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Neo-Keynesian and Neo-Classical Macroeconomic Models: Stability and Lyapunov Exponents

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Abstract:

The non-linear approach to economic dynamics enables us to study traditional economic models using modified formulations and different methods of solution. In this article we compare dynamical properties of Keynesian and Classical macroeconomic models. We start with an extended dynamical IS-LM neoclassical model generating behaviour of the real product, interest rate, expected inflation and the price level over time. Limiting behaviour, stability, and existence of limit cycles and other specific features of these models will be compared.

Keywords: Macroeconomic models, Keynesian and Classical model, nonlinear differential equations, linearization, asymptotical stability, Lyapunov exponents S

JEL: C00, E12, E13

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1. Macroeconomic Models

In this article we are trying to revive traditional models based on IS-LM structure. Such models are different from the models which utilize micro-foundation of macroeconomic theory or rational expectation and nowadays prevail in modern analysis but they are still the subject of analysis in many professional journals and books¹. We provide the non-linear reformulation of the models of IS-LM structure to comprehend better the nature of the economy which contradicts linear principles. In this way we get the non-linear models and try to analyse them with the help of appropriate methods.

For the non-linear model presented here we have found the inspiration in the book Chiarella C., Flaschel P., Groh G., Semmler W. (2000). In this book the IS-LM-PC model is introduced. PC denotes that IS-LM model is augmented by price-wage dynamics i.e. by the modified Philips curve including inflationary expectation. We will develop this model in the following way. We replace the price-wage dynamics by the price-marginal cost (PMC) dynamics. The modified model will be denoted by IS-LM-PMC.

The IS-LM-PMC model is considered as the four differential equations structure. The first equation describes the commodity market, the second one describes the money market and the third one describes the relation between marginal cost and prices. The fourth equation deals with expectation of inflation. We assume an adaptive expectation. Left hand side of the commodity market equation (1) contains a gap between demand (investment) and supply (savings) in the aggregated commodity market. Left hand side of the equation (2) contains a gap between money supply and money demand. There is a gap between price level and marginal cost in the left hand side of the equation (3). Notice that the IS-LM-PMC structure from general point of view could be common both Keynesian and neoclassical approaches. The difference lies only in the style of imputation the equalising factors to the model. The Keynesian approach states that the change of production equalise commodity market (IS), the

¹ In Turnovsky, S. (2000) we can find not only models of traditional macro-dynamics but also models of inter-temporal optimisation and rational expectation models. The last two exhibit the majority approach to the modern analysis of economic systems.

change of interest rate equalise money market (LM) and the change of price level equalise price level and marginal costs. Neoclassical approach assumes that the change of interest rate equalise commodity market (IS), the change of price equalise money market (LM) and the change of production equalise the price with marginal costs (PMC). The paper aims to analyse the consequences of Keynesian and neoclassical approach to the IS-LM-PMC structure for the dynamics of the related models.

We begin with the description of the Keynesian IS-LM-PMC model. Let (in continuous time $t \geq 0$) $Y(t)$, $S(\cdot, \cdot)$ and $I(\cdot, \cdot)$ be denote the real product, savings and real investments of the considered economy respectively. Recall that for the nominal interest $R(t)$ it holds $R(t) = r(t) + \pi^e(t)$ where $r(t)$ is the real rate of interest and $\pi^e(t)$ the expected inflation, in contrast to the inflation $\pi(t)$. The dynamics of the IS model is then given by the following differential equation (see e.g. A.Takayama (1994))

$$\dot{Y} = \alpha \{I(Y(t), r(t)) - S(Y(t), r(t))\}$$

or on taking logarithms by

$$\frac{dy(t)}{dt} = \alpha \{i(y(t), r(t)) - s(y(t), r(t))\} \quad (1)$$

where $y(t) = \ln Y(t)$, and $i(\cdot, \cdot) = \frac{I(\cdot, \cdot)}{Y(\cdot, \cdot)}$, $s(\cdot, \cdot) = \frac{S(\cdot, \cdot)}{Y(\cdot, \cdot)}$, is the so-called propensity to invest, to save respectively. Observe that for an equilibrium point $Y(t) \equiv Y^1$, $y(t) \equiv y^1$, $r(t) \equiv r^1$ we have $I(Y^1, r^1) = S(Y^1, r^1)$ or $i(y^1, r^1) = s(y^1, r^1)$.

Denoting by $p(t)$ the price level at time t , the dynamics of the money market is described by the following differential equation

$$\frac{dr(t)}{dt} = \beta \left\{ \ell(y(t), R(t)) - \ln \frac{M^s}{p(t)} \right\} = \beta \left\{ \ell(y(t), r(t) + \pi^e(t)) - (m^s - \bar{p}(t)) \right\} \quad (2)$$

where $\ell(y(t), R(t)) = \ln(L(Y(t), R(t)))$, $m^s = \ln M^s$, $\bar{p}(t) = \ln p(t)$; $L(\cdot, \cdot)$ and M^s is reserved for demand for money and money supply respectively. In (1), (2) α , β are positive constants signifying the speed of adjustment of the respective market.

To obtain a complete dynamic model of the economy we need to include equations for expected inflation $\pi^e(t)$ and the price level $p(t)$. According to Tobin (1975): for $\pi^e(t)$ the following adaptive equation is valid

$$\frac{d\pi^e(t)}{dt} = \gamma [\pi(t) - \pi^e(t)] \quad (3)$$

where γ is the coefficient of adaptation and $\pi(t)$ is the inflation. Recalling that $\pi(t) = \frac{\dot{p}(t)}{p(t)} = \frac{d}{dt} \bar{p}(t)$, from (3) we immediately get

$$\frac{d\pi^e(t)}{dt} = \gamma \left[\frac{d}{dt} \bar{p}(t) - \pi^e(t) \right]. \quad (4)$$

For what follows we need to express $\frac{d}{dt} \bar{p}(t)$. For this end we assume that the development of the price level $p(t)$ over time is in accordance with changes of the so-called cost function $C(y(t))$. In particular, the well-known condition of profit maximization

$p(t) - \frac{dC(y)}{dy} = 0$ is the base for the following adjustment formula for $p(t)$, where δ is a constant:

$$\frac{d\bar{p}(t)}{dt} = \delta \left(\frac{dC(y)}{dy} - e^{\bar{p}(t)} \right) \quad (5)$$

In fact, the above formula is in accordance with the traditional theory of perfectly competitive firms (see e.g. D. Laider and S. Estrin (1989)) and as such is interpreted in many treatises on monetary and price dynamics (cf. e.g. P. Flaschel, R. Franke, and W. Semmler (1997)).

In what follows we shall use shorthand notations only, i.e., we replace $\frac{d\bar{p}(t)}{dt}$ by $\dot{\bar{p}}$, similarly for the time derivatives \dot{y} , \dot{r} , $\dot{\pi}^e$, and $\frac{dC(y)}{dy}$ is replaced by $C'(y)$. Moreover, we shall often omit the argument t . Hence, (cf. (1), (2), (4), and (5)) using such a model the system describing an economy from the Keynesian point of view has the following form:

$$\left. \begin{aligned} \dot{y} &= \alpha[i(y, r) - s(y, r)], \\ \dot{r} &= \beta[\ell(y, r + \pi^e) - (m^s - \bar{p})], \\ \dot{\pi}^e &= \gamma[\dot{\bar{p}} - \pi^e], \\ \dot{\bar{p}} &= \delta[C'(y) - e^{\bar{p}}], \end{aligned} \right\} \quad (6)$$

where $i(y, r)$, $s(y, r)$, $\ell(y, r + \pi^e)$ and $C(y)$ are real investment, real savings, real money demand and cost functions respectively, depending on production y , rate of interest r , (expected) inflation π^e and the price level p .

Classical models that describe (commodity) price level, interest rate, production and expected inflation dynamics have similar structure of right hand sides (RHS) of differential equations, but left hand sides (LSD) are permuted as follows:

$$\left. \begin{aligned} \dot{r} &= \alpha[i(y, r) - s(y, r)], \\ -\dot{\bar{p}} &= \beta[\ell(y, r + \pi^e) - (m^s - \bar{p})], \\ \dot{\pi}^e &= \gamma[\dot{\bar{p}} - \pi^e]. \end{aligned} \right\} \quad (7)$$

Since for classical models the real product $y(t)$ is assumed to be constant, in (7) we ignore the equation $-\dot{y} = \delta[C'(y) - e^{\bar{p}}]$.

Just introduced models are the base for establishment of macroeconomic models of price and monetary dynamics. Recall that the vector $\mathbf{x}^* = (y^*, r^*, \pi^{e*}, \bar{p}^*)$ whose elements are obtained as a solution of the following set of equations:

$$\left. \begin{aligned} i(y, r) &= s(y, r), \\ \ell(y, r + \pi^e) &= m^s - \bar{p}, \\ e^{\bar{p}} &= C'(y), \end{aligned} \right\} \quad (8)$$

is the equilibrium point both of the Keynesian model given by the set of equations (6) and Classical models given by the set of equation (7). This equilibrium point is said to be (asymptotically) locally stable if every solution of the considered system, starting sufficiently close to \mathbf{x}^* converges to \mathbf{x}^* as $t \rightarrow \infty$. Similarly, \mathbf{x}^* is said to be (asymptotically) globally stable if every solution regardless the starting point converges to \mathbf{x}^* . It is well known (cf. e.g.

J. Guckenheimer and P. Holmes (1986) or A. Takayama (1994) that an equilibrium point (and also a stable point) of the system need not exist, hence the system is unstable. Recall that having found equilibrium points, the system need not converge to some or any of the equilibrium points (in the latter case the system is unstable). Furthermore, if the considered system is unstable and nonlinear, then the system can also exhibit limit cycles (i.e. its trajectory remains in a bounded region) or even chaotic behavior. In words, in contrast to above phenomena, stability is equivalent to monotone or oscillating convergence toward the equilibrium point.

To identify a chaotic behaviour of a macroeconomic model, it is plausible to compare dynamical behaviour of the macroeconomic model by an exponential divergence of nearby trajectories measured by the so-called Lyapunov exponents. The most important is the maximal Lyapunov exponent, negative for stable models, positive for unstable models and infinite for the chaotic behaviour – for details see H.-W. Lorenz (1993).

2. Approximation and Linearization of the Models

To find an analytical form of the output $y(t) = \ln Y(t)$, interest rate $r(t)$, expected inflation $\pi^e(t)$ and the price level $p(t)$ we need to assume that the functions $i(\cdot, \cdot)$, $s(\cdot, \cdot)$, $C(\cdot)$ are of a specific analytical form. As usual, the functions $s(\cdot, \cdot)$ as well as demand for money $\ell(y, R)$ can be well approximated by linear functions, whereas it is necessary to approximate $i(\cdot, \cdot)$ and sometimes also $C(\cdot)$ by suitable nonlinear functions. In what follows, we assume that savings $S(Y(t), r(t))$ can be well approximated by the following expression

$$S(Y(t), r(t)) = Y(t) \cdot [s_0 + s_1 \cdot y(t) + s_2 \cdot r(t)] \quad \text{with } s_0 < 0, \text{ and } s_1, s_2 > 0. \quad (9)$$

Hence the propensity to save $s(\cdot, \cdot) = S(\cdot, \cdot)/Y(\cdot)$ can be written as

$$s(Y(t), r(t)) \text{ def } = \bar{s}(y(t), r(t)) = s_0 + s_1 \cdot y(t) + s_2 \cdot r(t) \quad (10)$$

Similarly, the demand for money is described by the traditional Keynesian demand-for-money function being in the following form

$$\ell(y(t), R(t)) = \ell_0 + \ell_1 y(t) - \ell_2 R(t) - \ell_3 \pi^e(t) = \ell_0 + \ell_1 y(t) - \ell_2 [r(t) + \pi^e(t)] - \ell_3 \pi^e(t), \quad (11)$$

where the parameters $\ell_i > 0, i = 0, 1, 2, 3$ are given. On the other hand, it is convenient to assume that the propensity to invest $i(y(t), r(t))$ is a product of $\frac{1}{r(t)+1}$ and the so-called logistic function. Hence the propensity to invest is assumed to be given analytically as

$$i(y(t), r(t)) = \frac{1}{r(t)+1} \cdot \frac{k}{1+be^{-ay(t)}} \quad (12)$$

where the parameters $k, a > 0$ and b is an arbitrary real number. Similarly, we shall assume that the cost function $C(\cdot)$ is also a logistic function given analytically as

$$C(y(t)) = \frac{h}{1+de^{-cy(t)}} \quad (13)$$

where the parameters $h, c > 0$ and d is an arbitrary real number. Hence

$$\frac{dC(y)}{dy} = \frac{cdh}{(1+de^{-cy})^2} e^{-cy} \quad (14)$$

and we can assume that the "central" part of $C(y(t))$ can be well approximated by a linear function

$$C(y(t)) = d_0 + d_1 y(t) \quad (15)$$

Since $\pi^{e^i} = 0$ to calculate the values y^i, r^i, p^i , on inserting (10), (11), (12) and (13) into (8) we have

$$\frac{1}{r^i + 1} \cdot \frac{k}{1 + be^{-ay^i}} = s_0 + s_1 y^i + s_2 r^i \quad (16)$$

$$\ell_0 + \ell_1 y^i - \ell_2 r^i = m^s - \bar{p}^i \quad (17)$$

$$\bar{p}^i = -\ln d_1 def = -\bar{d}_1 \quad (18)$$

In virtue of (18) from (16), (17) the equilibrium values y^i, r^i can be found as a solution to

$$\frac{k}{1 + be^{-ay^i}} = (s_0 + s_1 y^i + s_2 r^i)(1 + r^i) \quad (19)$$

$$r^i = \frac{1}{\ell_2} (\ell_0 - (m^s + \bar{d}_1) + \ell_1 y^i) \iff y^i = \frac{1}{\ell_1} ((m^s + \bar{d}_1) - \ell_0 + \ell_2 r^i) \quad (20)$$

From (19), (20) we get

$$\left[s_0 + s_2 \left(\frac{\ell_0 - m^s + \bar{d}_1}{\ell_2} \right) + \left(s_1 + s_2 \frac{\ell_1}{\ell_2} \right) y^i \right] \cdot \left[1 + \frac{\ell_0 - m^s + \bar{d}_1}{\ell_2} + \frac{\ell_1}{\ell_2} y^i \right] = k \cdot \frac{1}{1 + be^{-ay^i}} \quad (21)$$

Hence finding the solution to (21) and inserting this value into (20) we immediately get the pair of equilibrium points y^i, r^i . We can observe that:

The RHS of (21) is the so-called logistic function (an increasing function having an inflection point at $y = \frac{1}{a} \ln b$ that is convex in the interval $(0, \frac{1}{a} \ln b)$ and concave in $(\frac{1}{a} \ln b, \infty)$);

The LHS of (21) is a quadratic function (in fact, for real-life models this function differs only slightly from a straight line).

Hence there exist at most three, in real models usually only one, pair(s) of equilibrium points y^i, r^i for $y \geq 0$. More insight in the properties of the equilibrium points, especially with respect to the stability, can be obtained by linearization around the neighborhood of the equilibrium point $(y^i, r^i, \pi^{ei}, \bar{p}^i)$ with $\pi^{ei} = 0$. To check stability of the linearized model, (i.e., that all eigenvalues of the matrix of the linearized system have negative real parts), let us recall that all eigenvalues of the matrix lay in the union of the Gershgorin's circles. The centers of circles are diagonal elements of the matrix and the radius is equal to the minimum of row of column sums of the absolute values of the corresponding off-diagonal elements. For details see e.g. Fiedler (1981).

3. Stability and Speed of Adjustment

1. Keynesian Model

In particular, on employing (16), (17), and (18) for the Keynesian model we have:

$$(22) \quad \begin{bmatrix} \frac{d(y(t) - y^i)}{dt} \\ \frac{d(r(t) - r^i)}{dt} \\ \frac{d(\pi^e(t))}{dt} \\ \frac{d(p(t) - p^i)}{dt} \end{bmatrix} = \begin{bmatrix} \alpha(D_y - s_1) & \alpha(D_r - s_2) & 0 & 0 \\ \beta\ell_1 & -\beta\ell_2 & -\beta(\ell_2 + \ell_3) & \beta \\ 0 & 0 & -\gamma & \gamma d_1 \\ 0 & 0 & 0 & -\delta d_1 \end{bmatrix} \begin{bmatrix} y(t) - y^i \\ r(t) - r^i \\ \pi^e(t) \\ p(t) - p^i \end{bmatrix}$$

where

$$D_y = \frac{1}{1+r} \Big|_{r=r^i} \cdot \frac{\partial}{\partial y} \frac{k}{1+be^{-ay(t)}} \Big|_{y=y^i}, \quad D_r = \frac{\partial}{\partial r} \frac{1}{1+r} \Big|_{r=r^i} \cdot \frac{k}{1+be^{-ay(t)}} \Big|_{y=y^i}.$$

and

$$k = \left[s_0 + s_2 \left(\frac{\ell_0}{\ell_2} - \frac{m^s + \bar{d}_1}{\ell_2} \right) + \left(s_1 + s_2 \frac{\ell_1}{\ell_2} \right) y^i \right] \cdot \left[1 + \frac{\ell_0}{\ell_2} - \frac{m^s + \bar{d}_1}{d_1 \ell_2} + \frac{\ell_1}{\ell_2} y^i \right] \cdot \left[1 + be^{-ay^i} \right].$$

To verify if the obtained equilibrium point is stable, we shall have a look at the eigenvalues of the matrix

$$\mathbf{A} = \begin{bmatrix} \alpha(D_y - s_1) & \alpha(D_r - s_2) & 0 & 0 \\ \beta\ell_1 & -\beta\ell_2 & -\beta(\ell_2 + \ell_3) & \beta \\ 0 & 0 & -\gamma & \gamma d_1 \\ 0 & 0 & 0 & -\delta d_1 \end{bmatrix} \quad (23)$$

Employing the "nearly" upper triangular structure of the matrix \mathbf{A} we can immediately conclude that the eigenvalues $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ of \mathbf{A} are equal to $\delta d_1, \gamma$ and the remaining two eigenvalues λ_3, λ_4 can be calculated as the two eigenvalues of the matrix

$$\tilde{\mathbf{A}} = \begin{bmatrix} \alpha(D_y - s_1) & \alpha(D_r - s_2) \\ \beta\ell_1 & -\beta\ell_2 \end{bmatrix} \quad (24)$$

In particular, if the following two equations Keynesian model

$$\begin{bmatrix} \frac{d(y(t) - y^i)}{dt} \\ \frac{d(r(t) - r^i)}{dt} \end{bmatrix} = \begin{bmatrix} \alpha(D_y - s_1) & \alpha(D_r - s_2) \\ \beta\ell_1 & -\beta\ell_2 \end{bmatrix} \begin{bmatrix} y(t) - y^i \\ r(t) - r^i \end{bmatrix} \quad (25)$$

is stable, then also our extended Keynesian model given by (22) must be stable. Obviously, the eigenvalues of $\tilde{\mathbf{A}}$ are (the symbols $\text{tr } \tilde{\mathbf{A}}$ and $\det \tilde{\mathbf{A}}$ are reserved for trace and determinant of $\tilde{\mathbf{A}}$)

$$\lambda_{3,4} = \frac{1}{2} \left(\text{tr } \tilde{\mathbf{A}} \pm \sqrt{(\text{tr } \tilde{\mathbf{A}})^2 - 4 \det \tilde{\mathbf{A}}} \right)$$

and $\det \tilde{\mathbf{A}}$ must be positive in order to exclude the possibility of a saddle point. For the asymptotic stability $\text{Re } \lambda_{3,4} < 0$, hence if $\text{tr } \tilde{\mathbf{A}} = \alpha(D_y - s_1) - \beta\ell_2 < 0$ both (23) and (24) are stable, in case that $\alpha(D_y - s_1) > \beta\ell_2$ the equilibrium is not asymptotically stable and the limit cycle occurs. In particular, sufficient conditions for the stability of the matrix \mathbf{A} of the considered four-equation Keynesian model are $D_y - s_1 < 0$ along with $D_y - s_1 > D_r - s_2$, $\ell_1 < \ell_2$ or $D_y - s_1 > \frac{\beta}{\alpha} \cdot \ell_1$, $D_r - s_2 > \frac{\beta}{\alpha} \cdot \ell_2$. An interesting case is when eigenvalues of $\tilde{\mathbf{A}}$ are purely imaginary, i.e. if $\alpha(D_y - s_1) = \beta\ell_0$.

Lyapunov exponents for the considered four-equation Keynesian model with the following

values of parameters,

$$\alpha = 20, \beta = 1, \gamma = 0.1, \delta = 0.02, a = 0.1, b = 1.5, s_0 = -0.16, s_1 = 0.07, s_2 = 0.016, \\ l_0 = 0.25, l_1 = 0.4, l_2 = -0.06, l_3 = -0.06, d = -1, d_2 = 0.3, m^s = 0.65, k = 0.4$$

are presented in Figure 1. The Lyapunov dimension of the Keynesian model attractor is equal 0. It means that real parts of all eigenvalues of the Keynesian model attractor are negative. Thus the Keynesian model is not a chaotic macroeconomic system.

2. Classical Model

In particular, on employing (16), (17), and (18) for the Classical model we have:

$$\begin{bmatrix} \frac{d(r(t) - r^i)}{dt} \\ \frac{d(p(t) - p^i)}{dt} \\ \frac{d(\pi^e(t))}{dt} \end{bmatrix} = \begin{bmatrix} \alpha(D_r - s_2) & 0 & 0 \\ \beta\ell_2 & -\beta & \beta(\ell_2 + \ell_3) \\ 0 & \gamma d_1 & -\gamma \end{bmatrix} \begin{bmatrix} r(t) - r^i \\ p(t) - p^i \\ \pi^e(t) \end{bmatrix} \quad (26)$$

where D_r and k take on the same values as in Section 3.1.

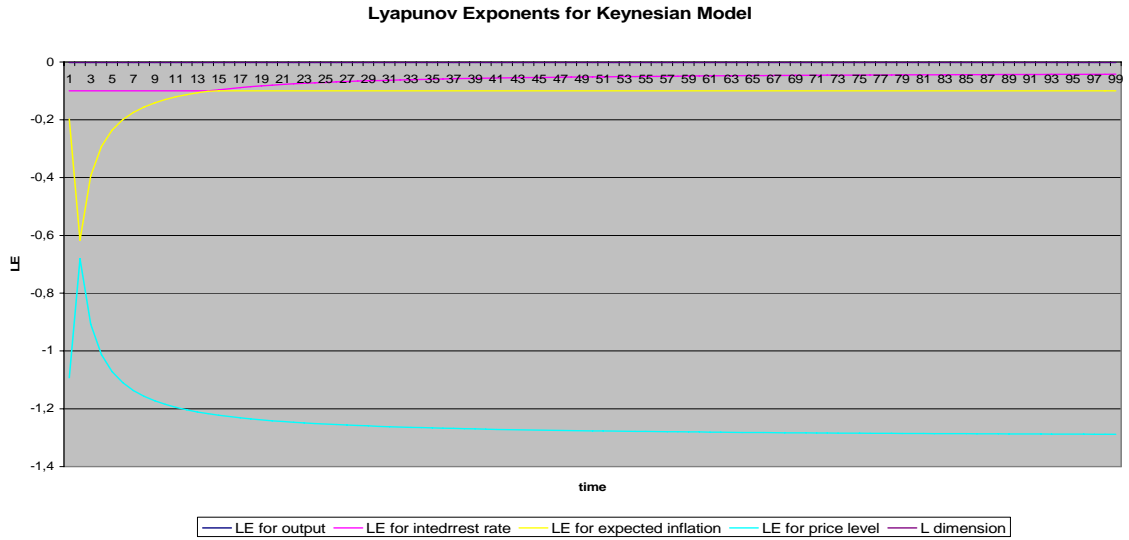


Figure 1: Lyapunov Exponents for Keynesian Model

Lyapunov exponents for the classical model with the following values of parameters,

$$\alpha = 200, \beta = 0.2, \gamma = \delta = 1, a = 0.1, b = 1.5, s_0 = -0.16, s_1 = 0.07, s_2 = 0.016,$$

$$l_0 = 0.25, l_1 = 0.4, l_2 = -0.06, l_3 = -0.06, d = -1, d_2 = 0.3, m^s = 0.65, k = 0.4, y = 4.5$$

are presented in Figure 2. It is shown one of the Lyapunov exponents for the classical model attractor is equal to 0. It means that one real part of eigenvalues is zero and the others real parts of eigenvalues of the classical model attractor are negative. The Lyapunov dimension for the classical model attractor is equal to 0 also. Thus the classical model can exhibit limit cycle.

Conclusions

Macroeconomic models, Keynesian model and classical model, were analyzed from view of its both stability and speed adjustment. It was shown, by different methods of analyzing, eigenvalues and Lyapunov exponents, the Keynesian model is not a chaotic macroeconomic system. On the contrary, it was shown that the classical model can exhibit a limit cycle.

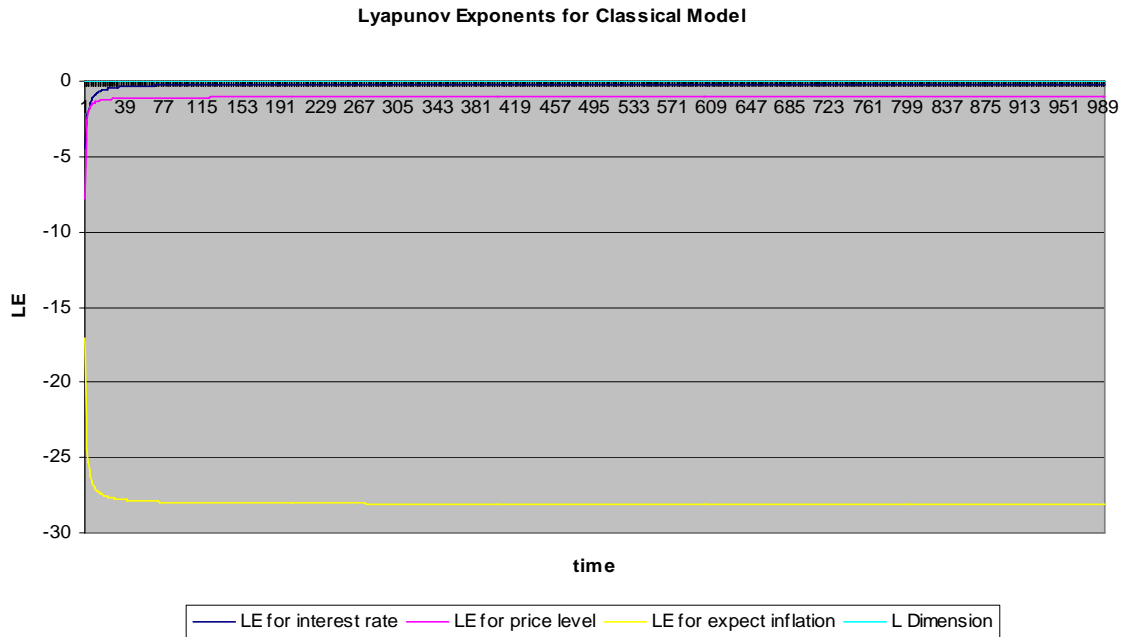


Figure 2: Lyapunov Exponents for Classical Model

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