentralization if, in each district i, a majority of citizens prefer the type of their representative to any other type $\lambda \in \langle 0, \overline{\lambda}\rangle$, given the type of the other district's representative λ_{-i}^*.

We assume that the elected representatives in the two regions will be of these majority preferred types. Further we assume that each citizen votes sincerely (according to his public goods preferences), all citizens always vote and have perfect information.

Citizens' preferences over types are single-peaked,[16] implying that a pair of representative types is majority preferred under decentralization if and only if it is a median pair; i.e. $(\lambda_1^*, \lambda_2^*) = (m_1, m_2)$. This yields:

the optimal strategy of the player who makes the last move of the game. Then, the optimal action of the next-to-last moving player is determined taking the last player's action as given. The process continues in this way backwards in time until all players' actions have been determined. Effectively, one determines the Nash equilibrium of each subgame of the original game.

[15] We assume that (i) candidates have no opportunity costs, i.e. any citizen can agree to be a candidate; and that (ii) representatives can only decide on the provision of public goods, i.e. there are no other perquisites of the office.

[16] Given any two types $\hat{\lambda}_i$ and λ_i' such that $\lambda_i' < \hat{\lambda}_i < \lambda$ or $\lambda < \hat{\lambda}_i < \lambda_i'$, type λ citizens always prefer type $\hat{\lambda}_i$ citizens.

Lemma 1. *Suppose that the assumptions of the political economy analysis are satisfied. Then the policy outcome under decentralization is*

$$(g_1, g_2) = \left(\frac{m_1(1-\kappa)}{p}, \frac{m_2(1-\kappa)}{p} \right).$$

These levels of local public goods respect the preferences of the median citizen within a region, which agrees with the standard local public finance analysis.

4.2 Policy Determination Under Centralization with Two Forms of Taxation

The policy determination process under centralization also has two steps: an election stage; and a policy selection stage. In the elections, one citizen from each district is chosen to serve in a common legislature. In the policy selection stage, the legislature determines public goods provision in each region. Our first method of capturing the decision making process in the legislature will be the minimum winning coalition view. Under this view, a coalition of just-above-50 % of the representatives forms to share the benefits of public spending among their districts. Regions whose representatives are outside the coalition are only allocated spending to the extent that this benefits coalition members. The logic is that, in a majority rule legislature, if there were any more than just-above-50 % of the representatives in the coalition supporting the spending bill, the majority of coalition members would benefit from expelling the surplus members and further concentrating spending on their own regions. Because there are many possible minimum winning coalitions, this view suggests that there will be uncertainty concerning the identity of the coalition that forms to determine expenditures.

In our model, we may capture this uncertainty by assuming that each representative can be thought of as a minimum winning coalition with equal probability. Thus, again using backward induction, if the representatives are of types λ_1 and λ_2, the policy outcome will be $(g_1^1(\lambda_1), g_2^1(\lambda_1))$ with probability 1/2 and $(g_1^2(\lambda_2), g_2^2(\lambda_2))$ with probability 1/2, where $(g_1^i(\lambda_i), g_2^i(\lambda_i))$ is the optimal choice of district i's representative.

4.2.1 Policy Determination with Uniform Taxation

With uniform taxation and representatives of types λ_1 and λ_2, the optimal choice of district i's representative is

$$(g_1^i(\lambda_i), g_2^i(\lambda_i)) = \arg\max_{(g_i, g_{-i})} \left\{ \lambda_i[(1-\kappa)\ln g_i + \kappa \ln g_{-i}] - \frac{p}{2}(g_i + g_{-i}) \right\}.$$

It is straightforward to verify that

$$(g_i^i(\lambda_i), g_{-i}^i(\lambda_i)) = \left(\frac{2\lambda_i(1-\kappa)}{p}, \frac{2\lambda_i\kappa}{p} \right), \quad i \in \{1, 2\}.$$

The level of public goods spending depends only on the decisive representative's preference for public goods and the level of spillovers. The stronger the preferences for public goods of the decisive representative are, the higher is the spending. Furthermore, spending for the representative's domestic public good varies inversely with spillovers while the other district's public good expenditures vary proportionally with spillovers.

When the representative types are λ_1 and λ_2, a citizen of type λ in region i obtains an expected public goods surplus of

$$\begin{aligned} \Delta U_{\lambda,i} = \frac{1}{2} \Bigg\{ & \lambda \left[(1-\kappa) \ln \frac{2\lambda_i(1-\kappa)}{p} + \kappa \ln \frac{2\lambda_i\kappa}{p} \right] - \lambda_i \\ & + \lambda \left[(1-\kappa) \ln \frac{2\lambda_{-i}\kappa}{p} + \kappa \ln \frac{2\lambda_{-i}(1-\kappa)}{p} \right] - \lambda_{-i} \Bigg\}. \end{aligned}$$

Again we assume that the representatives will be of the majority preferred types. A pair of representative types $(\lambda_1^*, \lambda_2^*)$ is majority preferred if and only if in each district i the median type prefers λ_i^* to any other type $\lambda \in \langle 0, \overline{\lambda} \rangle$, given the other district's representative type λ_{-i}^*.[17] This means that $(\lambda_1^*, \lambda_2^*)$ is majority preferred if and only if it is a Nash equilibrium of the two-player game in which each player has strategy set $\langle 0, \overline{\lambda} \rangle$ and player $i \in \{1, 2\}$ has payoff function

$$\begin{aligned} \Delta U_{m_i}(\lambda_i) = \frac{1}{2} \Bigg\{ & m_i \left[(1-\kappa) \ln \frac{2\lambda_i(1-\kappa)}{p} + \kappa \ln \frac{2\lambda_i\kappa}{p} \right] - \lambda_i \\ & + m_i \left[(1-\kappa) \ln \frac{2\lambda_{-i}\kappa}{p} + \kappa \ln \frac{2\lambda_{-i}(1-\kappa)}{p} \right] - \lambda_{-i} \Bigg\}. \end{aligned}$$

[17] If citizens of type λ prefer a type $\hat{\lambda}_i$ candidate to a type λ_i', where $\hat{\lambda}_i < \lambda_i'$ ($\hat{\lambda}_i > \lambda_i'$), then so must all citizens of types lower (higher) than λ. This implies that a majority of citizens in district i prefer a type $\hat{\lambda}_i$ candidate to a type λ_i' candidate if and only if the median type prefers a type $\hat{\lambda}_i$ candidate to a type λ_i' candidate.

Taking first-order conditions and solving yields

$$(\lambda_1^*, \lambda_2^*) = (m_1, m_2).$$

Thus, an elected pair of representatives will be of types (m_1, m_2) and will choose a policy which reflects their public goods preferences. So we have:

Lemma 2. *Suppose that the taxation is uniform and the assumptions of the political economy analysis are satisfied. Then the policy outcome under centralization with a minimum winning coalition view of the legislature is random, generating* $(g_1, g_2) = \left(\frac{2m_1(1-\kappa)}{p}, \frac{2m_1\kappa}{p}\right)$ *with probability* $1/2$ *and* $(g_1, g_2) = \left(\frac{2m_2\kappa}{p}, \frac{2m_2(1-\kappa)}{p}\right)$ *with probability* $1/2$.

This result illuminates the main drawbacks of centralization with a minimum winning coalition legislature and uniform taxation:

1) *Uncertainty:* each district faces uncertainty as to the amount of public good that it will receive, reflecting the uncertainty in the identity of the minimum winning coalition.

2) *Misallocation:* public expenditures across regions are skewed towards those inside the winning coalition.

4.2.2 Comparative Statics

The only situation in which centralization produces the surplus maximizing level is when the districts are identical and spillovers are maximal ($\kappa = 1/2$). When districts differ ($m_1 > m_2$) and spillovers are complete, spending is allocated equally across regions but district 1's representative over-provides local public goods, while district 2's representative under-provides them. While higher levels of spillovers still lead those in the minimum winning coalition to allocate public goods to districts outside the coalition, it is only to the extent that this benefits those inside the coalition.

For low levels of spillovers, the misallocation problem is at its worse. Public goods are over-provided to regions in the minimum winning coalition and under-provided to those districts that are outside the coalition, reflecting the budgetary externality created by common financing. However, this drawback is significantly suppressed when the non-uniform tax system is introduced.

4.2.3 Policy Determination with Non-uniform Taxation

With non-uniform taxation and representatives of types λ_1 and λ_2, the optimal choice of region i's representative is

$$(g_1^i(\lambda_i), g_2^i(\lambda_i)) = \arg \max_{(g_i, g_{-i})} \{\lambda_i[(1-\kappa)\ln g_i + \kappa \ln g_{-i}] - (pg_i(1-\kappa) + pg_{-i}\kappa)\}.$$

It is easily checked that

$$(g_1^i(\lambda_i), g_2^i(\lambda_i)) = \left(\frac{\lambda_i}{p}, \frac{\lambda_i}{p}\right), \quad i \in \{1, 2\}.$$

As above, if the representative types are λ_1 and λ_2, a citizen of type λ in district i obtains an expected public goods surplus of

$$\Delta U_{\lambda,i} = \frac{1}{2}\left\{\lambda\left[(1-\kappa)\ln\frac{\lambda_i}{p} + \kappa\ln\frac{\lambda_i}{p}\right] - \lambda_i + \lambda\left[(1-\kappa)\ln\frac{\lambda_{-i}}{p} + \kappa\ln\frac{\lambda_{-i}}{p}\right] - \lambda_{-i}\right\}.$$

Analogically to the case of uniform taxation, we arrive at the conclusion that an elected pair of representatives will be of types (m_1, m_2) and that they will choose a policy which reflects their preferences. This establishes:

Lemma 3. *Suppose that the tax system is non-uniform across districts and the assumptions of the political economy analysis are satisfied. Then the policy chosen under centralized system with a minimum winning coalition view of the legislature is random, generating* $(g_1, g_2) = \left(\frac{m_1}{p}, \frac{m_1}{p}\right)$ *with probability* $1/2$ *and* $(g_1, g_2) = \left(\frac{m_2}{p}, \frac{m_2}{p}\right)$ *with probability* $1/2$.

Compared to the case of uniform taxation, the problem of *uncertainty* remains due to the unknown identity of the coalition. However, the drawback of *misallocation* is significantly reduced reflecting the fact that each district is taxed according to its proportional consumption of both local public goods. This suppresses the incentives of the coalition members to allocate too much of the public goods to their districts while forgetting about the regions outside the coalition.

4.2.4 Comparative Statics

The levels of public goods are independent of spillovers. They only depend on preferences of the decisive representative which then chooses uniform provision of public goods. With identical representatives, centralization with non-uniform taxation produces the surplus maximizing levels of local public goods. When m_1 exceeds m_2 and spillovers are complete, region 1's representative over-provides local public goods, while district 2's representative under-provides them.

The misallocation problem is at its worse when the spillovers are lower than complete. The levels of public goods provided are further from the optimal, aggregate surplus enhancing levels. However, the extent of these misallocations is lower than that under the centralized system with uniform taxation.

4.3 Centralization vs. Decentralization

Decentralization produces the surplus maximizing public goods levels only when the spillovers do not occur. We have already seen that public goods levels under *centralization with uniform taxation* are surplus maximizing when the spillovers are complete and the districts are homogeneous. It follows that, in the case of identical districts, decentralization dominates when the spillovers are small and centralization is preferred when the spillovers are large.

Centralization with non-uniform taxation produces the surplus maximizing public goods levels when the districts are identical. This surplus is independent of spillovers and is higher than the surplus under decentralization for all κ except when the spillovers are absent. In such a case, both systems generate the surplus maximizing public goods levels. The next proposition and Figure 2 summarize these results.

Proposition 2. *Suppose that the assumptions of the political economy analysis are satisfied, the centralized decision making relies on the minimum winning coalition, and the districts are identical. Then*

(i) If the taxation is uniform, there is a critical value of κ, strictly greater than 0 but less than 1/2, such that a centralized system produces a higher level of surplus if and only if κ exceeds this critical level.

(ii) If the taxation is non-uniform across districts and spillovers are present ($\kappa > 0$), a centralized system produces a higher level of surplus than does decentralization. Absent spillovers ($\kappa = 0$), the two systems generate the same level of surplus.

(iii) Surplus under centralization with non-uniform taxation equals that under the centralized system in the standard analysis for all levels of spillovers. These surpluses are higher than that under centralization with uniform taxation except when the spillovers are maximal ($\kappa = 1/2$). In such a case, all three systems of centralization produce the same public goods surplus.

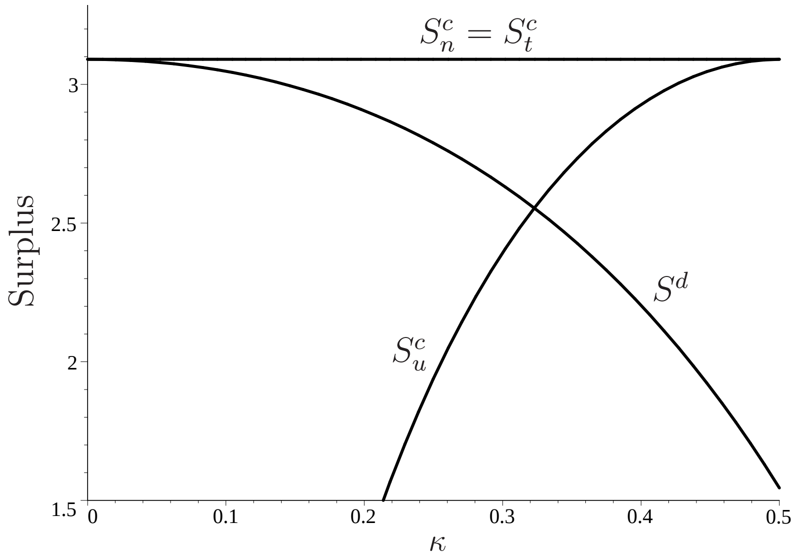


Figure 2: Aggregate public goods surpluses under decentralization (S^d), centralization in the standard analysis (S_t^c) and centralization with uniform (S_u^c) and non-uniform (S_n^c) taxation in the case of *identical districts*.

There are two comparisons which require analysis. First, comparing Proposition 2 with its counterpart in part (i) of the Proposition 1, there is one significant difference. With identical districts, centralized system in the standard analysis is supposed to dominate decentralization for all $\kappa > 0$. However, centralization based on minimum winning coalition and uniform taxation no more dominates for low levels of spillovers, as those inside the coalition have low incentives to provide public goods to the outside regions. This is further combined with the uncertain identity of the coalition. With higher spillovers, uncertainty remains but the decisive representatives have higher incentives to provide more public goods to both districts, which increases the surplus under centralization. Thus, political economy analysis weakens the case for centralization when the taxation is uniform.

Second, comparison of the two centralized systems under the political economy analysis generates a strong case for centralization with non-uniform taxation, which dominates for all $\kappa < 1/2$. This is due to the effects that each taxation has on the decisions about the allocation of public expenditures. When the taxation is uniform, each district pays the same head-tax independent of the received level of public goods. This motivates coalition members to allocate as much as they prefer to their districts. In contrast, under the centralized system with non-uniform taxation there is no such effect, as each district is taxed according to its proportional consumption of both local public goods. This balances the allocated levels and centralization with non-uniform taxation significantly dominates centralization with a uniform tax system. Furthermore, the resulting surplus under centralization with non-uniform taxation is the same as under centralization in the standard analysis.[18]

When the regions are heterogeneous, *centralized system with uniform taxation* still dominates decentralization for high levels of spillovers and its performance is increasing in spillovers. Thus, there is a critical value of κ under which decentralization is preferred and above which centralization dominates.

Centralization with non-uniform taxation is independent of spillovers, produces higher level of surplus than decentralization for maximal spillovers and lower level of surplus for zero spillovers. It follows that there exists a critical value of κ above which centralized system dominates and under which decentralization is preferred. However, this critical value is lower than that in the uniform taxation case. Again, the following proposition and Figure 3 summarize these findings.

Proposition 3. *Suppose that the assumptions of the political economy analysis are satisfied, the centralized decision making relies on the minimum winning coalition, and the districts are non-identical. Then*

(i) If the taxation is uniform, there is a critical value of κ, strictly greater than 0 *but less than* 1/2*, such that a centralized system produces a higher level of surplus if and only if κ exceeds this critical level. This critical level is higher than that in the standard analysis.*

[18]This is due to the fact that the provision of public goods and the actual taxes under centralization with non-uniform taxation are the same as under the centralized system in the standard analysis. With identical districts, each representative would choose such uniform public goods levels that would be chosen by all the other representatives, if elected.

(ii) *If the taxation is non-uniform across districts, there is a critical value of* κ*, strictly greater than* 0 *but less than* 1/2*, such that a centralized system produces a higher level of surplus if and only if* κ *exceeds this critical level. This critical level is higher than that in the standard analysis and lower than that under centralization with uniform taxation.*

(iii) *Surplus under centralization in the standard analysis is higher than surpluses under both centralized systems in the political economy analysis for all levels of spillovers. Furthermore, surplus under centralization with non-uniform taxation is higher than that under the centralized system with uniform taxation except when the spillovers are maximal* ($\kappa = 1/2$)*. In such a case, the two systems produce the same public goods surplus.*

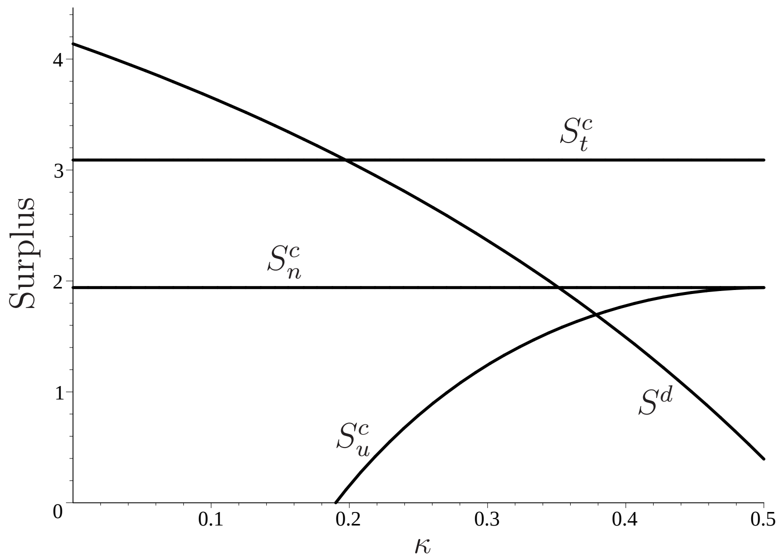


Figure 3: Aggregate public goods surpluses under decentralization (S^d), centralization in the standard analysis (S_t^c) and centralization with uniform (S_u^c) and non-uniform (S_n^c) taxation in the case of *non-identical districts*.

As above, two juxtapositions can be observed. First, comparing Proposition 3 with its relevant counterpart in part (ii) of the Proposition 1 reveals that centralization with non-cooperative legislature creates even larger incongruity when the districts are heterogeneous. This exacerbated misallocation problem combined with the persistent drawback of uncertainty results in a weakened case for centralization compared with the centralized system in the standard analysis. However, the fundamental qualitative conclusions remain unchanged under the political economy analysis as under the traditional one—for low spillover levels, decentralization dominates; when the spillovers are high, centralization is preferred.[19]

Second, comparison of the two centralized systems with different taxations remains as above. This is because the two systems' surpluses decrease in the same rate with increasing heterogeneity.[20] Thus, centralization with non-uniform taxation dominates centralization with uniform tax system for all $\kappa < 1/2$. When spillovers are complete, the two systems generate the same level of surplus. This reflects the fact that, in the uniform taxation case, representative in the winning coalition has incentives to provide the same level of public goods to both regions which corresponds to the case of non-uniform taxation.

Furthermore, surplus under centralization with non-uniform taxation is lower than that under centralization in the standard analysis. In the non-uniform taxation case, increasing heterogeneity causes the potential provisions of the two representatives to vary still more. This decreases the surplus under centralization with non-uniform taxation and because the surplus under centralized system in the traditional analysis is independent of heterogeneity, centralization with non-uniform tax system generates lower public goods surplus.[21] It follows

[19]What happens with both critical levels of spillovers as heterogeneity increases may be analyzed by letting $S^d(\kappa,\alpha)$, $S^c_u(\kappa,\alpha)$ and $S^c_n(\kappa,\alpha)$ denote surpluses under decentralization, centralization with uniform and non-uniform taxation, respectively, when $(m_1, m_2) = (\alpha\omega, (1-\alpha)\omega)$, where $\alpha \in \langle 1/2, 1\rangle$ measures the degree of heterogeneity between the regions.
The first critical level of κ, denoted $\kappa_1^*(\alpha)$, is uniquely defined by the equation $S^d(\kappa_1^*,\alpha) = S^c_u(\kappa_1^*,\alpha)$. To show that κ_1^* is an increasing function of α, it is necessary to show that for all $\alpha \in (1/2, 1)$, $\frac{\partial S^d(\kappa_1^*,\alpha)}{\partial\alpha} - \frac{\partial S^c_u(\kappa_1^*,\alpha)}{\partial\alpha} > 0$. Differentiating, we obtain

$$\frac{\partial S^d(\kappa,\alpha)}{\partial\alpha} - \frac{\partial S^c_u(\kappa,\alpha)}{\partial\alpha} = \underbrace{\omega\left(\frac{1}{2}-\kappa\right)}_{>0}\left(2\ln\frac{\alpha}{1-\alpha} - \frac{1-2\alpha}{\alpha(1-\alpha)}\right).$$

The expression in the latter parentheses equals zero when $\alpha = 1/2$ and is positive for all α in the range (1/2,1). Thus, the difference is positive for all $\alpha \in (1/2, 1)$ and $\kappa_1^* < 1/2$ which implies that the critical level of spillovers increases with increasing heterogeneity.
The second critical level of κ, denoted $\kappa_2^*(\alpha)$, is uniquely defined by the equation $S^d(\kappa_2^*,\alpha) = S^c_n(\kappa_2^*,\alpha)$. Due to the fact that $\frac{\partial S^c_u(\kappa,\alpha)}{\partial\alpha} = \frac{\partial S^c_n(\kappa,\alpha)}{\partial\alpha}$ for all κ and α, the critical level of spillovers increases with increasing heterogeneity for the non-uniform taxation case as well.

[20]See the previous footnote.

[21]Let $S^c_t(\kappa,\alpha)$ and $S^c_n(\kappa,\alpha)$ denote surpluses under centralization in the standard analysis and with non-uniform taxation, respectively, when $(m_1, m_2) = (\alpha\omega, (1-\alpha)\omega)$, where $\alpha \in (1/2, 1)$ measures the degree of heterogeneity. From the previous discussion we know that S^c_t is independent of heterogeneity while $\frac{\partial S^c_n(\kappa,\alpha)}{\partial\alpha} = \omega\frac{1-2\alpha}{2\alpha(1-\alpha)} < 0$ for all $\alpha \in (1/2, 1)$. This implies that increasing heterogeneity decreases surplus under centralization with non-uniform tax system which is then lower than that under a centralized system

that political economy analysis weakens the case for centralization when the taxation is non-uniform but not as considerably as in the case of a centralized system with uniform taxation.

5 Centralization with a Cooperative Legislature and Two Forms of Taxation

Under the minimum winning coalition view of legislative decision-making, policy outcomes are ex ante Pareto inefficient from the viewpoint of the representatives. Therefore, legislators may find a way around the inefficiency created by majoritarian decision criteria and prefer a less random outcome to the "feast or famine" implied by the minimum winning coalition theory. The representatives with power may, to a given extent, allocate benefits to those outside the coalition on the understanding that non-members would behave similarly if they were in power. However, there are many pairs of local public goods levels that are both efficient from the viewpoint of the representatives and that ex ante Pareto dominate minimum winning coalition outcomes.

Here we will assume the case when the representatives agree to the public goods allocation that maximizes their joint surplus, i.e. their behavior can be described by the *utilitarian bargaining solution.* This means that each representative now maximizes the same utility function as the others. They agree to form a coalition where everybody will have a weight in the decision making process, not just those who succeed to form a minimum winning coalition. This norm requires representatives to take into account the costs and benefits to their colleagues and would seem to offer centralization the best chance of dominating decentralization given our welfare criterion. But to what extent will centralization dominate decentralization will again depend on the form of taxation.

5.1 Policy Determination with Uniform Taxation

With uniform taxation and representatives of types λ_1 and λ_2, the policy outcome, $(g_1(\lambda_1, \lambda_2), g_2(\lambda_1, \lambda_2))$, will now maximize the representatives' joint surplus given by

$$\sum_{i=1}^{2} \{\Delta U_{\lambda_i}\} = \sum_{i=1}^{2} \Big\{\lambda_i[(1-\kappa)\ln g_i + \kappa \ln g_{-i}] - \frac{p}{2}(g_i + g_{-i})\Big\}.$$

in the traditional analysis.

It is straightforward to show that public goods levels maximizing this joint surplus are

$$(g_1(\lambda_1, \lambda_2), g_2(\lambda_1, \lambda_2)) = \left(\frac{\lambda_1(1-\kappa) + \lambda_2\kappa}{p}, \frac{\lambda_1\kappa + \lambda_2(1-\kappa)}{p} \right).$$

It is clear that if both districts elected representatives of the median types, the legislature would select the surplus maximizing public goods levels.

If the representative types are λ_1 and λ_2, a citizen of type λ in district i obtains public goods surplus of

$$\Delta U_{\lambda,i} = \lambda \left[(1-\kappa) \ln \frac{\lambda_i(1-\kappa) + \lambda_{-i}\kappa}{p} + \kappa \ln \frac{\lambda_{-i}(1-\kappa) + \lambda_i\kappa}{p} \right] - \frac{\lambda_1 + \lambda_2}{2}.$$

Turning to the election stage, we again assume that the pair of representatives will be of the majority preferred types that is defined in the by now familiar way. The main additional complication created by a cooperative legislature lies in finding the majority preferred types. This is because the public good level for each region depends on the type of legislator in both districts and, thereby, generates incentives for citizens in each district to delegate policy making strategically to a representative with different tastes than their own. This intention arises because sincere voting becomes suboptimal now.

To begin with, note that a pair of representative types $(\lambda_1^*, \lambda_2^*)$ is majority preferred if and only if in each district i the median type prefers λ_i^* to any other type $\lambda \in \langle 0, \overline{\lambda} \rangle$, given the other district's type λ_{-i}^*.[22] Thus, $(\lambda_1^*, \lambda_2^*)$ is majority preferred if and only if it is a Nash equilibrium of the two player game in which each player has strategy set $\langle 0, \overline{\lambda} \rangle$ and player $i \in \{1, 2\}$ has payoff function

$$U_i(\lambda_1, \lambda_2) = m_i \left[(1-\kappa) \ln \frac{\lambda_i(1-\kappa) + \lambda_{-i}\kappa}{p} + \kappa \ln \frac{\lambda_{-i}(1-\kappa) + \lambda_i\kappa}{p} \right] - \frac{\lambda_1 + \lambda_2}{2}.$$

In this game, the district i median citizen tries to manipulate λ_i so that he obtains something close to his preferred policy outcome anticipating the election outcomes in the other region and the subsequent working of the legislature.[23] He only has one degree of freedom, λ_i, but

[22] If district i elects a citizen of a higher type, then it receives more of both public goods. Then the same argument applies as in the footnote 17.

[23] To put it more rigorously, all citizens in region i now have an interest in manipulating λ_i to obtain something close to their preferred policy outcome. In other words, all voters in district i have the same interest in shifting λ_i according to their preferences and expectations of the election outcomes in the other regions and subsequent working of the legislature.

two objectives, (g_1, g_2). While raising λ_i always leads to an increase in g_i, if $\kappa > 0$ it also raises g_{-i}.

To state the equilibria, define $\hat{\kappa}$ as the solution to

$$\frac{m_1}{m_2} = \frac{\hat{\kappa}^3 + (1-\hat{\kappa})^3}{\hat{\kappa}(1-\hat{\kappa})}.$$

When the districts are identical, $\hat{\kappa} = 1/2$. In the non-identical districts case, $\hat{\kappa} < 1/2$. Then:

Lemma 4. *Suppose that the tax system is uniform across districts and the assumptions of the political economy analysis are satisfied. Then the policy chosen under a centralized system with a cooperative legislature is*

$$(g_1, g_2) = \left(\frac{2m_1[(1-\kappa)^4 - k^4]}{\left[(1-\kappa)^2 - \frac{m_1}{m_2}\kappa^2\right] p}, \frac{2m_1[(1-\kappa)^4 - k^4]}{\left[\frac{m_1}{m_2}(1-\kappa)^2 - \kappa^2\right] p} \right)$$

if $\kappa < \hat{\kappa}$, *and*

$$(g_1, g_2) = \left(\frac{2m_1(1-\kappa)}{p}, \frac{2m_1\kappa}{p} \right)$$

if $\kappa \geq \hat{\kappa}$.

It can be easily seen that the cooperative legislature does not select the surplus maximizing public goods levels. While a cooperative legislature deals with problems of uncertainty and misallocation that were present in the non-cooperative legislature, a new feature emerges:

- *Strategic delegation:* each district's median voter delegates policy-making to a representative of different type than median.

5.1.1 Comparative Statics

When the regions are identical ($m_1 = m_2 = m$), it follows from the Lemma that $g_1 = g_2 = 2m[(1-\kappa)^2 + \kappa^2]/p$. Recall that with identical districts, the surplus maximizing level of public goods is $g_1 = g_2 = m/p$. Thus, local public goods are over-provided in both regions for all $\kappa < 1/2$. The extent of this over-provision is decreasing with increasing spillovers and over-provision does not occur only when the spillovers are maximal ($\kappa = 1/2$). In such a case, local public goods are provided optimally.

The incentives to strategically delegate can be seen most clearly in the case of zero spillovers. Then, the optimal spending levels for the median voter from region 2 are $(g_1, g_2) = (0, 2m/p)$. Assume for a moment that both districts elect the median type representatives. This would lead to a policy outcome $(g_1, g_2) = (m/p, m/p)$. But if the district 2 elected a representative with a stronger taste for public spending, it would get more of its local public good with no impact on the district 1's public good level. Thus, each region is drawn to elect a type $2m$ representative.

As spillovers increase, the optimal spending levels in the two districts for each median voter converge. Electing a representative with a higher preference for public goods spending increases spending in the other region as well. Thus, the districts elect representatives with preferences closer to their median. When the spillovers are maximal, each region elects a median type representative and local public goods are provided at the surplus maximizing level.

With heterogeneity, an additional conflict over the *level* of public spending enters the picture, which can be seen most clearly in the case of complete spillovers. If $\kappa = 1/2$ and each region elects a representative of the median type, the public goods levels are $g_1 = g_2 = (m_1 + m_2)/2p$. This common level is too low for district 1's median voter and too high for region 2's. This gives district 1's median voter an incentive to have a higher representative type to boost public goods spending, while region 2's median voter desires a representative with lower public goods preferences. They pull in opposite directions until one or both districts has put in their most extreme type. Our assumption that $2m_1 < \overline{\lambda}$ implies that district 1 can obtain its preferred public goods level when district 2 has put in its most extreme type. Thus, district 1's median voter ends up getting his preferred outcome of $g_1 = g_2 = m_1/p$.

This additional conflict of interest creates a complex relationship between spillovers and public goods levels. Analyzing the solutions described in the Lemma, it can be shown that district 1's public good level is decreasing in the level of spillovers for κ sufficiently small and $\kappa > \hat{\kappa}$.[24] However, it is increasing in spillovers for κ sufficiently close to but less than $\hat{\kappa}$.

[24]This and the other claims concerning the public goods levels in Lemma 4 are established in T. Besley and S. Coate's discussion paper. In this paper we focus on the non-uniform taxation case which will be analyzed more analytically below.

This reflects the conflict over spending levels that arises as spillovers increase. To prevent district 2 from pulling down spending in both districts, district 1's median voter elects a representative with a higher public goods valuation, raising district 1's public good level. Region 2's public good level is decreasing in spillovers for $\kappa < \hat{\kappa}$ and increasing thereafter. It increases for spillover levels in excess of $\hat{\kappa}$, because it is now effectively controlled by district 1's median voter.

Comparing these outcomes with the surplus maximizing levels of public goods, district 1's public good level is always too high. The level provided to region 2 is too high for small κ and when κ is sufficiently large. However, it is less than the surplus maximizing level for κ sufficiently close to $\hat{\kappa}$. Note that this under-provision is in contrast to the over-provision results for the case of identical districts.

It is clear at this point that, although the legislature follows the utilitarian bargaining solution, the problem of strategic delegation causes that this solution may still be far from the surplus maximizing ideal. By introducing non-uniform taxation as defined above, we will nevertheless show that this problem is significantly suppressed.

5.2 Policy Determination with Non-uniform Taxation

If the taxation is non-uniform and the representatives are of types λ_1 and λ_2, the policy outcome $(g_1(\lambda_1, \lambda_2),\ g_2(\lambda_1, \lambda_2))$ will again maximize the representatives' joint surplus given by

$$\sum_{i=1}^{2} \{\Delta U_{\lambda_i}\} = \sum_{i=1}^{2} \{\lambda_i[(1-\kappa)\ln g_i + \kappa \ln g_{-i}] - p[(1-\kappa)g_i + \kappa g_{-i}]\}.$$

It is straightforward to verify that public goods levels maximizing this joint surplus are again

$$(g_1(\lambda_1, \lambda_2), g_2(\lambda_1, \lambda_2)) = \left(\frac{\lambda_1(1-\kappa) + \lambda_2\kappa}{p}, \frac{\lambda_1\kappa + \lambda_2(1-\kappa)}{p} \right).$$

Thus, as applicable also in the uniform taxation case, if both regions elected representatives of the median types, the legislature would select the surplus maximizing levels of public goods.

If the representatives are of types λ_1 and λ_2, a citizen of type λ in region i obtains public goods surplus

$$\Delta U_{\lambda,i} = \lambda \left[(1-\kappa)\ln\frac{\lambda_i(1-\kappa)+\lambda_{-i}\kappa}{p} + \kappa\ln\frac{\lambda_{-i}(1-\kappa)+\lambda_i\kappa}{p}\right] \\ - [\lambda_i(1-2\kappa+2\kappa^2)+\lambda_{-i}(2\kappa-2\kappa^2)].$$

As was the case in the previous section, the main complication lies in finding the majority preferred types when sincere voting is suboptimal. This complication is again due to the fact that public good level in each district depends on the type of legislator in both regions and, thereby, generates incentives for citizens in each region to strategically delegate policy making to a representative with different public goods preferences than their own.

A pair of representative types $(\lambda_1^*, \lambda_2^*)$ is majority preferred if and only if in each district i the median type prefers λ_i^* to any other type $\lambda \in \langle 0, \overline{\lambda}\rangle$, given the other region's type λ_{-i}^*.[25] Thus, $(\lambda_1^*, \lambda_2^*)$ is majority preferred if and only if it is a Nash equilibrium of the two player game in which each player has strategy set $\langle 0, \overline{\lambda}\rangle$ and player $i \in \{1, 2\}$ has payoff function

$$U_i(\lambda_1,\lambda_2) = m_i \left[(1-\kappa)\ln\frac{\lambda_i(1-\kappa)+\lambda_{-i}\kappa}{p} + \kappa\ln\frac{\lambda_{-i}(1-\kappa)+\lambda_i\kappa}{p}\right] \\ - [\lambda_i(1-2\kappa+2\kappa^2)+\lambda_{-i}(2\kappa-2\kappa^2)].$$

In this game, the district i median citizen tries to manipulate λ_i so that he obtains something close to his preferred policy outcome anticipating the election outcomes in the other region and the subsequent working of the legislature. While raising λ_i always leads to an increase in g_i, if $\kappa > 0$ it also raises g_{-i}.

To state the equilibria, define $\hat{\kappa}$ as the solution to

$$\frac{m_1}{m_2} = \frac{\hat{\kappa}^3 + (1-\hat{\kappa})^3}{\hat{\kappa}(1-\hat{\kappa})}.$$

When the districts are identical, $\hat{\kappa} = 1/2$. In the non-identical districts case, $\hat{\kappa} < 1/2$. Then we have:

Lemma 5. *Suppose that the tax system is non-uniform across districts and the assumptions of the political economy analysis are satisfied. Then the policy chosen under a centralized*

[25] If district i elects a citizen of a higher type, then it receives more of both public goods. Then the same argument applies as in the footnote 17.

system with a cooperative legislature is

$$(g_1, g_2) = \left(\frac{m_1[(1-\kappa)^2 - k^2]}{\left[(1-\kappa)^2 - \dfrac{m_1}{m_2}\kappa^2\right] p}, \frac{m_1[(1-\kappa)^2 - k^2]}{\left[\dfrac{m_1}{m_2}(1-\kappa)^2 - \kappa^2\right] p} \right)$$

if $\kappa < \hat{\kappa}$, *and*

$$(g_1, g_2) = \left(\frac{m_1(1-\kappa)}{[(1-\kappa)^2 + \kappa^2]p}, \frac{m_1\kappa}{[(1-\kappa)^2 + \kappa^2]p} \right),$$

if $\kappa \geq \hat{\kappa}$.

It is easily seen that the cooperative legislature does not always select the surplus maximizing public good levels. However, though *strategic delegation* occurs also when the taxation is non-uniform, it is significantly suppressed compared to the uniform taxation case in a way we will now explain.

5.2.1 Comparative Statics

With identical districts ($m_1 = m_2 = m$), the Lemma implies that $(g_1, g_2) = (m/p, m/p)$ which are the surplus maximizing public goods levels. Thus, strategic delegation is *completely eliminated* when the taxation is non-uniform and the districts are homogeneous. This is because neither district is drawn to elect a representative with stronger taste for public goods as each region knows that it will have to pay proportionally to its consumption. If it elected a higher type representative, the resulting increase in the provision of local public goods would be fully financed by the given district, which is in contrast to the uniform-taxation case where both districts participate at this increase in financing only by a half. Therefore, each district elects a median type representative and the local public goods are provided optimally, regardless of the level of spillovers.

Heterogeneity again gives rise to strategic delegation but in a lesser extent compared to the uniform taxation case. The only situation in which strategic delegation with non-uniform taxation is as strong as in the case of uniform taxation occurs when the spillovers are maximal. If $\kappa = 1/2$ and each district elects a median type representative, the policy outcome is $g_1 = g_2 = (m_1 + m_2)/2p$. But this level is too low for region 1's median voter and too high for that of region 2.[26] This gives district 1's median voter an incentive to

[26]The optimal spending levels for the district 1's median voter are $g_1 = g_2 = m_1/p$ whereas for the region 2's median voter are $g_1 = g_2 = m_2/p$.

elect a higher type representative and region 2's median voter an incentive to have a lower representative type. So they pull in opposite directions until one or both districts has put in their most extreme type. Under our assumption that $2m_1 < \overline{\lambda}$, region 1 can obtain its preferred public good levels when district 2 has put in its most extreme type.

The relationship between public goods levels and spillovers is again very complex. District 1's public good level is increasing in the level of spillovers for $\kappa < \hat{\kappa}$.[27] This appears puzzling as district 1's median voter's preferred public good level is actually constant in spillovers. The result reflects the conflict over spending levels. To prevent district 2 from pulling down spending in both regions, district 1's median voter elects a representative with a higher taste for public spending, raising region 1's public good level. Furthermore, district 1's public good level is decreasing for κ sufficiently close to 1/2. However, it can increase or decrease for κ sufficiently close to but higher than $\hat{\kappa}$. District 2's public good level is decreasing in the level of spillovers for $\kappa < \hat{\kappa}$ and increasing thereafter. It increases for spillovers in excess of $\hat{\kappa}$ because it is now effectively controlled by region 1's median voter.

Two comparisons require analysis here. Firstly, comparing these policy outcomes with the surplus maximizing levels, district 1's public good level is too high for all levels of spillovers except when $\kappa = 0$. In this case, region 1's public good is provided at the surplus maximizing level. The level provided to district 2 is too low for all $\kappa < \hat{\kappa}$ and for κ higher than but sufficiently close to $\hat{\kappa}$. The only exception here is again when $\kappa = 0$. In this case district 2's public good level is the surplus maximizing one. Moreover, district 2's public good level is too high for κ sufficiently close to 1/2. Secondly, comparing public goods levels in the two tax systems, each district's public good is provided at higher level when the taxation is uniform than under the non-uniform tax system, except when spillovers are maximal. In such a case, both systems generate the same public goods levels. Thus, non-uniform taxation suppresses, though not completely eliminates, the incentives to delegate policy-making strategically to representatives with higher preferences for public spending.

[27] This and the other claims concerning the public goods levels from Lemma 5 are established in the Appendix.

5.3 Centralization vs. Decentralization

We already know that decentralization produces the surplus maximizing public goods levels only in the case of zero spillovers. Public goods levels under *centralization with uniform taxation* are surplus maximizing only when spillovers are complete and the districts are identical. It follows that, in the case of identical regions, decentralization dominates when spillovers are small and centralization is preferred when spillovers are large. Surplus under centralization with uniform taxation increases with increasing κ and a critical value of spillovers exists above which centralization is welfare superior.

When the districts are identical, *centralization with non-uniform taxation* produces the surplus maximizing public goods levels for all spillover levels. This surplus is higher than that under decentralization for all κ except when the spillovers are absent. In such a case, both systems generate the surplus maximizing public goods levels. The next proposition and Figure 4 summarize these results.

Proposition 4. *Suppose that the assumptions of the political economy analysis are satisfied, the legislature is cooperative, and the districts are identical. Then*

(i) If the taxation is uniform, there is a critical value of κ, *strictly greater than* 0 *but less than* 1/2, *such that a centralized system produces a higher level of surplus if and only if* κ *exceeds this critical level.*

(ii) If the taxation is non-uniform across districts and spillovers are present ($\kappa > 0$), *a centralized system produces a higher level of surplus than does decentralization. Absent spillovers* ($\kappa = 0$), *the two systems generate the same level of surplus.*

(iii) Surplus under centralization with non-uniform taxation is higher than that under centralization with uniform taxation except when the spillovers are maximal ($\kappa = 1/2$). *In such a case, both systems produce the same public goods surplus.*

There are two important findings which require analysis. First, when the taxation is uniform, decentralization dominates when spillovers are low and centralization is preferred when spillovers are high, whereas a critical level of spillovers exists and is in the range $(0, 1/2)$. This is in line with the results obtained in the preceding sections when the legislature

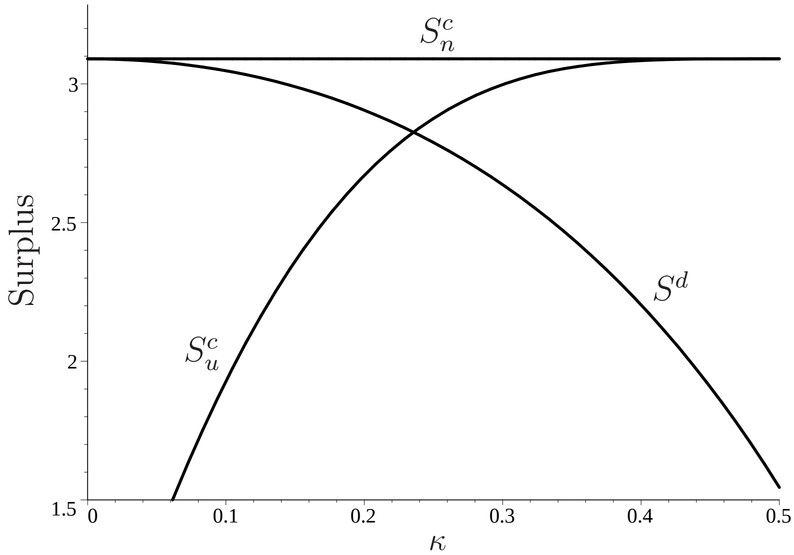


Figure 4: Aggregate public goods surpluses under decentralization (S^d), centralization with uniform (S_u^c) and non-uniform (S_n^c) taxation in the case of *identical districts*.

was based on minimum-winning coalition. Second, this does not hold, however, when the taxation under centralization is non-uniform. Such a system produces surplus maximizing public goods levels regardless of the level of spillovers and dominates decentralization for all $\kappa > 0$. Thus, non-uniform taxation is a significant tool for eliminating strategic delegation in the case of identical regions.

When the districts are heterogeneous, decentralization continues to dominate *centralization with uniform taxation* when spillovers are small and centralization is preferred when spillovers are large.

The case of *centralization with non-uniform taxation* is a bit more complicated when it comes to heterogeneous districts. Centralization still dominates decentralization when spillovers are large, but it dominates decentralization even when spillovers are small. Furthermore, it may be that centralization with non-uniform taxation produces a higher public goods surplus than does decentralization for all $\kappa > 0$. However, there is no general presumption that this is always so. Decentralization may dominate centralization when κ is sufficiently close to $\hat{\kappa}$. These findings are summarized in the following proposition and Figure 5.

Proposition 5. *Suppose that the assumptions of the political economy analysis are satisfied, the legislature is cooperative, and the districts are non-identical. Then* *(i) If the taxation is uniform, a decentralized system produces a higher level of surplus when spillovers are sufficiently small, while a centralized system produces a higher level when spillovers are sufficiently large.*

(ii) If the taxation is non-uniform, a centralized system produces a higher level of surplus than does decentralization when spillovers are sufficiently large and when spillovers are sufficiently small but positive. Absent spillovers, the two systems generate the same public goods surplus.

(iii) Surplus under centralization with non-uniform taxation is higher than that under centralization with uniform tax system except when the spillovers are maximal ($\kappa = 1/2$)*. In this case, both systems produce the same public goods surplus.*

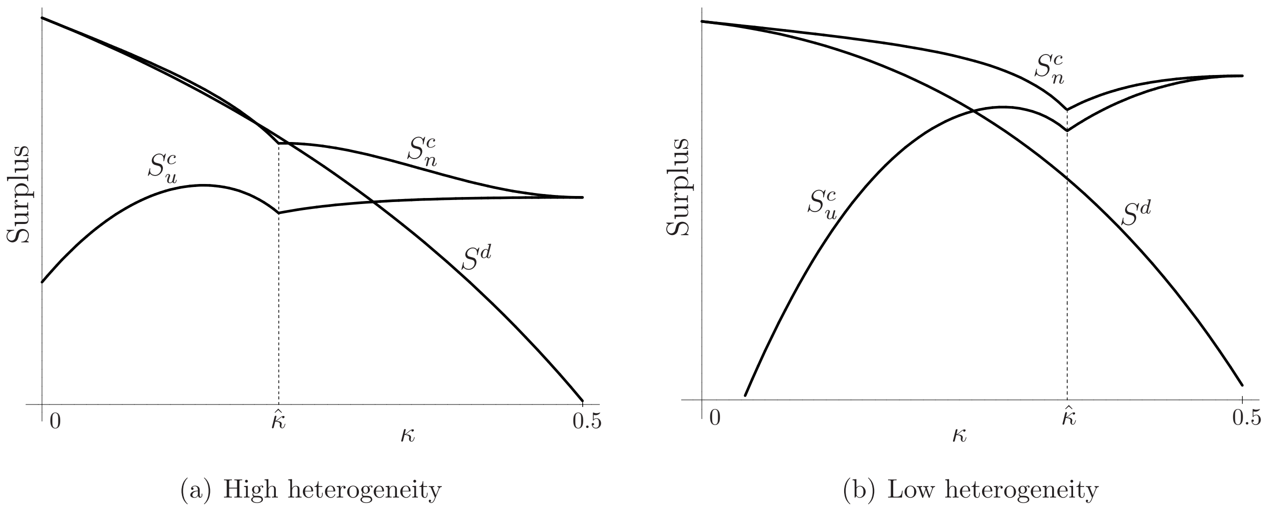


Figure 5: Aggregate public goods surpluses under decentralization (S^d), centralization with uniform (S_u^c) and non-uniform (S_n^c) taxation in the case of *non-identical districts*.

Three important lessons can be drawn from these statements. To begin with, the basic conclusions of part (i) of the Proposition 3 generalize to the case of a cooperative legislature. When the centralized system is financed by uniform taxation, decentralization dominates centralization for low spillover levels, while centralization dominates for high levels of spillovers. The only difference here is that we cannot show that there exists a critical level of spillovers. This reflects the fact that there is no general presumption that the relative performance of centralization is always increasing in spillovers. Surplus under centralization is decreasing in κ for κ sufficiently close to but lower than $\hat{\kappa}$.

Secondly, the just mentioned conclusions do not carry over to centralization with non-uniform tax system. Under this taxation, the centralized system produces higher surplus than does decentralization even for low levels of spillovers. This is due to the nature of a non-uniform tax system which means that the financing of public goods is not shared any more but is proportionally distributed between regions. However, we cannot show that centralization always dominates decentralization for all spillover levels. This reflects the fact that when the districts are sufficiently heterogenous, decentralized system produces higher surplus for κ sufficiently close to $\hat{\kappa}$ which is demonstrated in Figure 5a. On the other hand, when the regions do not differ very much in their public goods preferences, centralization with non-uniform taxation dominates decentralization for all $\kappa > 0$ (Figure 5b). Thus, although strategic delegation does arise under this system as well, it does so in a much lesser extent compared to the uniform taxation case. However, the policy outcomes produced under this system can still be improved in the direction towards the surplus maximizing ideal.

Finally, comparing the two tax systems, we must again conclude that centralization with non-uniform taxation dominates centralization with uniform tax system for all $\kappa < 1/2$. When the spillovers are maximal, the two systems produce the same public goods surplus. This is in line with the results obtained for centralization based on minimum winning coalition. We have thus generalized the conclusion that non-uniform tax system produces strictly better policy outcomes than does uniform taxation. In the case of cooperative legislature, centralization based on this taxation may even dominate decentralization for all $\kappa > 0$ which is a stunning result.

6 Conclusion

This paper has taken a fresh look at the relative merits of centralized and decentralized provision of local public goods. Allowing for a centralized system to provide non-uniform public goods and to use district-specific taxes has proved to be a significant tool for enhancing the performance of this kind of centralization relative to the one in which taxation is uniform across regions. Eliminating the feature of shared costs has a very positive effect on the performance of a centralized system in all of the studied cases. Specifically, centralized system with non-uniform taxation appears to dominate centralization with uniform tax system for all levels of spillovers except when the spillovers are maximal. This result holds regardless of heterogeneity in tastes and the basis on which the legislative behavior functions.

When decisions are made by a legislature of locally elected representatives, non-uniform tax system suppresses or completely eliminates the drawbacks created by a centralized system with uniform financing. If decisions on local public goods are made by a minimum winning coalition of representatives, non-uniform taxation significantly reduces (when the regions are non-identical) or completely eliminates (in the case of identical districts) the misallocation problem. Nevertheless, the uncertainty remains due to the unknown identity of the coalition in either case. If decisions are made on a more cooperative basis, then the strategic delegation is significantly suppressed (when the districts are non-identical) or completely eliminated (in the case of identical regions).

We have assumed throughout that the provision of local public goods is all the representatives are in power of. In reality, though, the representatives decide on numerous issues and shifting responsibility for one area of political decision-making from decentralized to centralized government may have other consequences as well which might entirely change the results. Other weakness is that, in reality, it is difficult to tax the districts according to their consumption of public goods. However, if this were possible, then such a tax system would lead to much more optimistic outcomes than a centralized system with uniform taxation.

Appendix - Proofs

Proof of Proposition 1. First note that the aggregate public goods surplus under decentralization is

$$
\begin{aligned}
S^d(\kappa) = {} & [m_1(1-\kappa) + m_2\kappa] \ln \frac{m_1(1-\kappa)}{p} \\
& + [m_2(1-\kappa) + m_1\kappa] \ln \frac{m_2(1-\kappa)}{p} - m_1(1-\kappa) - m_2(1-\kappa),
\end{aligned}
$$

while surplus under centralization in the traditional analysis is

$$S_t^c(\kappa) = [m_1 + m_2] \ln \frac{m_1 + m_2}{2p} - m_1 - m_2.$$

For part (i) (using a convention $m_1 = m_2 = m$), we establish three claims from which the proof will clearly follow.

Claim 1. $S_t^c(0) = S^d(0)$ and $S_t^c(1/2) > S^d(1/2)$.

Both statements are easily proven by inserting all the necessary variables into the functions of surpluses.

Claim 2. Surplus under centralization is independent of κ.

This claim is clearly true.

Claim 3. Surplus under decentralization is decreasing in κ.

Differentiating, we come to

$$\frac{\partial S^d}{\partial \kappa}(\kappa) = -\frac{2m\kappa}{1-\kappa} < 0.$$

Analogically, we prove part (ii) by proving three following claims.

Claim 1. $S_t^c(0) < S^d(0)$ and $S_t^c(1/2) > S^d(1/2)$.

Let (m_1, m_2) be given and suppose that $m_1 > m_2$. We can find $\omega > 0$ and $\alpha \in \langle 1/2, 1\rangle$ so that $(m_1, m_2) = (\alpha\omega, (1-\alpha)\omega)$. For the first inequality, this implies that

$$S_t^c\left(0, \alpha\right) = \omega \ln \frac{\omega}{2p} - \omega,$$

and

$$S^d(0, \alpha) = \alpha\omega \ln \frac{\alpha\omega}{p} + (1-\alpha)\omega \ln \frac{(1-\alpha)\omega}{p} - \omega.$$

Taking difference, we obtain

$$\begin{aligned} S^d(0,\alpha) - S_t^c(0,\alpha) &= \alpha\omega \ln \frac{\alpha\omega}{p} + (1-\alpha)\omega \ln \frac{(1-\alpha)\omega}{p} - \omega - \omega \ln \frac{\omega}{2p} + \omega \\ &= \alpha\omega \ln \alpha\omega + \omega \ln(1-\alpha)\omega - \alpha\omega \ln(1-\alpha)\omega - \omega \ln \omega + \omega \ln 2 \\ &= \alpha\omega \ln \alpha + \omega \ln(1-\alpha) - \alpha\omega \ln(1-\alpha) + \omega \ln 2 \\ &= \alpha\omega \ln \frac{\alpha}{1-\alpha} + \omega \ln(1-\alpha) + \omega \ln 2. \end{aligned}$$

Differentiating the difference with respect to α yields

$$\frac{\partial[S^d(0,\alpha) - S_t^c(0,\alpha)]}{\partial \alpha} = \omega \ln \frac{\alpha}{1-\alpha} > 0, \quad \text{for all } \alpha \in (1/2, 1\rangle.$$

Thus, the difference is increasing in α and since $S^d(0, 1/2) = S_t^c(0, 1/2)$, the inequality holds for all α in the relevant range.

For the latter inequality, surplus under centralization is independent of heterogeneity and therefore $S_t^c(0, \alpha) = S_t^c(1/2, \alpha)$. For the surplus under decentralization we have:

$$S^d\left(\frac{1}{2}, \alpha\right) = \frac{\omega}{2}\left[\ln \frac{\alpha\omega}{2p} + \ln \frac{(1-\alpha)\omega}{2p}\right] - \frac{\omega}{2}.$$

Calculating the difference, we have that

$$\begin{aligned} S_t^c\left(\frac{1}{2},\alpha\right) - S^d\left(\frac{1}{2},\alpha\right) &= \omega \ln \frac{\omega}{2p} - \omega - \frac{\omega}{2}\left[\ln\frac{\alpha\omega}{2p} + \ln\frac{(1-\alpha)\omega}{2p}\right] + \frac{\omega}{2} \\ &= \omega\ln\omega - \frac{1}{2}\omega\ln\alpha\omega - \frac{1}{2}\omega\ln(1-\alpha)\omega - \frac{1}{2}\omega \\ &= -\frac{1}{2}\omega\ln\alpha(1-\alpha) - \frac{1}{2}\omega. \end{aligned}$$

Differentiating the difference with respect to α, we obtain

$$\frac{\partial\left[S_t^c\left(\frac{1}{2},\alpha\right) - S^d\left(\frac{1}{2},\alpha\right)\right]}{\partial\alpha} = \frac{\omega(2\alpha-1)}{2\alpha(1-\alpha)} \geq 0.$$

Thus, this difference is non-decreasing in α. So if $S_t^c(1/2,1/2) > S^d(1/2,1/2)$, then the inequality holds for all α in the relevant range. But $\alpha = 1/2$ corresponds to the identical districts case and we already know that surplus under centralization is higher than under decentralization then.

Claim 2. Surplus under centralization is independent of κ.

This claim is clearly true.

Claim 3. Surplus under decentralization is decreasing in κ.

Differentiating, we obtain

$$\frac{\partial S^d}{\partial\kappa}(\kappa) = (m_2 - m_1)\ln\frac{m_1}{m_2} - (m_1+m_2)\frac{\kappa}{1-\kappa} < 0. \quad \blacksquare$$

Proof of Proposition 2. Aggregate public goods surplus under decentralization is as in the traditional analysis, while, in the case of identical districts ($m_1 = m_2 = m$), surplus under centralization

1. with uniform taxation is: $S_u^c(\kappa) = m\left(\ln\frac{2m(1-\kappa)}{p} + \ln\frac{2m\kappa}{p}\right) - 2m$;
2. with non-uniform taxation is: $S_n^c(\kappa) = 2m\ln\frac{m}{p} - 2m$.

We will prove the proposition via four following claims.

Claim 1. $S_u^c(0) < S^d(0)$ and $S_u^c(1/2) > S^d(1/2)$.

Note that $\lim_{\kappa\to 0^+} S_u^c(\kappa) = -\infty$ and $S^d(0) = 2m\ln\frac{m}{p} - 2m \in \mathbb{R}$, which implies the former statement.

For the latter statement, first observe that $S_u^c(1/2) = 2m\ln\dfrac{m}{p} - 2m$ and $S^d(1/2) = 2m\ln\dfrac{m}{2p} - m$. Rearranging the inequality, we come to the following one: $\ln 4 > 1$, which is clearly true.

Claim 2. $S_u^c(\kappa)$ is increasing in κ.

Differentiating, we obtain

$$\frac{\partial S_u^c}{\partial \kappa}(\kappa) = m\frac{1-2\kappa}{\kappa(1-\kappa)} > 0, \quad \text{for all } \kappa \in (0, 1/2).$$

Claim 3. $S_n^c(\kappa) = S_t^c(\kappa)$ for all $\kappa \in \langle 0, 1/2 \rangle$.

This statement is clear.

Claim 4. $S_u^c(1/2) = S_n^c(1/2)$.

This claim is clear as well.

It is straightforward to show that Claims 1 and 2 prove part (i). Due to the fact proved in Claim 3, part (ii) can be proven similarly as has been performed in Proposition 1. Finally, Claims 2, 3 and 4 imply part (iii) of the Proposition. ■

Proof of Proposition 3. When the districts are non-identical ($m_1 > m_2$), surplus under centralization

1. with uniform taxation is:

$$S_u^c(\kappa) = \frac{1}{2}[m_1(1-\kappa) + m_2\kappa]\left(\ln\frac{2m_1(1-\kappa)}{p} + \ln\frac{2m_2\kappa}{p}\right) + \frac{1}{2}[m_2(1-\kappa) + m_1\kappa]\left(\ln\frac{2m_2(1-\kappa)}{p} + \ln\frac{2m_1\kappa}{p}\right) - m_1 - m_2;$$

2. with non-uniform taxation is: $S_n^c(\kappa) = \dfrac{1}{2}(m_1 + m_2)\left(\ln\dfrac{m_1}{p} + \ln\dfrac{m_2}{p}\right) - m_1 - m_2$.

Again, we prove the proposition by establishing 7 claims from which the proof of the Proposition 3 will follow.

Claim1. $S_u^c(0) < S^d(0)$ and $S_u^c(1/2) > S^d(1/2)$.

$S_u^c(\kappa)$ tends to negative infinity as κ approaches zero from the right and $S^d(0)$ is a real number, which implies the former statement. Rearranging the latter one, we again come to the inequality $\ln 4 > 1$, which clearly holds.

Claim 2. $S_u^c(\kappa)$ is increasing in κ.

Differentiating, we come to

$$\frac{\partial S_u^c}{\partial \kappa}(\kappa) = (m_1 + m_2)\frac{1-2\kappa}{2\kappa(1-\kappa)} > 0, \quad \text{for all } \kappa \in (0, 1/2).$$

Claim 3. $S_u^c(\kappa) < S_t^c(\kappa)$.

We first prove that $S_u^c(1/2) < S_t^c(1/2)$. Let us compute:

$$\begin{aligned}
\frac{1}{2}(m_1+m_2)\left(\ln\frac{m_1}{p}+\ln\frac{m_2}{p}\right)-m_1-m_2 &< (m_1+m_2)\ln\frac{m_1+m_2}{2p}-m_1-m_2 \\
0 &< \underbrace{(m_1+m_2)}_{>0}\ln\frac{m_1+m_2}{2(m_1m_2)^{1/2}} \\
1 &< \frac{m_1+m_2}{2(m_1m_2)^{1/2}} \\
4m_1m_2 &< {m_1}^2+2m_1m_2+{m_2}^2 \\
0 &< (m_1-m_2)^2.
\end{aligned}$$

The last inequality holds due to the fact that $m_1 \neq m_2$ and since all the ?adjustments/modifications? were equivalent, the inequality $S_u^c(1/2) < S_t^c(1/2)$ is proven. Since Claim 2 proves that $S_u^c(\kappa)$ is increasing in κ and $S_t^c(\kappa)$ is constant in κ, it follows that $S_u^c(\kappa) < S_t^c(\kappa)$ for all $\kappa \in \langle 0, 1/2\rangle$.

Claim 4. $S_n^c(0) < S^d(0)$ and $S_n^c(1/2) > S^d(1/2)$.

For the former inequality, we may write $S_n^c(0) = \xi(1/2)$ and $S^d(0) = \xi(0)$, where

$$\xi(\phi) = [(1-\phi)m_1+\phi m_2]\ln\frac{m_1}{p}+[(1-\phi)m_2+\phi m_1]\ln\frac{m_2}{p}-m_1-m_2.$$

Observe that

$$\xi'(\phi) = \underbrace{(m_2-m_1)}_{<0}\underbrace{\ln\frac{m_1}{m_2}}_{>0}.$$

Since the function $\xi(\phi)$ is decreasing for all $\phi \in \langle 0, 1/2\rangle$, it holds that

$$S_n^c(0) = \xi(1/2) < \xi(0) = S^d(0).$$

For the latter statement, we may write $S_n^c(1/2) = \xi(1)$ and $S^d(1/2) = \xi(1/2)$, where

$$\xi(\phi) = \frac{1}{2}(m_1+m_2)\left(\ln\frac{\phi m_1}{p}+\ln\frac{\phi m_2}{p}\right)-\phi(m_1+m_2).$$

Differentiating, we obtain

$$\xi'(\phi) = (m_1+m_2)\left(\frac{1}{\phi}-1\right).$$

Since the function $\xi(\phi)$ is increasing for all $\phi \in \langle 1/2, 1\rangle$, it follows that

$$S_n^c(1/2) = \xi(1) > \xi(1/2) = S^d(1/2).$$

Claim 5. $S_n^c(\kappa)$ is independent of κ.

This claim is clearly true.

Claim 6. $S_n^c(\kappa) < S_t^c(\kappa)$.

This inequality follows from concavity of function $\ln(\cdot)$:

$$S_n^c(\kappa) = (m_1 + m_2)\left(\frac{1}{2}\ln\frac{m_1}{p} + \frac{1}{2}\ln\frac{m_2}{p}\right) < (m_1 + m_2)\ln\frac{m_1 + m_2}{2p} = S_t^c(\kappa).$$

Claim 7. $S_u^c(\kappa) < S_n^c(\kappa)$ for all $\kappa < 1/2$ and $S_u^c(1/2) = S_n^c(1/2)$.

The equality clearly holds. The facts that $S_u^c(\cdot)$ is increasing in κ and $S_n^c(\cdot)$ is constant in κ imply the inequality for all $\kappa < 1/2$.

For part (i) of the Proposition, the first two Claims imply the existence of a critical value of κ, strictly greater than 0 but less than 1/2, such that a centralized system with uniform taxation produces a higher level of surplus than does decentralization if and only if κ exceeds this critical level. The fact that surplus under decentralization is the same as in the standard analysis combined with Claim 3 imply that the critical level of κ is higher for centralization with uniform taxation than that implied by the traditional analysis.

As for the part (ii), Claims 4 and 5 imply the existence of a critical value of κ, strictly greater than 0 but less than 1/2, such that a centralized system with non-uniform taxation produces a higher level of surplus than does decentralization if and only if κ exceeds this critical level. Claim 6 and the fact that surplus under decentralization is the same as in the traditional analysis imply that the critical level of spillovers is higher in the centralized system with non-uniform taxation than under centralization in the standard analysis. Claim 7 further implies that the critical level of κ under a centralized system with non-uniform taxation is lower than that under centralization with uniform taxation.

Finally, part (iii) follows from Claims 3, 6 and 7. ■

Proof of Lemma 4. Due to close similarity between the proof of this Lemma and Lemma 5 and the fact that in this paper we focus more on non-uniform tax system, we refer to Besley and Coate (2003) for a thorough proof of this Lemma.

Proof of Lemma 5. As mentioned in the text, $(\lambda_1^*, \lambda_2^*)$ is majority preferred if and only if $(\lambda_1^*, \lambda_2^*)$ is a Nash equilibrium of the two player game in which each player has strategy set $\langle 0, \overline{\lambda} \rangle$ and player $i \in \{1, 2\}$ has payoff function $U_i(\lambda_1, \lambda_2)$. We prove the Lemma by calculating the set of equilibria of this game and computing the associated policy outcomes.

Note first that each player's payoff function is a twice continuously differentiable and strictly concave function of his strategy and each player's strategy set is compact and convex. Thus, the set of equilibria is non-empty. Moreover, $\partial^2 U_1 / \partial \lambda_1 \partial \lambda_2 < 0$ and $\partial^2 U_2 / \partial \lambda_2 \partial \lambda_1 < 0$, implying that types are strategic substitutes.

For $i = 1, 2$, let $r_i : \langle 0, \overline{\lambda} \rangle \to \langle 0, \overline{\lambda} \rangle$ denote the region i median voter's *reaction function.* By definition, for all $\lambda_2 \in \langle 0, \overline{\lambda} \rangle$,

$$r_1(\lambda_2) = \arg\max\{U_1(r_1, \lambda_2) : r_1 \in \langle 0, \overline{\lambda} \rangle\},$$

and for all $\lambda_1 \in \langle 0, \overline{\lambda} \rangle$,

$$r_2(\lambda_1) = \arg\max\{U_2(\lambda_1, r_2) : r_2 \in \langle 0, \overline{\lambda} \rangle\}.$$

Then, $(\lambda_1^*, \lambda_2^*)$ is an equilibrium of the game if and only if $(\lambda_1^*, \lambda_2^*) = (r_1(\lambda_2^*), r_2(\lambda_1^*))$.

Some general features of the reaction functions follow from the properties of the payoff functions. The fact that each player's payoff is a strictly concave and differentiable function of his strategy implies (i) that $r_1(\lambda_2) = 0$ if $\partial U_1(0, \lambda_2)/\partial \lambda_1 < 0$; (ii) that $r_1(\lambda_2) = \overline{\lambda}$ if $\partial U_1(\overline{\lambda}, \lambda_2) > 0$; and (iii) that otherwise $r_1(\lambda_2)$ is implicitly defined by the first-order condition $\partial U_1(r_1(\lambda_2), \lambda_2)/\partial \lambda_1 = 0$. In addition, the fact that types are strategic substitutes implies that $r_1(\lambda_2)$ is non-increasing. Analogical remarks apply to the district 2 median voter's reaction function.

It remains therefore to determine the details of each player's reaction function. Let $\overline{\lambda}_2(\overline{\lambda}_1)$ denote the level of $\lambda_2(\lambda_1)$ beyond which district 1's median voter (district 2's median voter) would like a type 0 representative. These levels are implicitly defined by the equalities

$$\partial U_1(0, \overline{\lambda}_2)/\partial \lambda_1 = 0,$$

and

$$\partial U_2(\overline{\lambda}_1, 0)/\partial \lambda_2 = 0.$$

Using the facts that

$$\frac{\partial U_1}{\partial \lambda_1} = m_1 \left[\frac{(1-\kappa)^2}{\lambda_1(1-\kappa) + \lambda_2 \kappa} + \frac{\kappa^2}{\lambda_2(1-\kappa) + \lambda_1 \kappa} \right] - (1 - 2\kappa + 2\kappa^2),$$

and

$$\frac{\partial U_2}{\partial \lambda_2} = m_2 \left[\frac{(1-\kappa)^2}{\lambda_2(1-\kappa)+\lambda_1\kappa} + \frac{\kappa^2}{\lambda_1(1-\kappa)+\lambda_2\kappa} \right] - (1-2\kappa+2\kappa^2),$$

we obtain

$$\overline{\lambda}_2 = m_1 \left\{ \frac{(1-\kappa)^3+\kappa^3}{\kappa(1-\kappa)(1-2\kappa+2\kappa^2)} \right\},$$

and

$$\overline{\lambda}_1 = m_2 \left\{ \frac{(1-\kappa)^3+\kappa^3}{\kappa(1-\kappa)(1-2\kappa+2\kappa^2)} \right\}.$$

Observe that $\frac{(1-\kappa)^3+\kappa^3}{\kappa(1-\kappa)(1-2\kappa+2\kappa^2)}$ is decreasing in κ, takes on the value 2 when $\kappa = 1/2$ and tends to infinity as κ goes to zero. This implies that $\overline{\lambda}_1 \geq 2m_2$ and $\overline{\lambda}_2 \geq 2m_1$.

Next, let $\lambda_1^{\times}$ ($\lambda_2^{\times}$) denote the highest type representative region 1's (region 2's) median voter would want. These levels are implicitly defined by the equalities

$$\partial U_1(\lambda_1^{\times}, 0)/\partial \lambda_1 = 0$$

and

$$\partial U_2(0, \lambda_2^{\times})/\partial \lambda_2 = 0,$$

which imply

$$\lambda_1^{\times} = \frac{m_1}{1-2\kappa+2\kappa^2}$$

and

$$\lambda_2^{\times} = \frac{m_2}{1-2\kappa+2\kappa^2}.$$

Note that $1/(1-2\kappa+2\kappa^2)$ is increasing in κ, takes on the value 1 when $\kappa = 0$ and value 2 when $\kappa = 1/2$. This implies that $\lambda_1^{\times} \leq 2m_1$ and $\lambda_2^{\times} \leq 2m_2$. By assumption, $2m_i < \overline{\lambda}$, so that the upper bound constraint on type choice is not binding here.

We may conclude from the above that for all $\lambda_2 \in \langle 0, \min\{\overline{\lambda}_2, \overline{\lambda}\}\rangle$, $r_1(\lambda_2)$ is implicitly defined by the first-order condition

$$\frac{\partial U_1(r_1(\lambda_2), \lambda_2)}{\partial \lambda_1} = 0$$

and for all $\lambda_2 \in (\min\{\overline{\lambda}_2, \overline{\lambda}\}, \overline{\lambda}\rangle$,

$$r_1(\lambda_2) = 0.$$

Further, we know that $r_1(0) = \lambda_1^{\times}$ and that $r_1(\lambda_2)$ is downward sloping on $\langle 0, \min\{\overline{\lambda}_2, \overline{\lambda}\}\rangle$.

Analogically, for all $\lambda_1 \in \langle 0, \min\{\overline{\lambda}_1, \overline{\lambda}\}\rangle$, $r_2(\lambda_1)$ is implicitly defined by the first-order condition

$$\frac{\partial U_2(\lambda_1, r_2(\lambda_1))}{\partial \lambda_2} = 0$$

and for all $\lambda_1 \in (\min\{\overline{\lambda}_1, \overline{\lambda}\}, \overline{\lambda}\rangle$,

$$r_2(\lambda_1) = 0.$$

Further, we know that $r_2(0) = \lambda_2^{\times}$ and that $r_2(\lambda_1)$ is downward sloping on $\langle 0, \min\{\overline{\lambda}_1, \overline{\lambda}\}\rangle$.

We can now prove the lemma. If $\kappa < \hat{\kappa}$, it follows from the definition of $\hat{\kappa}$ in the text that $(\kappa^3 + (1-\kappa)^3)/\kappa(1-\kappa) > m_1/m_2$. This in turn implies that

$$\overline{\lambda}_1 = m_2 \left\{ \frac{(1-\kappa)^3 + \kappa^3}{\kappa(1-\kappa)(1-2\kappa+2\kappa^2)} \right\} > \frac{m_1}{(1-2\kappa+2\kappa^2)} = \lambda_1^{\times}.$$

Observe further that

$$\overline{\lambda}_2 = m_1 \left\{ \frac{(1-\kappa)^3 + \kappa^3}{\kappa(1-\kappa)(1-2\kappa+2\kappa^2)} \right\} > 2m_2 \geq \lambda_2^{\times}.$$

These inequalities imply that there exists no boundary equilibria in which $\lambda_i^* = 0$ for one or more districts. If $\lambda_2^* = 0$, then $\lambda_1^* = r_1(0) = \lambda_1^{\times}$, but since $\lambda_1^{\times} < \overline{\lambda}_1$ we know that $r_2(\lambda_1^{\times}) > 0$ which contradicts the fact that $\lambda_2^* = 0$. If $\lambda_1^* = 0$, then $\lambda_2^* = r_2(0) = \lambda_2^{\times}$, but since $\lambda_2^{\times} < \overline{\lambda}_2$ we know that $r_1(\lambda_2^{\times}) > 0$ which contradicts the fact that $\lambda_1^* = 0$. Since $\max r_i(\lambda_{-i}) < \overline{\lambda}$, it is apparent that there can be no boundary equilibria in which $\lambda_i^* = \overline{\lambda}$ for one or more districts.

It follows that there must exist an interior equilibrium. Any such equilibrium $(\lambda_1^*, \lambda_2^*)$ must satisfy the first-order conditions $\partial U_i(\lambda_1^*, \lambda_2^*)/\partial \lambda_i = 0$ for $i \in \{1, 2\}$. Using the expressions for $\partial U_i/\partial \lambda_i$, $i \in \{1, 2\}$ from above, we may write these conditions as

$$m_1 \left[\frac{(1-\kappa)^2}{\lambda_1^*(1-\kappa) + \lambda_2^*\kappa} + \frac{\kappa^2}{\lambda_2^*(1-\kappa) + \lambda_1^*\kappa} \right] = (1-2\kappa+2\kappa^2),$$

and

$$m_2 \left[\frac{(1-\kappa)^2}{\lambda_2^*(1-\kappa) + \lambda_1^*\kappa} + \frac{\kappa^2}{\lambda_1^*(1-\kappa) + \lambda_2^*\kappa} \right] = (1-2\kappa+2\kappa^2).$$

Combining the two first-order conditions, we obtain

$$\frac{\lambda_1^*(1-\kappa) + \lambda_2^*\kappa}{\lambda_2^*(1-\kappa) + \lambda_1^*\kappa} = \frac{m_1(1-\kappa)^2 - m_2\kappa^2}{m_2(1-\kappa)^2 - m_1\kappa^2}.$$

Using this and the two first-order conditions for λ_1^* and λ_2^* yields

$$\lambda_1^*(1-\kappa)+\lambda_2^*\kappa = \frac{m_1 m_2[(1-\kappa)^4-\kappa^4]}{[m_2(1-\kappa)^2-m_1\kappa^2](1-2\kappa+2\kappa^2)} = \frac{m_1[(1-\kappa)^2-\kappa^2][(1-\kappa)^2+\kappa^2]}{\left[(1-\kappa)^2-\dfrac{m_1}{m_2}\kappa^2\right][(1-\kappa)^2+\kappa^2]}$$
$$= \frac{m_1[(1-\kappa)^2-k^2]}{\left[(1-\kappa)^2-\dfrac{m_1}{m_2}\kappa^2\right]}$$

and

$$\lambda_2^*(1-\kappa)+\lambda_1^*\kappa = \frac{m_1 m_2[(1-\kappa)^4-\kappa^4]}{[m_1(1-\kappa)^2-m_2\kappa^2](1-2\kappa+2\kappa^2)} = \frac{m_1[(1-\kappa)^2-\kappa^2][(1-\kappa)^2+\kappa^2]}{\left[\dfrac{m_1}{m_2}(1-\kappa)^2-\kappa^2\right][(1-\kappa)^2+\kappa^2]}$$
$$= \frac{m_1[(1-\kappa)^2-k^2]}{\left[\dfrac{m_1}{m_2}(1-\kappa)^2-\kappa^2\right]}.$$

Thus, as claimed, the policy outcome is

$$(g_1,g_2)=\left(\frac{m_1[(1-\kappa)^2-k^2]}{\left[(1-\kappa)^2-\dfrac{m_1}{m_2}\kappa^2\right]p},\frac{m_1[(1-\kappa)^2-k^2]}{\left[\dfrac{m_1}{m_2}(1-\kappa)^2-\kappa^2\right]p}\right).$$

If $\kappa\geq\hat{\kappa}$, it follows that $(\kappa^3+(1-\kappa)^3)/\kappa(1-\kappa)\leq m_1/m_2$, which in turn implies that

$$\overline{\lambda}_1 = m_2\left\{\frac{(1-\kappa)^3+\kappa^3}{\kappa(1-\kappa)(1-2\kappa+2\kappa^2)}\right\}\leq\frac{m_1}{(1-2\kappa+2\kappa^2)}=\lambda_1^{\times}.$$

This inequality implies that there exists a boundary equilibrium in which $(\lambda_1^*,\lambda_2^*)=(\lambda_1^{\times},0)$. This is because $r_2(\lambda_1^{\times})=0$ and $r_1(0)=\lambda_1^{\times}$. The same arguments from above imply that there exist no other boundary equilibria. We also claim that there are no interior equilibria. Any such equilibrium $(\lambda_1^*,\lambda_2^*)$ must satisfy the first-order conditions $\partial U_i(\lambda_1^*,\lambda_2^*)/\partial\lambda_i=0$ for $i\in\{1,2\}$. These first-order conditions imply that

$$\begin{aligned} & m_1[\lambda_2^*(1-\kappa)^3+\lambda_1^*\kappa(1-\kappa)^2+\lambda_1^*(1-\kappa)\kappa^2+\lambda_2^*\kappa^3] \\ = \;& m_2[\lambda_1^*(1-\kappa)^3+\lambda_2^*\kappa(1-\kappa)^2+\lambda_2^*(1-\kappa)\kappa^2+\lambda_1^*\kappa^3]. \end{aligned}$$

This means that

$$\lambda_2^*=\frac{[m_2((1-\kappa)^3+\kappa^3)-m_1\kappa(1-\kappa)]}{[m_1((1-\kappa)^3+\kappa^3)-m_2\kappa(1-\kappa)]}\lambda_1^*.$$

But the assumption that $\kappa \geq \hat{\kappa}$ implies that $\lambda_2^* \leq 0$ if $\lambda_1^* > 0$, which, in turn, is inconsistent with the hypothesis that $(\lambda_1^*, \lambda_2^*) > (0,0)$. Thus, the only equilibrium is that $(\lambda_1^*, \lambda_2^*) = (\lambda_1^\times, 0) = \left(\frac{m_1}{1-2\kappa+2\kappa^2}, 0\right) = \left(\frac{m_1}{(1-\kappa)^2+\kappa^2}, 0\right)$. The required policy outcomes follow from this equilibrium:

$$(g_1, g_2) = \left(\frac{m_1(1-\kappa)}{[(1-\kappa)^2+\kappa^2]p}, \frac{m_1\kappa}{[(1-\kappa)^2+\kappa^2]p}\right). \quad \blacksquare$$

Fact *Let $(g_1^{c-n}(\kappa), g_2^{c-n}(\kappa))$ and $(g_1^{c-u}(\kappa), g_2^{c-u}(\kappa))$ denote public goods levels described in Lemma 5 and 4, respectively, i.e. local public goods levels in district 1 and 2 under centralization with non-uniform and uniform taxation. Assume that $m_1 > m_2$. Then*

(i) $g_1^{c-n}(\kappa)$ is increasing for $\kappa < \hat{\kappa}$ and decreasing for κ sufficiently close to $1/2$, but can be increasing or decreasing for κ sufficiently close to but higher than $\hat{\kappa}$.

(ii) $g_2^{c-n}(\kappa)$ is decreasing for $\kappa < \hat{\kappa}$ and increasing thereafter.

(iii) $g_1^{c-n}(\kappa)$ is higher than the surplus maximizing level for $\kappa > 0$ and equals the surplus maximizing level only when $\kappa = 0$.

(iv) $g_2^{c-n}(\kappa)$ is lower than the surplus maximizing level for $\kappa \in (0, \hat{\kappa}\rangle$ and κ sufficiently close to but higher than $\hat{\kappa}$, but it exceeds the surplus maximizing level for κ sufficiently close to $1/2$. It equals the surplus maximizing level only when $\kappa = 0$.

(v) $(g_1^{c-u}(\kappa), g_2^{c-u}(\kappa)) > (g_1^{c-n}(\kappa), g_2^{c-n}(\kappa))$ for all $\kappa < 1/2$. When the spillovers are complete, $(g_1^{c-u}(\kappa), g_2^{c-u}(\kappa)) = (g_1^{c-n}(\kappa), g_2^{c-n}(\kappa))$.

Proof (i) For all $\kappa < \hat{\kappa}$ we have that

$$g_1^{c-n}(\kappa) = \frac{m_1[(1-\kappa)^2 - k^2]}{\left[(1-\kappa)^2 - \frac{m_1}{m_2}\kappa^2\right]p}.$$

Letting

$$\theta(\kappa) = ln\left\{m_1[(1-\kappa)^2 - k^2]\right\} - ln\left\{\left[(1-\kappa)^2 - \frac{m_1}{m_2}\kappa^2\right]p\right\},$$

we will show that $\theta'(\kappa) > 0$ for all $\kappa \in (0, \hat{\kappa})$. The derivative is

$$\theta'(\kappa) = 2\left\{\frac{-1}{1-2\kappa} + \frac{1-\kappa+\kappa\frac{m_1}{m_2}}{(1-\kappa)^2 - \frac{m_1}{m_2}\kappa^2}\right\},$$

and we need to show that for all $\kappa \in (0, \hat{\kappa})$

$$\frac{1-\kappa+\kappa\dfrac{m_1}{m_2}}{(1-\kappa)^2-\dfrac{m_1}{m_2}\kappa^2} > \frac{1}{1-2\kappa}.$$

Observe that the expression on the left hand side is increasing in $\dfrac{m_1}{m_2} \in (1, +\infty)$ for all $\kappa \in (0, \hat{\kappa})$, and that

$$\frac{1-\kappa+\kappa}{(1-\kappa)^2-\kappa^2} = \frac{1}{1-2\kappa},$$

which clearly holds. Thus, the desired inequality holds for all $\dfrac{m_1}{m_2} \in (1, +\infty)$ and $\kappa \in (0, \hat{\kappa})$.

For all $\kappa \geq \hat{\kappa}$, it follows from the Lemma that

$$g_1^{c-n}(\kappa) = \frac{m_1(1-\kappa)}{[(1-\kappa)^2+\kappa^2]p}.$$

Letting

$$\theta(\kappa) = ln\left\{m_1(1-\kappa)\right\} - ln\left\{[(1-\kappa)^2+\kappa^2]p\right\},$$

we will show that $\theta(\kappa)$ is increasing for small κ and decreasing for κ close to 1/2. The derivative of this expression is

$$\theta^{'}(\kappa) = \frac{2\kappa^2-4\kappa+1}{(1-2\kappa+2\kappa^2)(1-\kappa)}.$$

Calculating its value in 0 and 1/2, we come to $\theta^{'}(0) = 1$ and $\theta^{'}(1/2) = -2$. Putting the derivative equal to 0, we find out that $\theta(\kappa)$ is increasing for all $\kappa < 1-\sqrt{2}/2$ and decreasing thereafter. Thus, it depends on the value of $\hat{\kappa}$ whether $g_1^{c-n}(\kappa)$ is decreasing for all $\kappa \geq \hat{\kappa}$ or increasing and then decreasing. If $\hat{\kappa} \geq 1-\sqrt{2}/2$, $g_1^{c-n}(\kappa)$ is decreasing for all $\kappa \geq \hat{\kappa}$. If, on the other hand, $\hat{\kappa} < 1-\sqrt{2}/2$, $g_1^{c-n}(\kappa)$ is increasing for κ sufficiently close to but higher than $\hat{\kappa}$, and decreasing thereafter.

(ii) For all $\kappa < \hat{\kappa}$ it holds that

$$g_2^{c-n}(\kappa) = \frac{m_1[(1-\kappa)^2-k^2]}{\left[\dfrac{m_1}{m_2}(1-\kappa)^2-\kappa^2\right]p}.$$

Letting

$$\theta(\kappa) = ln\left\{m_1[(1-\kappa)^2-k^2]\right\} - ln\left\{\left[\frac{m_1}{m_2}(1-\kappa)^2-\kappa^2\right]p\right\},$$

we need to show that $\theta(\kappa)$ is decreasing in κ. Derivative of this expression is

$$2\left\{-\frac{1}{1-2\kappa}+\frac{\frac{m_1}{m_2}(1-\kappa)+k}{\frac{m_1}{m_2}(1-\kappa)^2-\kappa^2}\right\},$$

and we will show that for all $\kappa \in (0, \hat{\kappa})$

$$\frac{\frac{m_1}{m_2}(1-\kappa)+k}{\frac{m_1}{m_2}(1-\kappa)^2-\kappa^2}<\frac{1}{1-2\kappa}.$$

Observe that the expression on the left hand side is decreasing in $\frac{m_1}{m_2}$ for all $\kappa \in (0, \hat{\kappa})$, and that

$$\frac{1-\kappa+\kappa}{(1-\kappa)^2-\kappa^2}=\frac{1}{1-2\kappa}.$$

Thus, the inequality holds for all $\frac{m_1}{m_2} \in (1, +\infty)$ and $\kappa \in (0, \hat{\kappa})$.

For all $\kappa \geq \hat{\kappa}$, we have that

$$g_2^{c-n}(\kappa)=\frac{m_1\kappa}{[(1-\kappa)^2+\kappa^2]p}.$$

Letting

$$\theta(\kappa)=ln\{m_1\kappa\}-ln\{[(1-\kappa)^2+\kappa^2]p\},$$

differentiating this expression we obtain

$$\theta^{'}(\kappa)=\frac{1}{\kappa}-\frac{1}{1-2\kappa+2\kappa^2}=\frac{1-3\kappa+2\kappa^2}{\kappa(1-2\kappa+2\kappa^2)}.$$

Numerator of the last expression is positive for all $\kappa \in \langle 0, 1/2)$ and denominator is positive for all $\kappa \in (0, 1/2\rangle$. Thus, $g_2^{c-n}(\kappa)$ is increasing for all $\kappa \in (\hat{\kappa}, 1/2)$.

(iii) Suppose first that $\kappa \in (0, \hat{\kappa})$. Then, since the surplus maximizing public good level for district 1 is $[m_1(1-\kappa)+m_2\kappa]/p$, we must show that

$$g_1^{c-n}(\kappa)=\frac{m_1[(1-\kappa)^2-k^2]}{\left[(1-\kappa)^2-\frac{m_1}{m_2}\kappa^2\right]p}>\frac{m_1(1-\kappa)+m_2\kappa}{p}=g_1^*(\kappa).$$

We have already proven in Fact (i) that $g_1^{c-n}(\kappa)$ is increasing in $\kappa < \hat{\kappa}$. Next observe that the surplus maximizing public good level for region 1 is decreasing in κ. Differentiating the expression we obtain $m_2 - m_1 < 0$, which clearly holds. Since $g_1^{c-n}(0) = g_1^*(0)$,[28] the inequality is proven for all κ in the desired range.

Now suppose that $\kappa \geq \hat{\kappa}$. We must show that

$$g_1^{c-n}(\kappa) = \frac{m_1(1-\kappa)}{[(1-\kappa)^2 + \kappa^2]p} > \frac{m_1(1-\kappa) + m_2\kappa}{p} = g_1^*(\kappa).$$

First notice that $g_1^{c-n}(0) = g_1^*(0)$ and $g_1^{c-n}(1/2) > g_1^*(1/2)$. Secondly, we have shown in Fact (i) that $g_1^{c-n}(\kappa)$ is first increasing and then decreasing and that $g_1^*(\kappa)$ is decreasing in spillovers. Thus, the inequality is proven for all κ in the relevant range $\langle \hat{\kappa}, 1/2 \rangle$.

(iv) The surplus maximizing public good level for district 2 is $g_1^*(\kappa) = [m_2(1-\kappa) + m_1\kappa]/p$.

Suppose that $\kappa \leq \hat{\kappa}$. First observe that $g_2^{c-n}(0) = g_2^*(0)$.[29] Furthermore, it has already been shown that $g_2^{c-n}(\kappa)$ is decreasing for all $\kappa < \hat{\kappa}$ and it is clear that $g_2^*(\kappa)$ is increasing for all κ. If follows that $g_2^{c-n}(\kappa) < g_2^*(\kappa)$ for all $\kappa \in (0, \hat{\kappa} \rangle$.

If $\kappa > \hat{\kappa}$, then

$$g_2^{c-n}(\kappa) = \frac{m_1\kappa}{[(1-\kappa)^2 + \kappa^2]p}.$$

It is obvious that $g_2^{c-n}(1/2)$ exceeds the surplus maximizing level for κ sufficiently close to 1/2. Since $g_2^{c-n}(\kappa)$ is lower than the surplus maximizing level for all $\kappa \leq \hat{\kappa}$, it must still be lower for κ sufficiently close to but higher than $\hat{\kappa}$.

(v) If $\kappa < \hat{\kappa}$, we must show that

$$\frac{2m_1[(1-\kappa)^4 - k^4]}{\left[(1-\kappa)^2 - \frac{m_1}{m_2}\kappa^2\right]p} > \frac{m_1[(1-\kappa)^2 - k^2]}{\left[(1-\kappa)^2 - \frac{m_1}{m_2}\kappa^2\right]p},$$

and

$$\frac{2m_1[(1-\kappa)^4 - k^4]}{\left[\frac{m_1}{m_2}(1-\kappa)^2 - \kappa^2\right]p} > \frac{m_1[(1-\kappa)^2 - k^2]}{\left[\frac{m_1}{m_2}(1-\kappa)^2 - \kappa^2\right]p}.$$

To cancel out the denominators in the first inequality, we need to know whether they are positive of negative. Since the denominators are decreasing in $\frac{m_1}{m_2}$, it is sufficient to prove that they are positive in $\hat{\kappa}$:

$$(1-\hat{\kappa})^2 - \frac{(1-\hat{\kappa})^3 + \hat{\kappa}^3}{\hat{\kappa}(1-\hat{\kappa})}\hat{\kappa}^2 = \frac{(1-\hat{\kappa})^4 - \hat{\kappa}^4}{1-\hat{\kappa}} > 0,$$

[28]We obtain this result by simply calculating both public good levels in the point $\kappa = 0$.

[29]This can be again easily obtained by calculating the public good levels in $\kappa = 0$.

which is true. Thus, the first inequality is equivalent to

$$(1-\kappa)^2+\kappa^2>1/2,$$

which is true for all κ in the relevant range. The second inequality is equivalent to $(1-\kappa)^2+\kappa^2>1/2$ for all κ, which holds.

If $\kappa \geq \hat{\kappa}$, we need to show that for all $\kappa \in \langle \hat{\kappa}, 1/2)$

$$\frac{2m_1(1-\kappa)}{p} > \frac{m_1(1-\kappa)}{[(1-\kappa)^2+\kappa^2]p}$$

and

$$\frac{2m_1\kappa}{p} > \frac{m_1\kappa}{[(1-\kappa)^2+\kappa^2]p}.$$

Both inequalities are equivalent to

$$(1-\kappa)^2+\kappa^2>1/2,$$

which is true for all κ in the relevant range. If $\kappa = 1/2$, it is evident that the inequalities become equalities. ■

Proof of Proposition 4. When the regions are identical ($m_1 = m_2 = m$), surplus under centralization with cooperative legislature and

1. uniform taxation is: $S_u^c(\kappa) = 2m \ln \dfrac{2m(1-2\kappa+2\kappa^2)}{p} - 4m(1-2\kappa+2\kappa^2)$;
2. non-uniform taxation is: $S_n^c(\kappa) = 2m \ln \dfrac{m}{p} - 2m$.

We establish 6 claims from which the proposition will follow.

Claim 1. $S_u^c(0) < S^d(0)$ and $S_u^c(1/2) > S^d(1/2)$.

Calculating the first inequality, we obtain

$$S_u^c(0) = 2m \ln \frac{2m}{p} - 4m < 2m \ln \frac{m}{p} - 2m = S^d(0)$$
$$\ln \frac{2m}{p} - 2 < \ln \frac{m}{p} - 1$$
$$\ln 2 < 1,$$

which holds. Computing the second inequality, we come to

$$\ln 2 > 1/2,$$

which is true.

Claim 2. $S_u^c(\kappa)$ is increasing in spillovers.

Differentiating $S_u^c(\kappa)$, we obtain

$$\frac{\partial S_u^c}{\partial \kappa}(\kappa) = 4m(1-2\kappa)\left[\frac{(1-2\kappa)^2}{(1-\kappa)^2+\kappa^2}\right] > 0 \quad \text{for all } \kappa \in (0, 1/2).$$

Claim 3. $S^d(\kappa)$ is decreasing in κ.

This claim has already been proven in the Proof of Proposition 1.

Claim 4. $S_n^c(0) = S^d(0)$.

This statement clearly holds.

Claim 5. $S_n^c(\kappa)$ is constant in spillovers.

This claim is clearly true.

Claim 6. $S_u^c(1/2) = S_n^c(1/2)$.

This can be easily proven by inserting 1/2 into the functions of surpluses.

Part (i) of the proposition follows from Claims 1, 2 and 3. Claims 3, 4 and 5 imply part (ii) of the proposition. Finally, Claim 6, combined with Claims 2 and 5, imply part (iii) of the proposition. ■

Proof of Proposition 5. Considering non-identical districts ($m_1 > m_2$), let $(g_1^{c-u}(\kappa), g_2^{c-u}(\kappa))$ and $(g_1^{c-n}(\kappa), g_2^{c-n}(\kappa))$ be the policy outcomes under centralization with a cooperative legislature and a uniform and non-uniform tax system, respectively. Then the surplus

1. with uniform taxation is:

$$\begin{aligned} S_u^c(\kappa) = &[m_1(1-\kappa)+m_2\kappa]\ln g_1^{c-u}(\kappa) + [m_2(1-\kappa)+m_1\kappa]\ln g_2^{c-u}(\kappa) \\ &- p(g_1^{c-u}(\kappa)+g_2^{c-u}(\kappa)); \end{aligned}$$

2. with non-uniform taxation is:

$$\begin{aligned} S_n^c(\kappa) = &[m_1(1-\kappa)+m_2\kappa]\ln g_1^{c-n}(\kappa) + [m_2(1-\kappa)+m_1\kappa]\ln g_2^{c-n}(\kappa) \\ &- p(g_1^{c-n}(\kappa)+g_2^{c-n}(\kappa)). \end{aligned}$$

We prove the proposition via 5 claims.

Claim 1. $S_u^c(0) < S^d(0)$.

Computing this inequality we come to the following one: $\ln 2 < 1$, which clearly holds.

CLAIM 2. $S_u^c(1/2) > S^d(1/2)$.

Let (m_1, m_2) be given. We can find $\omega > 0$ and $\alpha \in (1/2, 1)$ such that $(m_1, m_2) = (\alpha\omega, (1-\alpha)\omega)$. In addition, since $\hat{\kappa} < 1/2$, we have that

$$g_1^{c-u}\left(\frac{1}{2}\right) = g_2^{c-u}\left(\frac{1}{2}\right) = \frac{\alpha\omega}{p}.$$

It follows that

$$S_u^c\left(\frac{1}{2}, \alpha\right) = \omega \ln \frac{\alpha\omega}{p} - 2\alpha\omega.$$

Under decentralization, surplus is given by

$$S^d\left(\frac{1}{2}, \alpha\right) = \frac{\omega}{2}\left[\ln \frac{\alpha\omega}{2p} + \ln \frac{(1-\alpha)\omega}{2p}\right] - \frac{\omega}{2}.$$

Calculating the difference, we obtain

$$\begin{aligned} S_u^c\left(\frac{1}{2}, \alpha\right) - S^d\left(\frac{1}{2}, \alpha\right) &= \omega \ln \frac{\alpha\omega}{p} - 2\alpha\omega - \frac{\omega}{2}\left[\ln \frac{\alpha\omega}{2p} + \ln \frac{(1-\alpha)\omega}{2p}\right] + \frac{\omega}{2} \\ &= \frac{\omega}{2}\left[\ln \frac{\alpha}{1-\alpha}\right] - 2\alpha\omega + \omega \ln 2 + \frac{\omega}{2}. \end{aligned}$$

Differentiating the difference with respect to α, we obtain

$$\frac{\partial\left[S_u^c\left(\frac{1}{2}, \alpha\right) - S^d\left(\frac{1}{2}, \alpha\right)\right]}{\partial\alpha} = \omega\frac{(1-2\alpha)^2}{2\alpha(1-\alpha)} \geq 0.$$

Thus, this difference is non-decreasing in α. So if $S_u^c(1/2, 1/2) > S^d(1/2, 1/2)$, then the inequality holds for all α in the relevant range. But $\alpha = 1/2$ corresponds to the identical districts case and we already know that surplus under centralization is higher than under decentralization then.

CLAIM 3. $S_n^c(0) = S^d(0)$.

This is easily verified by inserting 0 into the corresponding functions of surpluses.

CLAIM 4. $\dfrac{\partial\left[S_n^c(0) - S^d(0)\right]}{\partial\kappa} > 0$ for all $\kappa \in (0, \varepsilon)$, where $\varepsilon > 0$.

This claim holds but due to its algebraic difficulty we will not perform the proof here.

CLAIM 5. $S_n^c\left(\dfrac{1}{2}, \alpha\right) = S_u^c\left(\dfrac{1}{2}, \alpha\right)$.

This statement is easily checked. We leave this to the reader.

For the first half of part (i) of the Proposition 5, we employ Claim 1 and the fact that both surplus functions are continuous functions of κ. Then for each (m_1, m_2) there exists $\delta > 0$ such that $S_u^c(\kappa) < S^d(\kappa)$ for all $\kappa < \delta$. Similar logic with Claim 2 establish the second half of (i).

For part (ii), by utilizing Claims 5 and 2 we can prove that $S_n^c(1/2) > S^d(1/2)$. Since the surplus function for the non-uniform taxation case is a continuous function of κ, for each (m_1, m_2) we can find $\delta > 0$ such that $S_n^c(1/2-\kappa) > S^d(1/2-\kappa)$ for all $\kappa < \delta$. The latter half of (ii) is proven by employing Claims 3 and 4 and using the fact that both surplus functions are continuous functions of κ.

Part (iii) is algebraically very difficult to prove. We, therefore, leave this to the reader.

■

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