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Optimal Environmental Tax and the Double Dividend

Bakalářská práce

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Abstract

The aim of the thesis is to outline the main ideas in the development of the optimal environmental taxation theory and explain its connection to the double dividend hypothesis. Moreover, conditions under which the strong form of the double dividend holds are discussed. We conclude that the degree of substitution within consumption bundle as well as between polluting commodity and leisure is a fundamental factor for the validity of the strong double dividend and for the optimal level of environmental tax. Furthermore, a computable general equilibrium model is employed to assess outcomes of various tax reform policies and to test theoretical conclusions.

Key terms: optimal environmental tax, tax reform, double dividend, CGE model

JEL classification: C68, H21, H23

Abstrakt

Cílem této bakalářské práce je nastínit vývoj teorie optimální ekologické daně a objasnit její vztah k teorii dvojí dividendy. Jsou zde diskutovány podmínky, za kterých silná forma dvojí dividendy platí. Docházíme k závěru, že stupeň substituce ve spotřebním koši a mezi znečišťující komoditou a volným časem je rozhodujícím faktorem pro platnost silné dvojí dividedy a také pro stanovení optimální výše ekologické daně. V závěru práce je použit numerický model obecné rovnováhy k ohodnocení variant daňové reformy a k testování závěrů teoretických modelů.

Klíčová slova: optimální ekologická daň, daňová reforma, hypotéza dvojí dividendy, model obecné rovnováhy

JEL klasifikace: C68, H21, H23

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Prohlášení

Prohlašuji, že jsem předkládanou bakalářskou práci zpracovala samostatně a použila jen uvedené prameny a literaturu. Souhlasím, aby práce byla zpřístupněna veřejnosti pro účely výzkumu a studia.

V Praze dne 2.6.2008 Eva Brabcová

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Contents

1 Introduction

“Taxes grow without rain.”

- Jewish Proverb

Environmental taxes have become very popular in recent years and have been paid high attention by economists, politicians and others interested in green movements. Environmental taxes – often called “green” taxes – are defined as a means of promotion of the environment and the natural resources of the planet (Markandya, A., 2006). However, green taxes have been in existence for many years before this global interest in the environment. As history shows, such taxes have been a part of the complex system of taxation ranking an important position among tax revenues raising means. For example, in Russia in the Tsarist period of the 20th century around 90 % of local revenues came from taxes on the use of natural resources (Markandya, A., 2006) Moreover, it is worth noting that taxation of natural resources could be considered as a type of progressive taxation, as natural resources have always been in hands of wealthy people.

Despite the broad use of environmental taxes in the past, their only purpose was to collect revenues in a simple and well defined way. However, the present issue revolves around different target - to meet specific environmental objectives. Therefore, the main topic focuses on the reduction of environmental burdens while raising revenues in the least distortionary way.

These two aims are represented by two approaches to the theory of taxation. The first approach is to reduce environmental burdens and was developed by the economist Arthur Cecil Pigou in 1930s. The other approach discusses the question of raising revenues, so that least possible distortions are imposed on the economy. This method is connected with the name Frank Plumpton Ramsey.

Moreover, the idea of optimality from the point of view of tax revenues and environmental quality led to the origin of the double dividend theory. It states, that it is possible to achieve two dividends by substitution of distortionary taxes for environmental taxes. First, it is able to improve the environment by internalizing negative externalities, and second to reduce initial distortions resulting from taxes.

The aim of this study is to outline the main ideas in the development of the optimal environmental tax theory and explain its connection to the double dividend hypothesis. Moreover, I employ a simple CGE model to asses various policies from the double dividend perspective.

This thesis consists of three chapters. The first chapter is devoted to the environmental taxation and begins with the explanation of Pigouvian and Ramsey tax. Further, the subchapter on optimal environmental taxation provides an overview of the past literature and emphasizes its conclusions. Moreover, model developed by Fuest and Huber is used to present policy recommendations based on the kind of relationship between commodities themselves and leisure.

The second chapter covers the topic of the double dividend hypothesis and mentions the weak and the strong form. It also employs a model elaborated by Gouler, Parry and Butraw in 1996 to show the necessary assumptions for the validity of the double dividend hypothesis. Simultaneously, it points out the connection of the optimal environmental taxation to the theory of the double dividend.

In the last, third, chapter, a computable general equilibrium model is used to produce some empirical results concerning the type of recycling and the characteristics of initial tax system.

2 Environmental Taxation

"There is an increasing sense of what can be called "legal pollution.""

- Thomas Ehrlich

2.1 Pigouvian Tax

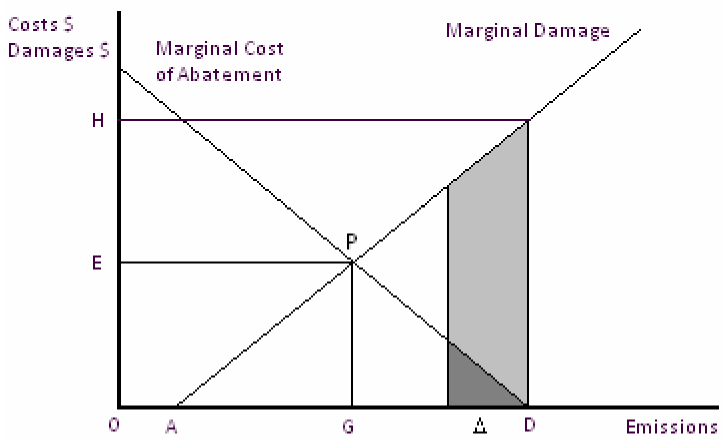
Pigouvian tax bears its name after the economist Arthur Cecil Pigou and is considered as a result of the discussion on "command and control regulation policy". With the onset of industrial revolution, the growing production began to generate non-negligible amount of external effects, which had a very strong impact on the environment and human being. Therefore, authorities were required to set some rules in order to protect the environment. Since the early 19^{th} century, the regulators struggled to find the best way of dealing with this problem and the measures taken were mainly based on passing a law, issuing an administrative order or requiring certain practices to be undertaken. This kind of policy is referred to as "command and control" regulation policy and is typical of directives that have to be carried out. Put differently, the political decisions were more powerful than economic incentives, which resulted in economists being not in favour of command and control policy and trying to find alternative methods of regulation that could achieve the same goals. The founder of the new solution was British economist Arthur Cecil Pigou. He investigated the area of externalities and suggested to set the taxes so that the negative externality is internalized.

In this context, the crucial position ranks the negative externality. "*Anthropogenic activities were said to create negative externalities when the actions of one person or group resulted in damages to another group and when the first group did not take proper account of such damages*" (Markandya, A., 2006, p.1). Pigou noted that if an activity generating negative externality could be taxed, the party responsible for the activity would reduce the amount or intensity of this activity. Moreover, by selecting the tax level suitably, any goal can be achieved in terms of the negative externality reduction. However, the purpose of this analysis is to reach the optimality of an externality generating activity. It is important to note, that optimality does not mean a complete removal of the external effect[1]. Optimality is defined as a point, where "*reduction in the additional damage caused*

[1] Graph 1 shows that complete removal of the externality is not optimal from the efficiency point of view.

by the activity is equal to the cost of abating that additional amount of the activity" (Markandya, A., 2006, p.3). The following Graph 1 is to clarify this.

Graph 1: Pigouvian tax



Source: Markandya, A. (2006), p.6

Graph 1 explains how the optimal level of external effects is achieved with the help of Pigouvian tax. The horizontal axis stands for the amount of emissions generated in production in a plant and the vertical axis stands for costs and damages expressed in money terms that are created by the emissions[2]. Moreover, there are two curves – marginal cost of abatement curve and marginal damage curve. The marginal damage curve is increasing as more emissions results in larger damages to the environment. The marginal cost of abatement curve is decreasing and the rationale behind it is following: the more emissions are to be cut down, the more has to be paid for it. Alternatively, it states the additional costs incurred if emissions are reduced by one unit.

If there are no regulations on the amount of emissions, the output of production is OD, because the enterprise has no incentive to decrease production due to emissions abatement. If the enterprise has to introduce any means of cleaner production, for example to adopt a new technology, it faces some abatement costs. So, if the output is OD, the production creates damages OH. If emissions are reduced by Δ, there is a decrease in damage and increase in costs.

However, in total, the result is positive as the dark shaded area plus the light shaded area denote what is gained and the dark shaded area denotes what is paid. Hence, the light shaded area shows the total net gain. It is obvious, that the total net gain exists, if the

[2] The underlying assumption is the possibility to measure these damages in money terms.

marginal damage curve lies above the marginal cost of abatement curve. This means, that the reduction in emissions is justified up to the point, where marginal cost equal marginal damage. It is the point P. This is exactly the point of optimal externality.

However, Pigouvian tax is not the only tool how to achieve the level of optimal externality. Markandya (2006) notes five approaches to deal with the reduction of externalities:

I. Command and control

This solution is suitable for one plant case only, because in case of more plants, every polluter faces his own marginal cost of abatement curve and hence, the total marginal cost of abatement curve is derived from all individual curves. This means that the amount of information required for optimal functioning of the more plant case is very large.

II. Tax or charge

A charge of OE per unit of emissions forces the emitters to reduce emissions to the point OG, because the costs of abatement are less than the charge. In other words, it is optimal to set the tax equal to the marginal social cost which is also equal to the marginal environmental damage in the point of intersection.

III. Subsidy

Subsidy works exactly as taxes. If a polluter receives a subsidy for each unit of reduction equal to OE, it is beneficial for the polluter to cut down emissions to the point OG, because the subsidy exceeds the abatement costs.

IV. Tradable permits

If tradable permits equal to OG are issued, polluters are allowed to pollute as much as they wish if they have a permit. This leads to the price of a permit equal to OE regardless of whether the permits are auctioned or given to the polluters.

V. Market for emission rights

Emission rights function in a very similar way as tradable permits. Laws would have to be passed defining these property rights for emissions and allowing them to be traded as desired.

These five above mentioned approaches represent solutions to the problem of negative externalities. However, Markandya, A. (2006) states three arguments to show the advantages of taxes as the best means of dealing with the externality problem. These arguments are not implications of Table 1, which shows that all tools lead to the same result. Indeed, the results from Table 1 are a bit imprecise, as imperfect information is not taken into account (e.g., one is not perfectly sure about the shape of the cost curve and the damage curve). This is the base of the following arguments.

First, taxes and permits[3] are preferred to subsidies. The reasons are that subsidies increase profitability of the enterprise, which can result in higher production and thus in increased volume of generated emissions. There is also a problem of corruption when allocating subsidies. Moreover, subsidies are financed from some sources, usually from tax revenues, which generate distortionary effects in the economy. Finally, there is a discussion on who should be given a subsidy.

Second, taxes and permits are better than command and control policy. This preference arises from the argument of flexibility. When the command and control policy is in action, polluters have less scope to choose an instrument to cut down negative effects. Hence, the final outcome can be more costly than necessary.

Third, the discussion whether taxes or permits depends on the considerations of uncertainty and on who bears the costs of adjustments. "*As Weitzmann (1974) showed, if we are not certain about the costs of abatement and impose too high tax rate, the costs will be borne by industry and we will miss the emission reduction target. On the other hand, if we use permits and are too strict in the number issued, we will over reach the emission target, but at a possibly higher cost in terms of cuts in output and employment.*"[4] Hence, the right decision depends on the slopes of the cost and damage curves. However, empirical studies insist that the marginal damage curve is relatively flat in many areas of investigation and thus a tax is more appropriate.

This section has provided basic information about dealing with negative externalities. It proves that Pigouvian tax is the most appropriate tool to internalize externalities. This is the reason Pigouvian tax plays an important role in the following analyses.

[3] Permits in this context are understood as tradable permits and emission rights together.
[4] Quoted from Markandya, A. (2006), p.5, Reference to Weitzman, M.L. (1974): "Prices versus Quantities", *Review of Economic Studies*, Vol. 41, pp.477-491.

2.2 Ramsey Tax

In the previous section, the problem of external effects was defined and the solution in the form of Pigouvian tax was proposed. However, the purpose of the Pigouvian tax was only to internalize externalities. The issue of raising tax revenues was not taken into account, even though the primary goal of taxes has always been to generate income for the government. Hence, this subchapter focuses on the optimal taxation regardless of the concern for environmental quality.

Frank Ramsey was the first economist, who was interested in optimal taxation. More precisely, he drew attention to the quest for such a tax system, that would generate the least possible distortions at a given level of tax revenues. In contrast to other economists of 1920s, he disregarded the trade-off between equity and efficiency and concentrated on efficiency only. He assumed a single-person economy - an economy with many identical people, whose preferences, income and productivity are the same; and omitted any equity considerations. In his analysis[5], there is a competitive economy that produces *N* consumption goods. Labour is the only input into production and each industry is assumed to produce a single output using a constant returns to scale technology. The preferences of the population are represented by an indirect utility function. Under these assumptions, the most efficient tax is a lump-sum tax that does not generate any distortions. However, the concern for the regressive impact of a lump-sum tax on the population leads to the desirability of distortionary taxation. Hence, only distortionary taxes are available in his model.

Ramsey discovers the following optimality condition. If a small amount of tax revenue should be raised, then taxes levied on each consumption good should cause equal proportional reductions in the demand for each good. Put differently, the amounts consumed of each good should decrease by the same proportion. Moreover, he also showed, that this result also holds for higher revenue requirements, but only if there are no income effects[6] and if the demand curves for each good are linear.

Unfortunately, Ramsey's "equal proportional reduction" rule does not reveal which goods should be taxed more heavily and which should be taxed less. Thus, an additional assumption is applied – the assumption of independent demands. This means, that there are

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[5] For further details, see Ramsey, P.F. (1927) or Myles, G., Hindricks, J.: "Intermediate Public Economics"
[6] „No income effects" means that the consumer is compensated in money terms to stay at the same utility curve, so that only substitution effects are taken into account.

no cross price elasticities between any two goods[7]. Under the assumption, Ramsey derived the "inverse elasticity rule": goods with higher price elasticity of demand should be taxed less than goods with lower price elasticity of demand. The rationale behind this suggestion is that goods with lower price elasticity of demand do not react to price changes as much as goods with higher price elasticity of demand and so revenues collected mainly from taxes levied on goods with lower price elasticity of demand do not cause large distortions.

Another approach to the optimal commodity taxation was formulated by Corlett and Hague in 1953[8]. They looked at the problem from a different point of view. They assumed a uniform taxation of two consumption goods and investigated the possibility of efficiency improvement of the tax system by introduction of any non-uniformity in the taxation (raising tax rate on one good and simultaneously lowering it on the other). They came to a surprising result: if the two goods differed in the degree of substitution or complementarity to leisure, it was more efficient to impose a higher tax rate on the good that was more complementary to leisure and to levy a lower tax rate on the good that was more substitutable with leisure. The background of this result is following. Uniform commodity taxation has the same effect as income taxation, where distortionary effect discourages labour supply and encourages leisure. Hence, by taxation of the good, that is more complementary to leisure, labour supply is encouraged and leisure discouraged. This encouragement of labour supply partly offsets the original distortions of the tax.

There are two implications of this result. First, non-uniformity cannot improve efficiency of the tax system if uniform taxation is optimal. Second, uniform taxation is optimal if all consumption goods have the same degree of substitution and complementarity to leisure[9].

It is important to note, that the result by Ramsey and the result by Corlett and Hague come from a very similar model, so one can expect these models to lead to the same conclusions. Moreover, the relationship between them was diagrammatically depicted by Heady in 1987[10]. The following quotation clarifies all.

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[7] Formally, $\partial D_i / \partial p_j = 0$ for $i \neq j$.

[8] Corlett, W.J., Hague, D.C. (1953): "Complementarity and the Excess Burden of Taxation", *The Review of Economic Studies*, Vol. 21, Issue 1, pp. 21-30.

[9] This relationship is often called "weak separability" between consumption goods and leisure.

[10] Heady, C.J. (1987): "A diagrammatic approach to optimal commodity taxation", *Public Finance*, vol. 42, pp. 547-563

"All that need be noted here is that in the "inverse elasticity" case with independent demands, it can be shown that the good which is most complementary to leisure will also be the good with the most inelastic demand curve. Thus the two results pick the same good to be most heavily taxed." Heady (1994), p. 33

2.3 Optimal Environmental Tax

This section is devoted to the theory of optimal environmental taxation. A brief overview of the past development of the theory is outlined which consists of the main contributions. This overview is characterized by the discussion on the optimal level of environmental taxation. As the optimal environmental tax consists of the Pigouvian and Ramsey component, the fundamental question is whether the tax lies above or below the Pigouvian tax. As will be shown later, the results depend on the type of model used (partial equilibrium versus general equilibrium), on the characteristics of utility function, on the relationship between leisure, polluting consumption good and non-polluting consumption good (degree of substitutability and complementarity), on the elasticity of labour supply, and, finally, on the underlying assumptions of a given model and relaxation of these assumptions.

2.3.1 Sandmo 1975[11]

The first economist, who elaborated the theory of optimal environmental taxation, was Angar Sandmo in 1975. He distinguished two different settings in which the optimal environmental taxes differ - first-best setting and second-best setting.

In a model, the first best setting is defined as a system, where all optimality conditions hold. If a tax system is considered, only lump-sum taxes are available as they do not distort agent's optimal decisions. On the contrary, the second best setting is typical of the non-fulfilment of these conditions. In particular, if lump-sum taxes are not available because of the regressive impact on economic agents with different incomes, the second best setting exists. Therefore it may be beneficial for the government not to practice laissez

[11] Sandmo A. (1975): "Optimal Taxation in the Presence of Externalities", *The Swedish Journal of Economics*, Vol. 77, No. 1, pp. 86-98

faire policy, but to apply such measures that the market failures, resulting from the unavailability of lump-sum taxes, cancel each other out.

In the first best setting, he showed that the optimal environmental tax is equal to the Pigouvian tax, as the only purpose is to correct for externalities. In the second best setting[12], he suggested, that the tax should be composed of the Pigouvian tax to internalize negative external effects of pollution and the Ramsey tax to raise government revenues in the least possible distortionary way.

In the model, it is assumed that consumers have identical preferences and the production side of the economy is treated in an aggregate fashion with simple linear production structure. All individuals are equal substitutes in production as everyone has the same productivity. There are *N* consumers and *m+1* consumer goods, from which *m-1* goods are clean denoted C and the *mth* good is dirty denoted *D*. The consumption of good *D* creates a negative externality which is a function of the total consumption of *D*, denoted *ND*. Moreover, *L* denotes hours worked and leisure *V* is expressed as *1-L*. The utility of a representative consumer is

(2.1) $u = u(1-L, C_1,..C_{m-1},D, ND)$.

The utility function is quasi-concave[13] and differentiable with the following first partial derivatives[14] $u_i>0$ for $i = 0, 1, ..., m$ and $u_{ND}<0$.

In the first best setting[15], Sandmo suggests to set the optimal environmental tax levied on the polluting consumption good *D* equal to the Pigouvian tax. The formulae is following:

(2.2) $\theta_{DP} = -N\frac{u_{ND}}{u_D}$

"*From the formulae we see that the optimal tax rate reflects the marginal social damage as the sum of the marginal rates of substitution between good D as a private good and as a public good*". (Sandmo A. (1975), p. 90)

Furthermore, there is a zero tax levied on the non-polluting good *i*, where $i = 0,1,...,m-1$, because the government needs to correct for externalities only and not to raise any revenues.

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[12] For further analyses only the second-best setting will be considered.

[13] Quasi-concavity of the utility function has to be fulfilled if one requires explicit solution and local optimum being identical with global optimum.

[14] The subscript *i* indicates the partial derivative according to the i_{th} commodity.

[15] For the complete optimisation, see Sandmo A. (1975), p. 87-90

In the second best setting[16], there is a government's tax revenue requirement R. Thus, the optimal tax rate formulas are as follows:

$$(2.3)\ \theta_{C_k} = (1-\eta)\left[-\frac{1}{p_k}\frac{\sum_{i=1}^{m-1} C_i J_{ik} + DJ_{mk}}{J}\right] \text{ for } k \neq m$$

$$(2.4)\ \theta_D = (1-\eta)\left[-\frac{1}{p_k}\frac{\sum_{i=1}^{m-1} C_i J_{ik} + DJ_{mk}}{J}\right] + \eta\left[-N\frac{u_{ND}}{u_D}\right]$$

Where p_k is the consumer price of good k, J_{ik} (J_{im}, respectively) is the cofactor of the element in the *ith* row, *kth* column (*mth* column, respecively) of the Jacobian matrix J^* of the demand functions for the taxed goods[17], J is the determinant of the Jacobian matrix J^*. The expression η is equal to λ/μ, where λ is the marginal utility of income and μ is the effect of an increase in the tax requirement R on social utility. Therefore η might be interpreted as the marginal rate of substitution between private and public income[18].

There are three main conclusions from this analysis. First, the Pigouvian tax component enters the formulae for the polluting consumption commodity only, regardless of the pattern of complementarity and substitutability. *"Thus, the fact that a commodity involves a negative externality is not in itself an argument for taxing other commodities which are complementary with it, nor for subsidizing substitutes."* (Sandmo, A., 1975, p. 92).

Second, the optimal tax rate levied on the polluting commodity consists of two components – of the Pigouvian component and the Ramsey component. This can be shown to hold, if the case of independent demands is considered, so that $\partial C_i / \partial p_k = 0$ for $i \neq k$. The determinant J is then equal to the sum of diagonal elements and the optimal tax formulas are[19]:

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[16] For the complete optimisation, see Sandmo, A. (1975), p. 90-92

[17] $$J^* = \begin{pmatrix} \frac{\partial C_1}{\partial P_{C1}} & \cdots & \frac{\partial D}{\partial P_{C1}} \\ \vdots & \ddots & \vdots \\ \frac{\partial C_1}{\partial p_D} & \cdots & \frac{\partial D}{\partial p_D} \end{pmatrix}$$

[18] Later, the expression $1/\eta$ was called marginal costs of public funds.

[19] See Appendix A for derivation.

$$(2.5)\ \theta_{C_k} = (1-\eta)\left(-\frac{1}{\varepsilon_{C_k C_k}}\right) \quad \text{for } k \neq m$$

$$(2.6)\ \theta_D = (1-\eta)\left(-\frac{1}{\varepsilon_{DD}}\right) + \eta\left(-N\frac{u_{ND}}{u_D}\right)$$

Where ε_{kk} is the own price elasticity of demand for the good k[20]. This is exactly the inverse elasticity rule derived by Ramsey. So the optimal tax rate on the polluting good is a weighted average of the inverse elasticity rule and the marginal social damage.

Third, as η increases, the marginal value of private income is higher compared to public income, and the weight of the Ramsey component decreases and the Pigouvian component inceases. In case $\eta = 1$, we are back in the first best setting.

Since 1980s, the debate on the optimal environmental tax is closely connected to the hypothesis of the double dividend. This hypothesis claims that a revenue neutral swap from labour taxes to pollution taxes can reap two dividends: first, to improve the environment by internalizing negative externality, second, to alleviate the initial distortions in the economy.

2.3.2 Terkla 1984, Lee and Misiolek 1986[21]

These authors were among the first economists who were interested in environmental policy. The main feature of their research resides in the partial equilibrium models that are used by all of them and that significantly influenced their results. Terkla's main topic is not optimal environmental taxation, although he deals with it indirectly, too. Instead, he rediscovers the ideas of Tullock[22] and focuses on tax substitutions, which later led to the origin of "the double dividend hypothesis". Terkla suggests that substituting pollution taxes for labour income taxes may result in increased efficiency in the economy. "*Terkla finds that 630 millions to 3 billions $ in improved efficiency benefits could be achieved if federal taxes on particulates and sulphur oxides were substituted for labour income taxes, and 1 to 5 billion $ in benefits if the same taxes were substituted for*

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[20] It is assumed, that $\varepsilon_{kk} < 0$ for all *k*.

[21] Terkla D. (1984): "The Efficiency Value of Effluent Tax Revenues", *Journal of Environmental Economics and Management*, Vol. 11, pp. 107-123, Lee, D.R., Misiolek, W.S. (1986): "Substituting Pollution Taxation for General Taxation: Some Implications for Efficiency in Pollution Taxation", *Journal of Environmental Economics and Management*, Vol. 13, pp. 338-347

[22] Tullock G. (1967): "Excess benefit", *Water resources resources*, Vol. 3

corporate revenue tax." (Lee, D.R., Misiolek, W.S., 1986, p.1). Moreover, Terkla speculates that there might be some efficiency gains in raising tax rates above the level at which marginal damages equal marginal abatement cost if this increases tax revenues.

The issue of the tax rate level was elaborated by Lee and Misiolek two years later. The analysis is based on the maximization of $B(P)-C(P)+E(R(P))$, where B denotes benefits of pollution, C costs of pollution, E stands for the tax substitution benefit and R is tax revenues; all depend on the pollution level P. Hence, the first order condition $B'(P)-C'(P)+E'(R)R'(P)=0$ has to hold. This means, that the level of pollution should be reduced by increasing the pollution tax rate until the net environmental benefit together with tax substitution benefit equal zero.

However, the question whether the optimal pollution tax is higher or lower than the Pigouvian tax, depends on the elasticity of pollution demand $\bar{\varepsilon}_{DD}$[23] (the marginal benefits of pollution stand for the demand for pollution). If $\bar{\varepsilon}_{DD}=1$, the optimal environmental tax is equal to the Pigouvian tax and the tax revenues does not change. If $\bar{\varepsilon}_{DD}>1$, the tax rate is lower than the Pigouvian tax and the tax revenues rise. Finally, if $\bar{\varepsilon}_{DD}<1$, the optimal tax is higher than the Pigouvian tax and the tax revenues fall[24].

Moreover, Lee and Misiolek investigated, which possibility is most probable to occur. They came from the situation of linear marginal benefit curve and linear marginal cost curve and investigated how the net benefits alter relative to the changes of tax revenues. Further, they relaxed the linearity assumptions and generally stated: "*Even in the most general case, however, it remains true that the larger the net benefit derived from unrestrained pollution, the more likely tax revenues shrink and the optimal tax pollution is above the Pigouvian tax*" (Lee, D.R., Misiolek, W.S., 1986, p.345).

Hence, the fundamental message of this analysis asserts that the tax imposed on the polluting commodity lies above the Pigouvian tax, if partial equilibrium model is applied. However, the following analyses employ general equilibrium models only to depict the influence on other markets of the economy.

[23] Elasticity of pollution demand is analogous to price elasticity of demand, but the price is the tax rate. If e>1, an increase in the pollution tax lowers the tax revenues, if e<1 an increase in the pollution tax increases the tax revenues and if e=1 then change in the pollution tax does not alter the tax revenues.

[24] See Appendix A for graphs.

2.3.3 Bovenberg and van der Ploeg 1994[25]

These authors have summed up the theory of the optimal environmental taxation up to the year 1994 including the results derived by Sandmo. They described the optimal tax structure under command economy, under internalization of externalities, under least distortionary way of raising revenues and finally connected the problem of externalities to the distortionary taxation issue. Moreover, they considered the optimal tax structure under some restrictions on the utility function.

The economy is composed of N representative households. Each of them derives utility U from consumption of clean C and dirty D goods, leisure V, clean X and dirty Y public goods and from the environmental quality E. The utility function can be expressed as $U = u(C, D, V, X, Y, E)$. Thus there are no weak separability assumptions[26]. The quality of the environment E is dependent on the total consumption of dirty private ND, on the public good Y and on the government's abatement activities A, i.e. $E = e(ND, Y, A)$. The environmental quality E deteriorates with consumption of D and Y, i.e $e_{ND}, e_Y < 0$, and improves with the abatement activity A, i.e. $e_A > 0$. Moreover, households neglect the negative effects of dirty consumption when they decide on optimal amounts of each good consumed. The production uses constant returns to scale technology with labour L as the only input. Furthermore, the producer wage is denoted ω and the total time endowment fulfils the condition $V + L = 1$. See Appendix A for complete derivation of this model.

To deal with external effects, they used the same procedure as Pigou and derived the following formulae for optimal tax to internalize negative externalities under first best setting and zero tax rate on labour and clean consumption good:

$$(2.7)\quad t_{DP} = -\frac{N u_E e_{ND}}{u_C}$$

Where N is the number of households, u_E stands for the first derivate of the utility function according to the environmental quality and e_{ND} stands for the marginal change in environmental quality due to a change in total consumption of the polluting good.

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25 Bovenberg A.L., van der Ploeg F. (1994): "Environmental Policy, Public Finance and the Labour Market in a Second-Best World", *Journal of Public Economics*, Vol. 55, p. 349-390

26 Particularly, the quality of the environment influences household's decisions on consumption and leisure.

To collect revenues in the least distortionary way, they assumed that labour tax has the same impact as uniform commodity tax and the purpose of the dirty tax is to discourage from consumption of dirty good. Thus, they employed the wage tax system[27] and looked for optimal tax rates on the dirty good and labour, expressed as effective tax rates. The results are following:

$$(2.8)\ \theta_D = \left(\frac{t_D}{1+t_D}\right) = \left(\frac{\varepsilon_{CL}-\varepsilon_{DL}}{\varepsilon_{CD}-\varepsilon_{DD}}\right)\theta_L$$

$$(2.9)\ \theta_L = \left(\frac{t_L}{1-t_L}\right) = \left(\frac{\varepsilon_{LD}-\varepsilon_{DD}}{-\varepsilon_{DD}\varepsilon_{LL}+\varepsilon_{LD}\varepsilon_{DL}}\right)\left(\frac{\mu-\lambda'}{\mu}\right)$$

Where t_D denotes the tax levied on the polluting good, t_L labour income tax, ε is the compensated cross price elasticity of demand, μ stands for the marginal disutility of financing public spending and λ' is the marginal social utility of private income. The effective dirty tax rate is positive if $\varepsilon_{CL} > \varepsilon_{DL}$, i.e. if clean good is better substitute for leisure, so that dirty good is relative complement to leisure. Hence, it is optimal to levy a uniform commodity tax in the form of a labour tax and to tax the good which is more complementary to leisure.

To connect the issue of externalities to the revenue raising optimality condition, they claim, as Sandmo in 1975, that the optimal environmental tax consists of the externality correcting term (the second term on the right hand side) and the revenue raising term (the first term on the right hand side), as suggested by equation (2.10).

$$(2.10)\ \theta_D = \left(\frac{\varepsilon_{CL}-\varepsilon_{DL}}{\varepsilon_{CD}-\varepsilon_{DD}}\right)\theta_L + \theta_{DP}; \theta_{DP} = \frac{t_{DP}}{1+t_D}$$

However, to derive precise results for the polluting commodity tax rate, it is necessary to employ the characteristics of the labour tax and Pigouvian tax under second best setting. Thus the outcome is following:

$$(2.11)\ \theta_D = \left(1-\eta'^{-1}\right)\left(\frac{(\varepsilon_{CL}-\varepsilon_{DL})(1-\alpha_D)}{-\varepsilon_{DD}\varepsilon_{LL}+\varepsilon_{DL}\varepsilon_{LD}}\right) + \eta^{-1}\left(\frac{-Ne_{ND}u'_E}{(1+t_D)u_C}\right)$$

Where α_D is budget share of the dirty product in the private consumption bundle. The weight of the revenue-raising term $\left(1-\eta'^{-1}\right)$ is positive, if the government revenues from the clean consumption good are positive. On the other hand, the weight of the externality

[27] See subchapter "Schöb 1997".

correcting term η^{-1} decreases with the marginal cost of public funds η[28]. As a result, if public funds become scarcer, the revenue raising term dominates the tax rate. This is exactly the same results as derived by Sandmo in 1975.

In a market economy, the marginal cost of public funds MCPF are expressed as follows:

$$(2.12)\ \eta = \left(\frac{1}{1-(\theta_D - \theta_{DP})\alpha_D \varepsilon_{DL}^U - \theta_L \varepsilon_{LL}^U} \right)$$

where ε^U denotes the uncompensated cross price elasticity of demand. The marginal cost of public funds MCPF exceeds unity if:

- $\varepsilon_{LL}^U > 0$
- $\theta_D > \theta_{DP}$ and consequently $\varepsilon_{DL} > 0$

However, to be more specific, Bovenberg and van der Ploeg imposed some restrictions on the utility function – they wrote the utility function in a weak separable form:

$$(2.13)\ U = u(C,D,V,X,Y,E) = U(M(Q(C,D),V),G(X,Y),E)$$

Moreover, it is assumed that the utility functions *U, M, Q* and *G* are homothetic[29]. This has strong consequences for the results and the most important implications are following. First, the elasticities ε_{CL} and ε_{DL} are identical. This means that the optimal tax is composed of the externality correcting term only.

$$(2.14)\ \theta_D = \eta^{-1}\left(-\frac{Nu_E e_{ND}}{(1+t_D)u_C} \right)$$

Second, because θ_D is equal to the externality correcting term only, the marginal cost of public funds is expressed as:

$$(2.15)\ \eta = \left(\frac{1}{1-\theta_L \varepsilon_{LL}^U} \right).$$

Hence, the answer to the question whether the marginal cost of public funds is above or under unity depends only on the uncompensated wage elasticity of labour supply and on the labour taxation.

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[28] $\eta = \frac{\mu}{\lambda}; \eta' = \frac{\mu}{\lambda'}$

2.3.4 Bovenberg and de Mooij 1994[30]

This analysis responds to the conclusions outlined by Bovenberg and van der Ploeg, described in the previous subsection. The main purpose was to state the optimal level of environmental tax relative to the Pigouvian tax.

The basic assumptions of the model are the same as in the previous model: labour *L* is the only input into production, productivity is constant and is equal to wage ω, output is used for production of clean commodity *C*, polluting commodity *D* and public consumption *G*. The number of households is denoted *N*. The total time endowment can be devided between leisure *V* and labour supply *L*, so that $1 = L + V$. The household utility function is expressed in a weak separable fashion $U = u(G, E, H(V, Q(C, D)))$, where the amount of *G* and *E* is given. The environmental distortions depend on the dirty consumption, so that $E = e(ND), e_{ND} < 0$. The labour market distortions stem from the distortionary labour tax t_L. There is a pollution tax t_D levied on the dirty consumption commodity *D*.

Based on these assumptions, they derive the following formula for changes in household'utility:

(2.16) $\frac{dU}{\lambda} = \omega t_L dL + [t_D - Nu_E(-e_{ND})/\lambda]dD$

However, to define the impact of environmental tax on household's utility, it is crucial to evaluate the impact on the labour supply. Hence, they use an equation showing the change in labour supply:

(2.17) $\Delta\tilde{L} = -\varepsilon_{LL}^{U} t_D a_D (1 - \alpha_D)\varepsilon_{CD}\tilde{t}_D$, where $\Delta = 1 - (t_L + a_D t_D)(1 + \theta_l) > 0$

Where a_D stands for the output share of dirty.

Under first best setting, the labour tax is zero, and therefore household's welfare does not change if the environmental tax is equal to the Pigouvian tax. However, if the initial labour tax is positive, welfare would rise. According to the equation (2.17), if the uncompensated elasticity of labour supply is positive and the environmental tax rate declines, labour supply boosts. Hence, equation (2.16) performs rising household's utility due to higher labour supply, while the second term on the right hand side is zero. As a

[30] Bovenberg, A.L., de Mooij, R.A. (1994): "Environmental Levies and Distortionary Taxation", *American Economic Review*, Vol. 84, pp.1085-108

result, it is necessary to decrease the environmental tax rate sufficiently below Pigouvian tax, so that the positive employment effect is offset.

To sum up, Bovenberg and de Mooij perceive the labour market as the fundamental factor of the optimal level of the environmental tax rate. In particular, they claim: "*An increase in the pollution tax reduces employment if the uncompensated wage elasticity of labour supply is positive*."(Bovenberg, A.L., de Mooij, R.A., 1994, p. 1087).

Moreover, this paper served as a basis for another paper whitten by Bovenberg in 1999[31]. Bovenberg used the formula (2.15) derived in 1994 together with van der Ploeg[32] to clarify previous results if weak separability is assumed.

He showed, that the optimal environmental tax is equal to the Pigouvian tax weighted by the marginal cost of public funds. The MCPF η can be expressed as a ratio between the marginal disutility of raising public revenue μ and the marginal utility of private income λ, or by equation $\eta = \left[1 - t_L \varepsilon_{LL}^U\right]^{-1}$.

The MCPF is equal to unity, if the distortionary taxes are absent (i.e. under first best setting, t_L=0) or if the labour supply is completely inelastic (ε_{LL}^U=0). On the other hand, the MCPF exceeds unity, if the uncompensated wage elasticity of labour supply is possitive and the labour tax is possitive, because the Pigouvian taxes are not sufficient source of government revenues.

Hence, he sumps up and connects outcomes of his two previous analyses, which are further developed in his paper with Goulder (2002) [33].

2.3.5 Parry 1995[34]

Another approach to the theory of optimal environmental taxation was presented by Parry in his analysis from 1995. He understood the tax as a result of two powers: revenue recycling effect and tax interaction effect.

[31] Bovenberg A.L. (1999), Green Tax Reforms and the Double Dividend: an Updated Reader´s Guide, International Tax and Public Finance, Volume 6
[32] See subchapter "Bovenberg and van der Ploeg 1994"
[33] Bovenberg A.L., Goulder L.H. (2001), Environmental Taxation and Regulation, NBER Working Paper No. 8458
[34] Parry, I.W.H. (1995): "Pollution Taxes and Revenue Recycling", *Journal of Environmental Economics and Management*, Vol. 29, pp. S64-S77

"*The revenue recycling effect refers to the welfare gain from using environmental tax revenues to cut distortionary taxes, relative to the case where revenues are returned lump-sum (and have no efficiency consequences)*" (Parry, I.W.H., 1995, p. 64).

Similarly, the definition of the tax interaction effect is as follows: *"This refers to likely exacerbation of pre-existing tax distortions caused by the environmental tax"* (Parry, I.W.H., 1995, p.65). Put another way, he claims, that commodity tax does not affect only this commodity market, but also other commodity markets, labour market etc. These interactions have, in general, negative impact on total distortions in the economy.

Hence, according to Parry, the optimal environmental tax is determined by marginal values of revenue recycling and tax interaction effects. Thus, the optimal tax formula is following[35]:

(2.18) $t^* = MB_{RE} + t_{DP} - MC_{IE}$

where MB_{RE} is the marginal benefit from revenue recycling effect, t_{DP} is the Pigouvian tax which indicates the negative externality and MC_{IE} is the marginal cost of tax interaction effect. Hence, if both marginal effects cancel each other, the optimal environmental tax is the Pigouvian tax. Using the equations derived by Parry, the optimal tax as a proportion of the Pigouvian tax can be written as:

(2.19) $\bar{t}^* = \dfrac{1 + (Z / t_{DP}\varepsilon_{DD})\{1 - (\varepsilon_{DL} / \varepsilon_{LL})\}}{1 + 2Z}$

Where Z stands for the efficiency value per dollar of changes in aggregate tax revenues. Moreover, if the assumption of weak separability is applied (i.e. $\varepsilon_{DL} = \varepsilon_{LL}$), the equation (2.19) simplifies to:

(2.20) $\bar{t}^* = \dfrac{1}{1 + 2Z}$

Parry also conducted a numerical research concerning the question, whether the optimal tax is lower or higher than the Pigouvian tax. He estimates the median of Z to be 0.29 with the lower and upper bounds of 0.14 and 0.59. This implies that the optimal tax lies below the Pigouvian tax.

[35] See Appendix A for graphs and derivations.

2.3.6 Schöb 1997[36]

As has already been outlined, the debate on the optimal level of environmental tax emphasized mainly the points of view of those economists, who are in favour of polluting taxes being lower than Pigouvian taxes. Therefore it may seem that a consensus has been reached in the academic club.

However, Schöb in his model proves, that this apparent consistency of views and policy recommendations stems from one particular definition of second-best pollution taxes. He claims, that the definition is fundamentally dependent on the normalization of the tax system and defines two basic frameworks. A wage tax system arises if the tax on clean commodity is normalized to zero, so that only dirty tax and labour tax are in place. On the other hand, if the labour tax is chosen to be equal to zero, the system is called commodity tax system and only dirty and clean consumption taxes are introduced.

"*Consequently, the approach of commodity tax system includes a tax component which is raised for non-environmental reasons in its definition of the second-best optimal pollution tax while the approach of labour tax system defines the tax on a polluting good as the tax component which is imposed for environmental reasons only"* (Schöb, R., 1997, p.168).

To prove these results, he uses the same model as Bovenberg and van der Ploeg in 1994. Under the labour tax system, the optimal taxes are as follows:

$$(2.21)\ t_L = \frac{(1-\nu)(LS_{DD} + DS_{DL})}{S_{DD}S_{LL} - S_{DL}S_{LD}} = t_L^R$$

$$(2.22)\ t_D = \frac{-(1-\nu)(DS_{LL} + LS_{LD})}{S_{DD}S_{LL} - S_{DL}S_{LD}} - \frac{\lambda}{\mu} N \frac{u_E}{\lambda} e_{ND} = t_D^R + \frac{\lambda}{\mu} t_{DP}$$

Where the term ν denotes the net social marginal utility of private income of a representative household and is defined as

$$(2.23)\ \nu = \frac{\lambda}{\mu} + \sum_{I,i=L,D} t_i \frac{\partial I}{\partial Y} + \frac{\lambda}{\mu} N \frac{u_E}{\lambda} e_{ND} \frac{\partial D}{\partial Y}$$

Where Y stands for disposable private income.

The optimal tax on labour consists of one term only, which is the Ramsey term providing the efficiency of the tax system. The optimal pollution tax, however, consists of two components – the Ramsey term and environmental term. The environmental

36 Schöb, R. (1997): "Environmental Taxes and Pre-Existing Distortions: The Normalization Trap", *International Tax and Public Finance*, Vol. 4, pp. 167-176

component is the first-best Pigouvian tax weighted by the marginal cost of public funds. As in Bovenberg and de Mooij 1994, weak separability of the utility function between environmental quality, public good, leisure and consumption is assumed as well as homotheticity of the consumption utility function. This implies zero Ramsey term in the pollution tax and hence, the optimal pollution tax is equal to the Pigouvian tax weighed by the MCPF. Defining the inverse of MCPF as:

(2.24) $$\frac{\lambda}{\mu} = 1 + \frac{t_L}{L}\frac{\partial L}{\partial t_L}$$

it concludes, that if the labour tax is positive ($t_L > 0$) and the labour supply curve upward-sloping ($\partial L / \partial t_L < 0$), the inverse MCPF is less than unity. As a result, it implies that the optimal environmental tax lies below the Pigouvian tax.

However, the situation is different if commodity tax system is considered. The optimal tax structure is following:

(2.25) $$t_C = \frac{-(1-\hat{v})(CS_{DD} - DS_{DC})}{S_{DD}S_{CC} - S_{DC}S_{CD}} = t_C^R$$

(2.26) $$t_D = t_D^R - \frac{\hat{\lambda}}{\mu} N \frac{u_E}{\lambda} e_{ND} = t_D^R + \frac{\hat{\lambda}}{\mu} t_{DP}$$

Where $\hat{\lambda}$ is the marginal utility of income under the commodity tax structure, which differes from the labour tax system, and hence the net social marginal utility of income $\hat{v}$ is different. The marginal utility of public expenditures μ remains identical. Nonetheless, from equation (2.26) it is no longer obvious, whether the dirty tax lies below or above the Pigouvian tax. However, Schöb states, that the optimal pollution tax exceeds the Pigouvian tax, if the curve representing optimal commodity tax structure in the presence of externalities is increasing in both tax rates[37].

To sum up, Schöb has outlined two main conclusions resulting from different normalizations. First, the optimal dirty tax is lower than the Pigouvian tax if a labour tax system is used; second, the pollution tax lies above the Pigouvian tax as long as commodity tax system is in place. Schöb also warns not to rely on these normalizations only. "*For example, we can normalize the sum of consumer prices to unity or even normalize the tax on dirty good to zero. In each case, we get different looking answers on what should be the optimal tax on polluting good*" (Schöb, R., 1997, p. 174).

[37] See Figure 1, Schöb, R., 1997, p. 173

2.3.7 Fuest and Huber 1999[38]

This paper contributes to the discussion on the optimal level of environmental tax compared to the Pigouvian tax with some new results. Fuest and Huber suggest, that the tax rate is determined by the substitutability between labour supply and consumption of the dirty good or by the substitutability between the consumption of polluting and non-polluting good, depending of the normalization chosen. So the weak separability between leisure and consumption goods is not assumed. Moreover, they show that the result derived by Bovenberg and de Mooij is a special case and thus should not be interpreted as depending primarily on the slope of the labour supply curve. The model is described in detail in the section 2.3.2.

2.4 Model

The model presented here was developed by Fuest and Huber in 1999. First of all, I make the following definitions:

Definition 1: *If* $\partial L/\partial t_D > 0$*,* *(*$\partial L/\partial t_D < 0$*), leisure and polluting consumption are gross complements (substitutes).*

Definition 2: *If* $\partial C/\partial t_D > 0$*,* *(*$\partial C/\partial t_D < 0$*), clean consumption and polluting consumption are gross substitutes (complements).*

The utility function U of a representative agent[39] is following

(2.27) $U = U(L, C, D, G, E(D))$

Where *L* denotes labour, *C* clean consumption good, *D* polluting consumption good, *E(D)* stands for the quality of the environment and *G* for public expenditure, which is assumed to be fixed. The amounts of *E* and *G* are given and the agent maximizes utility *U* over *L*, *C* and *D*, subject to the budget constraint

(2.28) $p_C C + p_D D = wL$

[38] Fuest, C., Huber, B. (1999): "Second-best Pollution Taxes: An Analytical Framework and Some New Results", *Bulletin of Economic Research*, Vol. 51, Issue 1, pp.31-38

[39] It is assumed, that the representative agent is a group of N identical households.

Where p_C and p_D are consumer prices of the two consumption goods and ω is the consumer wage. The Lagrangean is

$$(2.29) H(C,D,L,\lambda) = U(L,C,D,G,E(D)) - \lambda(p_C C + p_D D - wL)$$

With follwing first order conditions:

$$(2.30)\ \frac{\partial H}{\partial C} = \frac{\partial U}{\partial C} - \lambda p_C = 0 \quad \frac{\partial U}{\partial C} = \lambda p_C$$

$$(2.31)\ \frac{\partial H}{\partial D} = \frac{\partial U}{\partial D} - \lambda p_C = 0 \quad \frac{\partial U}{\partial D} = \lambda p_D$$

$$(2.32)\ \frac{\partial H}{\partial L} = \frac{\partial U}{\partial L} + \lambda \omega = 0 \quad \frac{\partial U}{\partial L} = -\lambda \omega$$

As the production side of the economy regards, a linear technology is considered with labour as the only input. The producer prices of *C* and *D* and the producer wage are normalized to unity. The government may set a labour tax t_L and taxes on clean and dirty goods t_C and t_D. Equation (2.28) implies

$$(2.33)\ p_C \frac{\partial C}{\partial t_i} + p_D \frac{\partial D}{\partial t_i} - \omega \frac{\partial L}{\partial t_i} + i = 0 \qquad i = C, D, L$$

However, one of the three taxes is redundant and can be normalized to zero. For $t_C = 0$, a wage tax system is in place and for $t_L = 0$, a commodity tax system is considered.

The government budget constraint is

$$(2.34)\ R(t_C, t_D, t_L) = t_C C + t_D D + t_L L = G$$

Where R denotes the aggregate tax revenue. Moreover, we assume

$$(2.35)\ \frac{\partial R}{\partial j} > 0 \ \ j = t_C, t_D, t_L$$

This means that the slope of the Laffer curve is possitive. The government´s optimal tax problem is to maximize (2.27) subject to (2.34). The Lagrangean is

$$(2.36)\ \Gamma(t_C, t_D, t_L, \eta) = U(L,C,D,G,E(D)) + \mu(t_C C + t_D D + t_L L - G)$$

The first order conditions can be written as

$$(2.37)\ \frac{\partial \Gamma}{\partial j} = \frac{\partial U}{\partial C}\frac{\partial C}{\partial j} + \left(\frac{\partial U}{\partial D} + \frac{\partial U}{\partial E}\frac{\partial E}{\partial D}\right)\frac{\partial D}{\partial j} + \frac{\partial U}{\partial L}\frac{\partial L}{\partial j} + \mu \frac{\partial R}{\partial j} = 0 \qquad j = t_C, t_D, t_L$$

Using $1 + t_C = p_C$, $1 + t_D = p_D$ and $\omega + t_L = 1$, and (2.30), (2.31), (2.32), the expressions (2.37) can be written as

$$(2.38)\ \lambda(1 + t_C)\frac{\partial C}{\partial j} + [\lambda(1 + t_D) + u_E e_D]\frac{\partial D}{\partial j} - \lambda(1 - t_L)\frac{\partial L}{\partial j} = -\mu \frac{\partial R}{\partial j}$$

The tax which internalizes the marginal environmental damage t_{DP} is

(2.39) $t_{DP} = -\frac{1}{\lambda} u_E e_D$

Equation (2.33) can be written as

(2.40) $\frac{\partial C}{\partial j} + \frac{\partial D}{\partial j} - \frac{\partial L}{\partial j} + \frac{\partial R}{\partial j} = 0 \qquad j = t_C, t_D, t_L$

Substituting (2.39) into equations (2.38), dividing by λ and using (2.40) yields

(2.41) $t_C \frac{\partial C}{\partial j} + t_L \frac{\partial L}{\partial j} + (t_D - t_{DP}) \frac{\partial D}{\partial j} = \left(1 - \frac{\mu}{\lambda}\right) \frac{\partial R}{\partial j} \quad j = t_C, t_D, t_L$

The discussion about the second-best optimal environmental taxation mainly revolves around the relationship with the Pigouvian tax. Particularly around the question, whether the environmental tax is higher or lower than the Pigouvian tax. However, as has already been found out by Schöb (1997), the sign of $(t_D - t_{DP})$ depends on the normalization of the tax rates. Hence, the model is described under both tax systems.

2.4.1 Wage Tax System

The wage tax system implies zero tax rate on clean consumption good. Hence, equations (2.41) can be written as

(2.42) $t_L \frac{\partial L}{\partial t_D} + (t_D - t_{DP}) \frac{\partial D}{\partial t_D} = \left(1 - \frac{\mu}{\lambda}\right) \frac{\partial R}{\partial t_D}$

(2.43) $t_L \frac{\partial L}{\partial t_L} + (t_D - t_{DP}) \frac{\partial D}{\partial t_L} = \left(1 - \frac{\mu}{\lambda}\right) \frac{\partial R}{\partial t_L}$

A positive uncompensated wage elasticity of labour supply is assumed, implying negative terms on the right hand side of equations (2.42) and (2.43)[40]. Thanks to the cross-price effects it can be seen how results of previous models change. The equation (2.42) gives the following proposition.

<u>Proposition 1:</u> *If the wage tax is positive and the uncompensated wage elasticity of labour supply is positive ($\partial L / \partial t_L < 0$):*

[40] Revenue raising taxes are typical of second best setting, which implies marginal costs of public funds being larger than unity, and therefore the expression $1 - \mu/\lambda$ is negative.

(i) *a sufficient condition for* $t_D^* > t_{DP}$ *is that leisure and polluting consumption are gross complements (*$\partial L / \partial t_D > 0$*) and hence*

(ii) *a necessary condition for* $t_D^* < t_{DP}$ *is that leisure and polluting consumption are gross substitutes (*$\partial L / \partial t_D < 0$*).*

Proposition 1 shows that if the labour supply curve is upward-sloping, the result depends on the gross substitutability between dirty consumption goods and leisure. If leisure and dirty good are gross complements, an increase in the dirty tax encourages agents to supply more labour and thus alleviate the labour market distortions due to the labour tax. Conversely, if leisure and dirty consumption are gross substitutes, higher pollution tax even exacerbates the distortions in the economy.

Moreover, this result can be used to clarify the statement by Bovenberg and de Mooij. They claim, that the environmental tax is lower than the marginal social damage because of the positive uncompensated wage elasticity of labour supply. If this interpretation is considered within the model by Fuest and Huber, some additional assumptions come out. First, leisure and polluting consumption have to be gross substitutes. Second, the term on the right hand side of eq. (2.42) has to be smaller than the first term on the left hand side, to guarantee the relationship $t_D^* < t_{DP}$. Hence, Fuest and Huber prove that the decisive point is not only positive uncompensated wage elasticity of labour supply, but also the substitutability between leisure and polluting good.

2.4.2 Commodity Tax System

If the commodity tax system is applied, the tax rate levied on labour income is zero. So, equations (2.41) yield

(2.44) $$t_C \frac{\partial C}{\partial t_D} + \left(t_D - t_{DP}\right)\frac{\partial D}{\partial t_D} = \left(1 - \frac{\mu}{\lambda}\right)\frac{\partial R}{\partial t_D}$$

(2.45) $$t_C \frac{\partial C}{\partial t_C} + \left(t_D - t_{DP}\right)\frac{\partial D}{\partial t_C} = \left(1 - \frac{\mu}{\lambda}\right)\frac{\partial R}{\partial t_C}$$

Under the commodity tax system, the fundamental relationship is the substitutability between clean and dirty consumption goods. Assuming that C is normal ($\partial C / \partial t_C < 0$), the equation (2.44) implies the following proposition.

Proposition 2: *If the tax on clean consumption is positive:*

(i) *a sufficient condition for $t_D^{**} > t_{DP}$ is that clean and polluting consumption are gross substitutes ($\partial C / \partial t_D > 0$) and hence,*

(ii) *a necessary condition for $t_D^{**} < t_{DP}$ is that clean and polluting consumption are gross complements ($\partial C / \partial t_D < 0$).*

The intuition behind this result suggests that if clean and dirty consumption are gross complements, an increase in the dirty tax rate boosts the clean consumption and thus revenues from taxation of clean consumption raise, which mitigates the distortions of higher pollution tax. On the other hand, if the clean and polluting consumption are complements, a higher pollution tax discourages clean consumption and distortions in the economy even deteriorate.

2.5 Conclusion

To sum up, this chapter has examined Pigouvian and Ramsey tax and answered the question of how these taxes are related to the optimal environmental taxation. In the first best setting with no need for tax revenues, the only purpose is to correct for externalities. Therefore, the optimal environmental tax equals the Pigouvian tax. In the second best setting, distortionary taxes are present ensuring given tax revenues. In this case, the optimal tax consists of the revenue-raising and externality correcting term, weighted by the marginal costs of public funds.

An overview of the optimal taxation debate was outlined resulting in the description of the model by Fuest and Huber. This model incorporates the relationship between leisure and polluting commodity and between clean and dirty commodity, as the crucial determinants of the optimal rate. Hence, the issue whether the optimal environmental tax is higher or lower than the Pigouvian tax, is solved by this model with respect to normalization chosen.

3 The Double Dividend

"We contend that for a nation to try to tax itself into prosperity is like a man standing in a bucket and trying to lift himself up by the handle."

- Winston Churchill

The double dividend hypothesis is closely connected to the environmental tax reform. The target is to "punish" consumption of polluting final and intermediate goods by increasing the tax rate levied on these goods. This incentive motivates people to substitute other consumption goods for dirty goods and therefore to improve the quality of the environment. Consequently, taxation, in general, is perceived as harmful as it creates distortions in various markets.

These two main ideas lead economists to devise such a system, in which the "bad" would be penalized and the "good" supported. Hence, the definition of the double dividend hypothesis is following:

"The double dividend hypothesis, which explains two-way benefits of environmental taxation, suggests that increased taxes on polluting activities can provide two kinds of benefit. The first dividend is an improvement in the environment, and the second dividend is an improvement in economic efficiency from the use of the environmental tax revenues to reduce other taxes such as income taxes that distort labour supply and saving decisions." Pak (2000), p.42

The concept of the double dividend was first presented in the analysis by Tullock in 1967 called "Excess Benefit"[41]. However, nobody paid attention to the new emerging tool of economic policy making. Few years later, in 1984, Terkla continued and assessed the impacts of environmental tax swaps in partial equilibrium model. Up to 1990s, only few economists were seriously interested in tax substitution yielding more benefits. The concept "double dividend" was first used by Pearce in 1991[42] and since then, the double dividend issue occupies many pages of economic journals. However, two explanations of the second dividend developed. The definition stated above handles the efficiency improving dividend, while in the second half of 1990s economists discussed so called employment dividend or even the concept of triple dividend.

[41] Tullock, G. (1967): *"Excess Benefit",* Water Resources Research, Vol. 3, pp. 643-644
[42] Pearce, D.W. (1991): „The Role of Carbon Taxes in Adjusting to Global Warming", *Economic Journal*, Vol. 101, pp. 938-948

The employment dividend has been paid a considerably wide attention in Europe, in order to harmonize environmental requirements with issues of the welfare state. The employment dividend claims, that under certain assumptions, the environmental tax reform can boost employment in economies, which are typical of unemployment in equilibrium and a strong role of trade unions. However, I will focus on the efficiency dividend and leave the literature on the employment dividend to interested readers[43].

Moreover, I find it useful to mention the purpose of this issue in a broader context. There is a widespread agreement about the ability of environmental taxes to achieve the first dividend (environmental improvements), even though the magnitude of this dividend is highly uncertain. This has sowed the seeds of the current about what (if any) second dividend can be reached by the environmental taxes. The occupation with this question stems from the uncertainty of the first dividend. Thus, the crucial idea behind is whether the environmental tax reform can be introduced in a costless way. As it is very difficult to compare the uncertain environmental improvements with abatement costs, it is more beneficial to be sure about the sign of the second dividend as all debates head towards the desire to make safe judgments about the environmental reforms in the presence of uncertainty. (Goulder, 1995)

To be able to deal with the double dividend hypothesis in the policy making process, Goulder (1995) distinguishes two forms of the double dividend – weak form and strong form.

3.1 Weak Double Dividend

3.1.1 Definition

"By using revenues from the environmental tax to finance reductions in marginal rates of an existing distortionary tax, one achieves cost savings relative to the case where the taxrevenues are returned to taxpayers in lump-sum fashion." Goulder, 1995, p. 4

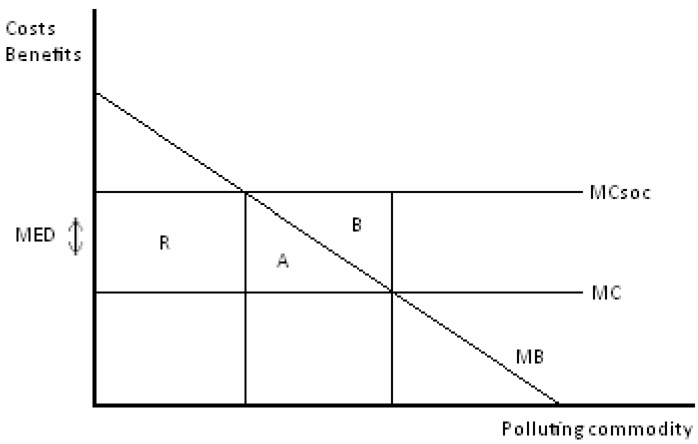
[43]Koskela, E., Schöb, R. (1998) consider various characteristics of the labour market and summarize the necessary conditions for the validity of the employment dividend. In particular, they assume a unionized labour market with unemployment in equilibrium and wages determined endogenously between trade unions and employers. Moreover, various institutional arrangements are considered, e.g. types of taxation of unemployment benefits, price indexation of unemployment benefits, etc. They find out that results are highly sensitive to institutional arrangements and conclude: "*A revenue neutral green tax reform will boost employment if unemployment benefits are nominally fixed and taxed at a lower rate than labour income*." (Koskela, E., Schöb, R. ,1998, p. 1723). Furthermore, see Heady, C.J., Markandya, A. (2000), Markandya, A. (2006), Schöb, R. (2006), Bosquet, B. (2000), Carraro, C., Galleotti, M., Gallo, M. (1996).

As defined above, the weak form of the double dividend discusses the question of using the tax revenues from the environmental tax. Put another way, from the social welfare point of view, the environmental tax reform is more efficient if the tax revenues are returned through cuts in existing distortionary taxes than through lump sum payments to residents.

This statement can be shown both mathematically and graphically. Let $C(t_D, \Delta T)$ be the gross costs of a given tax system after an introduction of new environmental tax t_D and consequent cut in lump-sum tax T, if the revenue neutrality holds. Similarly, let $C(t_D, \Delta t_x)$ denote the gross cost of the tax system after an introduction of environmental tax t_D accompanied by a decrease in a distortionary tax t_X. Then the weak form claims: $C(t_D, \Delta T) < C(t_D, \Delta t_x)$. Under this proposition, the second dividend is the lower distortionary cost in the former case (left hand case) relative to the latter case (right hand side).

The Graph 2 offers a typical partial equilibrium analysis of the impact of environmental tax on social welfare.

Graph 2: Weak double dividend



Source: Goulder, L.H. (1995), p.37

The horizontal axis denotes the amount of the polluting good and the vertical axis stands for the marginal costs. There are three curves: marginal cost curve *MC* represents private marginal cost borne by an individual, while the marginal social cost curve *MCsoc* denotes the marginal cost borne by the society. The marginal social cost incorporates private marginal cost and marginal external damage *MED*. The marginal benefit curve *MB* stands for the demand for the polluting good. If tax equal to the marginal external damage is levied on the polluting good, the marginal private and social cost balance and social welfare increases by area *B* (the area of environmental improvement *A*+*B* minus dead weight loss *A*).

The weak double dividend claims, that the revenues from the environmental tax (area *R*) are recycled back to the economy with different efficiency consequences. If the area *A* represents the costs of the tax system with the revenue recycling through lump sum payments, then the costs of the tax system under revenue recycling through cuts in distortionary taxes is represented by area smaller than *A*.

However, partial equilibrium analysis does not include all relevant information for outcome assessment as it relates to given market only and disregards the influence of other markets. Therefore, the area A does not need to be a valuable indicator.

The validity of the weak double dividend hypothesis can be proved theoretically and is supported by many numerical simulations. However, the consensus about its validity is conditioned by an important assumption – the assumption of only one distortionary tax in the economy.

3.1.2 Weak Double Dividend under Second Best

This subchapter provides more general results about the weak double dividend, as its validity is investigated in case there is more than one distortionary tax. It is based on model by Metcalf, Babiker, and Reilly developed in 2002. They show that the weak double dividend hypothesis does not have to hold in an economy with multiple distortions.

They assume a model with two goods *C* and *D*. The good *D* creates pollution *P*, which is understood as an input into production function. Taxes t_C and t_D are commodity taxes to generate revenue for the government, tax t_P is environmental tax[44]. There is a constant return to scale production function using labour and pollution as inputs. The utility of the representative agent is derived from consumption of clean *C* and dirty *D* consumption good, leisure *V* and environmental quality *E*, which deteriorates with the amount of pollution *P*. Moreover, leisure and environmental quality are weakly separable from each other and from consumption of goods *C* and *D*. An analytical general equilibrium model is used to assess different policies.

[44] Notice that the environmental tax is not imposed on the polluting commodity itself as in all previous models, but on emissions from the commodity.

As shown by Table 1, tax revenues from environmental taxes can be recycled through lump sum payments or cuts in distortionary taxes t_C or t_D, so that total tax revenues remain stable.

Table 1: Revenue recycling options

Policy	Fixed	Recycled
1	t_C, t_D	T
2	t_D, T	t_C
3	t_C, T	t_D

Source: Metcalf G.E., Babiker M.H., Reilly J. (2002), p.12

If increase in welfare resulting from Policies 2 and 3 is higher that the welfare increase from Policy 1, the weak form holds.

Metcalf et al (2002) set the environmental tax t_P equal to 10 % exogenously and investigate, how welfare changes with respect to different values of t_C and t_D and with respect to the kind of recycling of the tax revenues. The results for Policy 1 and 2 are shown in Tables 2 to 5.

Table 2: Welfare change (in %) – Policy 1

		t_C				
		0	0.1	0.2	0.3	0.4
t_D	0	0.25	0.32	0.39	0.47	0.56
	0.1	0.05	0.13	0.20	0.28	0.36
	0.2	-0.15	-0.07	0.01	0.09	0.17
	0.3	-0.35	-0.27	-0.19	-0.11	-0.03
	0.4	-0.55	-0.48	-0.40	-0.32	-0.23

Source: Metcalf G.E., Babiker M.H., Reilly J. (2002), p.12

Table 2 demonstrates the impact of the environmental tax reform on welfare in case of revenue recycling through lump sum payments. The figures indicate, that welfare significantly decreases if the polluting commodity *D* is taxed more that the clean commodity *C*.

Table 3: Welfare change (in %) – Policy 2

		t_C				
		0	0.1	0.2	0.3	0.4
t_D	0	0.30	0.60	0.91	I.24	I.57
	0.1	0.00	0.30	0.61	0.94	I.27
	0.2	-0.32	-0.01	0.30	0.62	0.96
	0.3	-0.64	-0.34	-0.02	0.30	0.63
	0.4	-0.97	-0.67	-0.35	-0.03	0.30

Source: Metcalf G.E., Babiker M.H., Reilly J. (2002), p.13

The same result is presented in Table 3. If tax rate imposed on polluting good *D* is much higher than the tax rate levied on clean good *C*, and simultaneously the revenue recycling is conducted through cuts in the clean commodity tax rate, welfare deteriorates.

Table 4: Welfare Change (in %) - Comparison of Policies 1 and 2

		t_C				
		0	0.1	0.2	0.3	0.4
t_D	0	0.054	0.283	0.520	0.764	1.019
	0.1	-0.056	0.174	0.411	0.655	0.909
	0.2	-0.172	0.058	0.294	0.538	0.790
	0.3	-0.293	-0.064	0.172	0.414	0.664
	0.4	-0.415	-0.188	0.046	0.286	0.534

Source: Metcalf G.E., Babiker M.H., Reilly J. (2002), p.13

Finally, the values in Table 4 are calculated as a difference between social welfare in Policy 2 and 1 (i.e. the difference between recycling through cuts in t_C and lump-sum recycling). If the values are positive, social welfare is higher under recycling through cuts in distortionary taxes and the weak double dividend exists. The first row represents the case, when only one distortionary tax exists. There is a wide consensus about its validity which is supported by these values. Weak double dividend does not hold if the tax rate on the clean good is much below the tax rate on polluting good.

In general, economy with multiple distortions gives rise to weak double dividend only if revenue recycling results in converging tax rates levied on consumption goods. On the contrary, if the revenue recycling makes the tax rates diverge, distortions in economy increase and weak double dividend does not exist. In such a case, it is more efficient to use the tax revenues from environmental taxes for lump sum transfers.

However, question which emerges is whether such conclusion arises from optimal commodity taxation system. Thanks to the assumption of weak separability between leisure, environmental quality and consumption goods, the Corlett-Hague results described in chapter 2 imply a uniform commodity system. Hence, any means that support convergence of tax rates are welfare improving, while divergence of tax rates has the opposite effect.

This optimal commodity taxation implication is also proved by Table 3. If the tax t_C is markedly higher than the tax t_D and revenue recycling lowers rate the t_C, the social welfare change is positive as the tax system approaches optimality. If both tax rates are equal, the change in social welfare is constant as it does not depend on the actual value of the tax rate, but the equality plays role. If the tax rate t_D is higher than t_C and recycling is through cuts in t_C, social welfare deteriorates as the tax system moves far from optimality.

3.2 Strong Double Dividend

3.2.1 Definition

The strong form of the double dividend is a statement about the ability of environmental tax reform to achieve several goals. The definition is following:

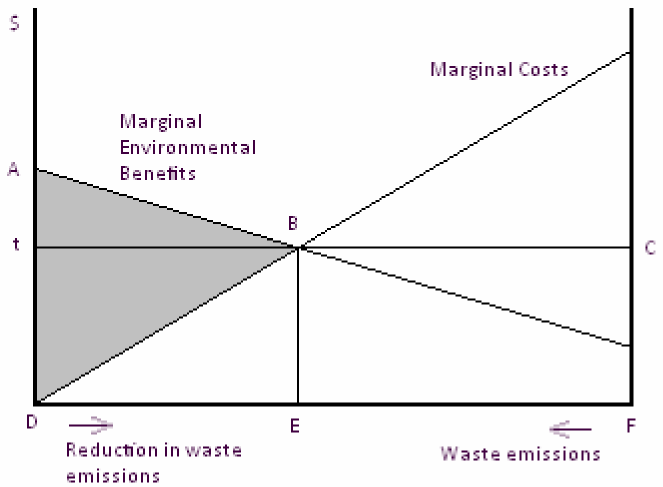
"The revenue neutral substitution of the environmental tax for typical or representative distortionary taxes involves a zero or negative gross cost." Goulder, 1995, p.4

Put another way, the strong double dividend asserts that a green tax reform not only improves environmental quality, but also increases non-environmental welfare, i.e. reduces the total deadweight loss of the tax system. If the second dividend exists, the environmental tax reform is so called "no regret" option; even though one is not sure about the magnitude of environmental improvements (first dividend), the green tax reform may yield desired outcomes (Bovenberg, 1999).

However, the validity of the strong form of double dividend hypothesis depends on three distinct effects on economic welfare, as suggested by Parry (2000). It is the revenue recycling effect, tax interaction effect and primary welfare gain[45]. The primary welfare gain simply stems from the net benefits of environmental quality improvements due to internalization of negative effects. This effect can be shown in Graph 3.

[45] These effects have already been mentioned on page 27 in the subsection about Parry 1995.

Graph 3: Primary welfare gain and revenue recycling effect



Source: Parry, I.W.H., Oates, W.E., (2000), p.607

The primary welfare gain is the shaded area 0AB. This gain is largest if the Pigouvian tax is in place, so marginal environmental benefits equal marginal cost of emission abatement. Moreover, the revenues from the environmental tax are represented by the area EFCB and can be used to cut distortionary labour tax so that revenue neutrality is maintained. This effect is called revenue recycling effect and is considered to be positive, as the wedge between gross and net wage gives the incentive to supply more labour. Finally, the tax interaction effect involves the way in which the environmental tax interacts with other pre-existing taxes in the economy, particularly with the labour income tax.

However, as has already been mentioned, comparison of environmental improvements with efficiency improvements involves large approximation and may yield inaccurate conclusions. Therefore, the sign of the second dividend plays role and is determined by the revenue recycling effect and tax interaction effect, so that the total impact of environmental tax reform on economic welfare depends crucially on the net effect of these two powers. Numerical findings from analytical models indicate that the costs of the tax interaction effect outweigh the benefits of revenue recycling effect. In other words, the reduction of real wage due to higher distortions is larger than the gain from lower tax on labour. As a result, economic agents are worse off and the strong double dividend fails to hold.

Nonetheless, this conclusion perfectly coincides with the optimal taxation theory by Ramsey. The efficiency based theory is built on the use of broad-based taxes that are preferred to narrow-based taxes. As there is more scope for economic agents to substitute broad-based goods for narrow-based goods, the former distortions even deteriorate. This is exactly the case of broad-based labour tax and narrow-based environmental tax.

However, the validity of the strong form of double dividend depends on a particular model used. Thus, the next section outlines an overview of the double dividend issue is outlined in connection to the optimal environmental tax overview.

3.2.2 Literature Overview

The first economist who discovered the advantages of tax swaps was Gordon Tullock in his analysis from year 1967. Approximately twenty years later, Terkla, Lee and Misiolek were interested in this issue bringing some new outcomes to the beginning of the theory. Even though they did not publish together, they used the same assumptions and derived very similar results. They tried to evaluate the impacts of revenue neutral switch of labour tax for environmental tax on the efficiency of the tax system. As has already been depicted, only partial equilibrium models were used ignoring the influence of the green tax reform on other markets in the economy. However, they considered the revenue recycling effect only and completely omitted any tax interaction effects. Hence, they assumed only the positive side of the coin indicating that strong double dividend holds and the environmental tax reform is able to achieve great efficiency improvements.

Bovenberg and van der Ploeg look at the double dividend issue from the employment point of view and investigate the changes in labour supply as an indicator of distortionary effects[46]. They consider two cases – an upward sloping and backward bending labour supply curve. They claim, that "*if the uncompensated wage elasticity of labour supply is positive, the change in the tax structure away from distortionary towards pollution taxes reduces the marginal cost of public funds, thereby expanding the size of the social sector at the expense of private welfare and thus the consumption wage*[47]" (Bovenberg, A.L., van der Ploeg, F., 1994, p. 376). Hence, under the upward sloping labour supply curve, the lower wage depresses the supply of labour and thus employment.

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[46] The labour market is the only distorted market and thus changes in distortions are measured as changes in employment.

[47] For relationships between distortionary taxation, public spending and marginal cost of public funds, see Atkinson, A.B., Stern, N.H. (1974): "Pigou, Taxation and Public Goods", *Review of Economic Studies*, Vol. 41, Issue 1; and Waldasin, D.E. (1984): "On Public Good Provision with Distortionary Taxation", *Economic Enquiry*, Vol. 22, Issue 2; and Ballard, C.L., Fullerton, D. (1990): "Wage Tax Distortions and Public Good Provision", National Bureau of Economic Research, Working Paper No. 3506

On the other hand, if the wage elasticity is negative, "*the change in the tax structure away from distortionary taxation raises rather than reduces the costs of public funds. Hence, the social sector contracts while the private welfare expands. As the labour supply bends backwards the associated higher consumption wage reduces labour supply*" (Bovenberg, A.L., van der Ploeg, F., 1994, p. 376). To sum up, only if labour supply is completely inelastic, employment does not change and green tax reform does not boost more distortions.

To assess the validity of double dividend hypothesis, Bovenberg and de Mooij also find out the impacts on labour supply. They claim: "An increase in the pollution tax from a positive initial level reduces employment if the uncompensated wage elasticity of labour supply is positive" (Bovenberg, A.L., de Mooij, R.A., 1994, p. 1087). The idea behind this statement is following. There is a drop in the real after-tax wage because the cut in labour tax does not fully compensate workers for the effect of the pollution levy on their income. This means, that the revenue neutrality on the government level does not necessarily imply unchanged real after-tax wage of workers. This decline in real after tax wage induces workers to supply less labour and if the wage elasticity of labour supply is positive, it causes larger distortions. As the marginal utility is composed of labour market and environmental distortions, welfare decreases and the strong double dividend does not hold.

Another approach is presented by Parry in his analysis from 1995. He defines the first dividend as primary welfare gain stemming from the net benefits of environmental improvements. The second dividend is understood as a result of revenue recycling effect and tax interaction effect. However, calculating the optimal environmental tax according to Parry, marginal values of these effects were used. Now, it is the absolute value that plays role. Parry presents an intuitive diagrammatic approach and shows the gains and losses in graphs. Then he calculates the "area of improvements" and "area of distortions" and compares them. Hence, the benefit from the revenue recycling effect relative to the cost from tax interaction effect is defined as

$$(3.1)\quad \frac{D(t)}{D^0}\frac{\varepsilon_{LL}}{\varepsilon_{DL}}$$

Where D^0 is the initial value of consumption of the polluting good, $D(t)$ is the amount consumed after levying the dirty tax, ε_{LL} is the compensated wage elasticity of labour supply and ε_{DL} is the compensated price elasticity of demand for D with respect to the price of leisure. However, $D(\mathrm{t}) < D^0$ because of the decrease in consumption due to the tax imposition. Hence, the relationship $\varepsilon_{LL} > \varepsilon_{DL}$ is required for the validity of the strong double dividend and the relationship has to be strong enough to outweigh the decline in consumption of *D* and thus make the eq. (3.1) greater than unity. As D is a sufficiently weak substitute for leisure (i.e. $\varepsilon_{DL} < \varepsilon_{LL}\left(D/D^0\right)$), the tax interaction effect outweighs the revenue recycling effect. In addition, the usual assumption about weak separability between consumption goods and leisure implies, that D is an average substitute for leisure and thus $\varepsilon_{LL} = \varepsilon_{DL}$. Finally, under these restrictive assumptions the strong form of the double dividend fails to hold, as eq. (1) changes to D/D^0, which is always less than unity.

Bovenberg in his paper from 1999 sums up the conditions for validity of the strong double dividend, based on Bovenberg and de Mooij 1994. Defining the marginal utility function and labour supply change function, he comes to two conclusions. First, if the initial environmental tax is zero and is increased to a positive value, the real after tax wage does not change, since the higher pollution tax exactly offsets the effect of a lower labour tax on the overall wedge between before and after tax wage. Second, increasing environmental tax from a positive initial level reduces employment[48]. This is because of the insufficient compensation of the lower income tax to offset the higher pollution levy as a result of the narrower tax base. Hence, real after tax wage declines and so does employment. To sum up, the larger the magnitude of pre-existing tax distortions, the higher become the gross distortionary costs of revenue neutral pollution taxes. This implies invalidity of the strong form of the double dividend.

However, another approach taking the relationships of complementarity and substitutability into account and deriving more general results is presented in the following subchapter.

[48] See equation (2.17). Furthermore, it is assumed that the uncompensated wage elasticity of labour supply is positive.

3.3 Model[49]

3.3.1 Model Description

This model was developed by Goulder, Parry and Butraw in 1996 and is based on the use of revenue recycling and tax interaction effect in order to set the conditions under which the strong form of the double dividend holds. The representative agent utility function is defined as

(3.2) $U(u(C, D, V), E)$

Where C and D denote market goods, V is leisure and E is the quality of the environment. The utility functions U and u satisfy the continuity and local quasi-concavity properties. The weak separability between C, D, V and E means, that the demands for goods C and D and for leisure V is independent of the quality of the environment[50].

Goods C and D are produced under perfect competition with the constant returns to scale production function that is unaffected by E. The only input into production is labour L. Normalizing transformation rates to unity, the economy´s resource constraint is following

(3.3) $TE = C + D + V$

Where TE is agent's total time endowment and TE-V is labour supply.

The quality of the environment is harmed by the good D which generates pollution.

(3.4) $E = E(D)$

The environmental quality decreases with the amount of D consumed.

(3.5) $\frac{\partial E}{\partial D} = E_D < 0$

From equation (3.2) and (3.4), one can define the marginal environmental damage from production of the good D as

(3.6) $P(D) = -\frac{1}{\lambda} U_E E_D = t_{DP}$

Where λ is the marginal utility of income. Furthermore, $P'(D) \geq 0$ is assumed.

[49] Butraw D., Goulder L.H., Parry I.W.H. (1996): "Revenue Raising versus Other Approaches to Environmental Protection: the Critical Significance of Preexisting Tax Distortions", National Bureau of Economic Research, Working Paper No. 5641

[50] Notice that the weak separability does not apply for the relationship consumption and leisure.

Finally, the government requires tax revenue R and levies proportional tax t_L on labour income and a pollution tax t_D on good D. It does not matter what is R used for, however, I will assume it is used for sump sum transfers T. Normalizing the gross wage to unity, agent's budget constraint can be expressed as

(3.7) $(1+t_D)D + C = (1-t_L)(TE - V) + T$

Agent maximizes the utility over the budget constraint taking the environmental quality E as given. Hence, the Lagrangian is as follows:

(3.8) $L(C,D,V,\lambda) = U(u(C,D,V),E) - \lambda((1+t_D)D + C - (1-t_L)(TE-V) - T)$

The first order conditions yield:

(3.9) $\frac{\partial L}{\partial D} = \frac{\partial U}{\partial D} - \lambda(1+t_D) = 0 \qquad U_D = \lambda(1+t_D)$

(3.10) $\frac{\partial L}{\partial C} = \frac{\partial U}{\partial C} - \lambda = 0 \qquad U_C = \lambda$

(3.11) $\frac{\partial L}{\partial V} = \frac{\partial U}{\partial V} - \lambda(1-t_L) = 0 \qquad U_V = \lambda(1-t_L)$

From the equations (3.7) and (3.8) demand functions for D, C and V can be derived.

(3.12) $C(t_D,t_L); D(t_D,t_L); V(t_D,t_L)$

Moreover, the government's balanced budget requires validity of the expression

(3.13) $R = t_L(TE - V) + t_D D$

3.3.2 Revenue Neutral Tax Swap

Now, a revenue neutral change in the tax system is considered, so that an increase in t_D is accompanied by a decrease in t_L. Substituting (3.12) into (3.13) and totally differentiating while keeping R constant gives

(3.14) $$\frac{dt_L}{dt_D} = -\frac{D + t_D \frac{\partial D}{\partial t_D} - t_L \frac{\partial V}{\partial t_D}}{TE - V - t_L \frac{\partial V}{\partial t_L} + t_D \frac{\partial D}{\partial t_L}}$$

This equation expresses the increase in t_D with simultaneous decrease in t_L, so that the government budget is kept constant. Differentiating the resource constraint (3.3) yields the following condition for the aggregate quantity effects of the policy change.

(3.15) $\dfrac{\partial D}{\partial t_D}+\dfrac{\partial C}{\partial t_D}+\dfrac{\partial V}{\partial t_D}+\left(\dfrac{\partial D}{\partial t_L}+\dfrac{\partial C}{\partial t_L}+\dfrac{\partial V}{\partial t_L}\right)\dfrac{dt_L}{dt_D}=0$

Using (3.2), (3.3) and (3.12), the welfare effect from a marginal change in t_D allowing for changes in demands and in t_L is

(3.16)

$$\frac{dU}{dt_D}=U_D\frac{\partial D}{\partial t_D}+U_C\frac{\partial C}{\partial t_D}+U_V\frac{\partial V}{\partial t_D}+U_EU_D\frac{\partial D}{\partial t_D}+\left(U_D\frac{\partial D}{\partial t_L}+U_C\frac{\partial C}{\partial t_L}+U_V\frac{\partial V}{\partial t_L}+U_EE_D\frac{\partial D}{\partial t_L}\right)\frac{dt_L}{dt_D}$$

Substituting (3.6), (3.9), (3.10), (3.11) and (3.15) into (3.16) yields

(3.17) $\dfrac{1}{\lambda}\dfrac{dU}{dt_D}=-(t_{DP}-t_D)\dfrac{\partial D}{\partial t_D}-t_L\dfrac{\partial V}{\partial t_D}-\left[(t_{DP}-t_D)\dfrac{\partial D}{\partial t_L}+t_L\dfrac{\partial V}{\partial t_L}\right]\dfrac{dt_L}{dt_D}$

Substituting (3.14) into (3.17) gives

(3.18)

$$\frac{1}{\lambda}\frac{dU}{dt_D}=-(t_{DP}-t_D)\frac{\partial D}{\partial t_D}+\frac{\left\{t_L\frac{\partial V}{\partial t_L}+(t_{DP}-t_D)\frac{\partial D}{\partial t_L}\right\}\left\{D+t_D\frac{\partial D}{\partial t_D}\right\}-\left\{TE-V+t_{DP}\frac{\partial D}{\partial t_L}\right\}t_L\frac{\partial V}{\partial t_D}}{TE-V-t_L\frac{\partial V}{\partial t_D}+t_D\frac{\partial D}{\partial t_L}}$$

However, the equation (17) can be expressed as

(3.19) $\dfrac{1}{\lambda}\dfrac{dU}{dt_D}=\underbrace{(t_{DP}-t_D)\left(-\dfrac{\partial D}{\partial t_D}\right)}_{\partial W^P}+\underbrace{M\left(X+t_D\dfrac{\partial D}{\partial t_D}\right)}_{\partial W^R}-\underbrace{(1-M)t_L\dfrac{\partial V}{\partial t_D}}_{\partial W^I}$

Where *M* is equal to

(3.20) $M=\dfrac{t_L\dfrac{\partial V}{\partial t_L}-(t_{DP}-t_D)\left(-\dfrac{\partial D}{\partial t_L}\right)}{TE-V-t_L\dfrac{\partial V}{\partial t_L}+t_D\dfrac{\partial D}{\partial t_L}}$

To comment on the expression (3.20), the numerator in *M* denotes the welfare loss because of an increase in the t_L , which is composed of two parts: an increase in leisure multiplied by the labour tax t_L (which is the difference between gross and net wage) less decrease in *D* multiplied by the difference between marginal social cost $1+t_{DP}$ and demand price $1+t_D$. Moreover, *D* and leisure are supposed to be substitutes. The denominator of *M* is the total increase in government revenue collected due to higher labour tax t_L. Therefore,

M can be explained as efficiency cost of raising an additional dollar of revenue by increasing t_L; the marginal welfare cost of labour taxation.

Furthermore, the equation (3.19) splits the total welfare change into three single components; the Pigouvian effect, the revenue recycling effect and the tax interaction effect which are characterized by $\partial W^P, \partial W^R, \partial W^I$, respectively. The Pigouvian effect is the welfare improvement from a marginal decrease in D as a result of higher t_D multiplied by the difference between social and private cost (i.e. $1+t_{DP}-1-t_D=t_{DP}-t_D$, the marginal social benefit). The second term shows the revenue recycling effect which is the product of the efficiency value per dollar of tax revenue and the incremental pollution tax revenue. Finally, the tax interaction effect reflects the welfare loss due to two subeffects – if D and leisure are substitutes, an increase in t_D alleviates consumption of leisure and therefore worsens the welfare cost of labour taxation by $t_L \, \partial V / \partial t_D$. Consequently, this cuts down the labour tax revenues. Hence, the tax interaction effect is the result of these two mentioned effects.

3.3.3 Tax Interaction Effect

The tax interaction effect can be written as

$$(3.21)\ \partial W^I = \frac{M\left(t_L \left(\frac{\partial V^C}{\partial t_D} - \frac{\partial V}{\partial Y} D \right) \left(TE - V + t_{DP} \frac{\partial D}{\partial t_L} \right) \right)}{t_L \left(\frac{\partial V^C}{\partial t_L} - \frac{\partial V}{\partial Y} (TE - V) \right) + \left(t_{DP} - t_D \right) \frac{\partial D}{\partial t_L}}$$

Using Slutsky symmetry $\partial V^C / \partial t_D = -\partial D / \partial t_L$, $\partial V^C / \partial t_L = -\left(\partial D^C / \partial t_L + \partial C^C / \partial t_L \right)$ and differentiating equation (3.3) and substituting into equation (3.21) yields

(3.22)

$$\partial W^I = \frac{MD\left\{ \left[\frac{\partial D^C}{\partial t_L} \frac{1-t_L}{D} + \frac{\partial V}{\partial Y} \frac{(1-t_L)(TE-V)}{TE-V} \right] \left[1 + \frac{\partial D}{\partial t_L} \frac{1-t_L}{D} \frac{t_{DP} D}{(1-t_L)(TE-V)} \right] \right\}}{\left\{ \frac{\partial D^C}{\partial t_L} \frac{1-t_L}{D} \frac{D}{TE-V} + \frac{\partial C^C}{\partial t_L} \frac{1-t_L}{C} \frac{C}{TE-V} + \frac{\partial V}{\partial Y} \frac{(1-t_L)(TE-V)}{TE-V} \right\} - \frac{\partial D}{\partial t_L} \frac{(1-t_L)}{D} \frac{(t_{DP} - t_X) D}{t_L (TE-V)}}$$

Moreover, let us denote

(3.23) $\varepsilon_{DL} = -\frac{\partial D^C}{\partial t_L}\frac{1-t_L}{D}$

to be the compensated elasticity of demand for good *D* with respect to leisure

(3.24) $\varepsilon_{CL} = -\frac{\partial C^C}{\partial t_L}\frac{1-t_L}{C}$

To be the compensated elasticity of demand for good *C* with respect to leisure

(3.25) $\varepsilon_{LY}^U = -\frac{\partial V}{\partial Y}\frac{(1-t_L)(TE-V)}{TE-V}$

To be the income elasticity of labour supply

(3.26) $\varepsilon_{DL}^U = -\frac{\partial D}{\partial t_L}\frac{1-t_L}{D}$

To be the uncompensated elasticity of demand for good *D* with respect to leisure.

Substituting equations (3.23)-(3.26) into equation (3.22) gives the following formula for the tax interaction effect

(3.27) $$\partial W^I = \frac{MD(\varepsilon_{DL} + \varepsilon_{LY}^U)\left(1 - \varepsilon_{DL}^U\left(\frac{t_{DP}D}{(1-t_L)(TE-V)}\right)\right)}{\left(\varepsilon_{DL}\left(\frac{D}{D+C}\right) + \varepsilon_{CL}\frac{C}{D+C} + \varepsilon_{LY}^U\right) - \varepsilon_{DL}^U\frac{(t_{DP}-t_D)D}{t_L(TE-V)}}$$

Moreover, expression (3.27) can be simplified into

(3.28) $\partial W^I = \phi_D MD$, where ϕ_D denotes the degree of substitution between *D* and leisure relative to the total consumption and leisure:

(3.29) $$\phi_D = \frac{\varepsilon_{DL} + \varepsilon_{LY}^U}{\frac{D}{D+C}\varepsilon_{DL} + \frac{C}{D+C}\varepsilon_{CL} + \varepsilon_{LY}^U}$$

The simplification is possible because "ε_{DL}^U *in the numerator is trivial when environmental damages from D are small relative to aggregate net labour income and* ε_{DL}^U *in the denominator is trivial when environmental damages net of environmental tax revenues is small relative to aggregate labour tax revenues*" (Goulder, 1996, appendix A-2).

Finally, having defined all three effects, some conclusions can be derived. The value of ϕ_D depends crucially on the degree of substitution between good *D* and leisure, *C* and leisure, resp. ϕ_D is equal to unity if goods *D* and *C* have the same degree of complementarity to leisure (i.e. $\varepsilon_{DL} = \varepsilon_{CL}$). ϕ_D exceeds unity if the good *D* is stronger substitute to leisure than the good C (i.e. $\varepsilon_{DL} > \varepsilon_{CL}$). Finally, ϕ_D is less than unity if the good *D* is weaker substitute to leisure than the good *C*.

3.3.4 Comparison

Furthermore, it is possible to make comparison of the revenue recycling and tax interaction effect and investigate, which one is larger and under what conditions; and what impact it has on the validity of the double dividend hypothesis. Tax-interaction effect: $\partial W^I = \phi_D MD$, Revenue-recycling effect: $M\left(X + t_D\,\partial D/\partial t_D\right)$.Both effects are in balance if both goods are equal substitutes to leisure, i.e. ϕ_D=1 and the tax rate levied on good *D* is zero, i.e. $\partial W^R = MX$. However, if the tax on *D* is positive and both goods are equal substitutes to leisure, the tax interaction effect dominates the revenue recycling effect, because expression $t_D\,\partial D/\partial t_D$ is negative, and the double dividend fails to hold. Hence, the strong double dividend holds only if the revenue recycling effect overweighs the tax interaction effect. This occurs only if the good *D* is sufficiently weak substitute to leisure so that ϕ_D is sufficiently below unity.

3.4 Conclusion

This chapter has discussed the double dividend hypothesis and distinguished its two forms – the weak form and the strong form. The hypothesis states that the weak double dividend holds, in general. However, if the tax swap increases the difference between tax rates in the economy, the weak form does not need to exist, as it moves the tax system far from optimality. On the other hand, the strong form is much more controversial. Special attention is paid to the model derived by Goulder, Parry and Butraw in 1996, which is based on the relationship between consumption goods and leisure. It concludes that the strong double dividend occurs only if the polluting good is sufficiently weak substitute to leisure.

However, this is perfectly compatible with results derived by Fuest and Huber and conforms to the case of wage tax system. Fuest and Huber claim, that if the wage tax is positive, the uncompensated wage elasticity of labour supply is positive and leisure and polluting consumption good are gross complements, the optimal environmental tax lies above the first best Pigouvian tax.

Moreover, one can find even more general interpretation: according to Corlett and Hague, it is efficient for the economy to tax goods that are more complementary to leisure than others. This enhances welfare by lowering the distortions in the tax system. Therefore, if leisure and polluting good are gross complements, it is efficient to tax the polluting good more, as indicated by Corlett and Hague and proved by Fuest and Huber. This consequently moves the tax system closer to optimum, as the initial tax distortions in the economy lessen, which, in turn, gives better prospects for the existence of the strong double dividend. Hence, the validity of the strong double dividend is not surprising as it has strong foundations in the theory of optimal taxation.

4 CGE Model

"Tax reform is taking the taxes off things that have been taxed in the past and putting taxes on things that haven't been taxed before."

- Art Buchwald

This chapter focuses on empirical issues related to the theory of environmental tax reform. A simple computable general equilibrium model is used to assess outcomes of various recycling policies and to test the validity of some theoretical models described in the previous chapter. First, the structure of the model is explained, followed by several simulations considering two types of tax revenues recycling, three types of initial tax system and three levels of initial tax rates. Second, conclusions derived by theoretical models in previous chapters are tested.

The construction of the CGE model is based on the model presented by Christoph Böhringer in his paper "Green Tax Reforms and Computational Economics: A Do-it-yourself Approach"[51]. The model incorporates different energy tax rates in the private and production sectors within a complex system of other initial tax distortions. It is designed for closed economy with no foreign trade which is based on the assumption of full employment.

4.1 Model Specification

4.1.1 Household Sector

A representative household[52] is assumed, which demands four goods: the use of energy in household sector denoted E_H, a non-energy consumption good Y_H, leisure l and public good G. The utility from consumption of these goods is expressed as $u(Y_H, E_H, l, G)$ with all first partial derivatives being positive. Moreover, energy is also used in production of the good Y, which is denoted as E_Y. The environmental damages created by emissions from total consumption of energy $E_H + E_Y$ is captured by the

[51] Böhringer, Ch., Wiegard, W., Starkweather, C., Ruocco, A. (2003): "Green Tax Reforms and Computational Economics: A Do-it-yourself Approach", *Computational Economics,* Number 22

[52] The representative household is considered as an aggregate of many identical households.

function $v(E_H + E_Y)$, having negative first partial derivative. A linear relationship between energy consumption and emissions is assumed.

The total household's utility U consists of the utility u from consumption and disutility v from environmental damages and reflects the separability assumption between both functions:

(4.1) $U = u(Y_H, E_H, l, G) + v(E_H + E_Y)$

This type of utility function indicates that the representative household neglects the individual contribution of energy consumption to the total energy consumption, which, in turn, does not change the household's optimal decisions.

The representative household chooses the optimal quantities of E_H, Y_H and l in order to maximize the utility U, given the budget constraint:

(4.2) $p_Y Y_H + p_E E_H + w(1-t_w)l = w(1-t_w)\overline{T} + r(1-t_K)\overline{K}$ [53]

Where p_Y and p_E are consumer prices of consumption goods Y_H and E_H, resp., w denotes the wage rate, t_w is the labour income tax, t_K is the tax rate on capital income according to the residence principle and r is the interest rate. Finaly, $\overline{T}$ is total time endowment given exogenously that is used for leisure l and labour supply L^S, so that $\overline{T} - l = L^S$. $\overline{K}$ is exogenous capital endowment.

The household's optimization gives the following demand functions for Y_H, E_H and l [54]:

(4.3) $Y_H = Y_H\left(p_Y, p_E, w(1-t_w), r(1-t_K)\overline{K}\right)$

(4.4) $E_H = E_H\left(p_Y, p_E, w(1-t_w), r(1-t_K)\overline{K}\right)$

(4.5) $l = l\left(p_Y, p_E, w(1-t_w), r(1-t_K)\overline{K}\right)$

The provision of the public good G need not be included in the demand function, as its provision is derived from the tax revenues (see equation 21).

4.1.2 Production Sector

There are three production sectors in the model. The consumer good industry produces the non-energy consumption good Y with the use of three production inputs:

[53] The labour income from total time endowment on the right hand side is not surprising as leisure is perceived as consumption good and stands on the left hand side.

[54] See Appendix B for the derivation of demands and price indexes.

capital K_Y, labour L_Y and energy E_Y. The sectors producing energy and public good use capital K_E, K_G, resp., and labour L_E, L_G, resp. The linear-homogenous production functions are following:

(4.6) $Y = f_Y(K_Y, L_Y, E_Y)$

(4.7) $E = f_E(K_E, L_E)$

(4.8) $G = f_G(K_G, L_G)$

In the economy, taxation of all production factors is possible. The tax rate on the use of energy in the production of Y is denoted t_E^Y, the payroll tax is t_L and the tax rate on capital is t_K.

Furthermore, perfect competition on all goods markets is assumed implying zero economic rent. Hence, the zero-profit conditions for the three production sectors are following:

(4.9) $q_Y Y = w(1+t_L)L_Y + r(1+t_K)K_Y + q_E(1+t_E^Y)E_Y$

(4.10) $q_E E = w(1+t_L)L_E + r(1+t_K)K_E$

(4.11) $q_G G = w(1+t_L)L_G + r(1+t_K)K_G$

where q_Y, q_E, q_G denote the producer prices of goods Y, E and G.

All three sectors minimize costs of given production, which determines following production input demand functions[55]:

(4.12) $L_Y = L_Y(w(1+t_L), r(1+t_K), q_E(1+t_E^Y), Y)$

(4.13) $K_Y = K_Y(w(1+t_L), r(1+t_K), q_E(1+t_E^Y), Y)$

(4.14) $E_Y = E_Y(w(1+t_L), r(1+t_K), q_E(1+t_E^Y), Y)$

(4.15) $L_E = L_E(w(1+t_L), r(1+t_K), q_E(1+t_E^Y), E)$

(4.16) $K_E = K_E(w(1+t_L), r(1+t_K), q_E(1+t_E^Y), E)$

(4.17) $L_G = L_G(w(1+t_L), r(1+t_K), q_E(1+t_E^Y), G)$

(4.18) $K_G = K_G(w(1+t_L), r(1+t_K), q_E(1+t_E^Y), G)$

Moreover, the relationship between consumer and producer price is expressed as:

(4.19) $p_Y = (1+t_C)q_Y$

(4.20) $p_E = (1+t_C)(1+t_E^H)q_E$

[55] See Appendix B for the derivation of demands and price indexes.

Where t_C stands for a uniform consumption tax rate and t_E^H represents the tax on energy consumption by households. The public good remains untaxed.

4.1.3 Public Sector

To keep the public sector balanced, expenditures and revenues have to be equal. Therefore the budget constraint is:

(21) $q_G G = t_C \left[q_Y Y_H + q_E \left(1 + t_E^H\right) E_H \right] + t_w w\left(\overline{T} - l\right) + t_L w\left(L_Y + L_E + L_G\right) + t_K r\overline{K} + t_E^H q_E E_H + t_E^Y q_E E_Y$

The left hand side represents the expenditures in the form of provision of public goods. The right hand side stands for the revenues collected from consumption tax, labour income tax, payroll tax, capital taxe and energy taxes.

4.1.4 Market Equilibrium Conditions

Market equilibrium conditions depend on the characteristics of the economy. If closed economy with full employment is assumed, the conditions are as follows:

(24) $Y = Y_H$

All non-energy consumption good has to be consumed by domestic household as foreign trade is not assumed.

(25) $E = E_H + E_Y$

All energy produced has to be consumed by households or by firms as an input into the production of non-energy consumption good.

(26) $\overline{K} = K_Y + K_E + K_G$

(27) $L^S \equiv \overline{T} - l = L_Y + L_E + L_G \equiv L$

Moreover, all domestic capital has to be used in production. The same applies to labour, as there is no labour mobility.

4.1.5 Functional Forms

The utility and production functions are given as constant-elasticity-of substitution CES functions.

First, utility is a composite which combines leisure with an aggregate consumption good *C*, composed of energy and non-energy good. Hence, there is a weak separability between consumption and leisure. The parameters π_C and π_U represent the elasticity of substitution in consumption, i.e. between energy and non-energy good in the consumption bundle, resp. the elasticity of substitution between aggregate consumption and leisure. Put differently, these elasticities determine the shape of indifference curves in $Y_H - E_H$ space and $C - l$ space. The parameter β is a share parameter, which expresses the proportion of Y_H, E_H and leisure l in the utility function U. The price index P of goods Y_H and E_H is also derived from the structure of the CES function.

Second, CES function is used for production in all three sectors. To produce the non-energy good, capital and labour is combined in a production factor bundle *Q*, which is further combined with energy to yield output *Y*. The production of E and G is a standard use of labour and capital combined in the CES function. The symbol σ stands for elasticity of substitution between production inputs in given sector indicated by the subscript. The share parameter α represents the weight every input has in production.

Finally, the environmental damage function v has no empirical foundation and is designed to perform useful characteristics – linear first derivative and a positive gradient.

Table 5: Functional forms

Utility function	$C = \left[\beta_{YH}^{1/\pi_C} Y_H^{\phi_C} + (1-\beta_{YH})^{1/\pi_C} E_H^{\phi_C}\right]^{1/\phi_C}$
$u(C(Y_H, E_H), l)$	Where $\phi_C = (\pi_C - 1)/\pi_C$
	$u = \left[\beta_C^{1/\pi_u} C^{\phi_u} + (1-\beta_C)^{1/\pi_u} l^{\phi_u}\right]^{1/\phi_u}$
	Where $\phi_u = (\pi_u - 1)/\pi_u$
Consumer goods price index	
$p_C(p_Y, p_E)$	$p_C = \left[p_{YH}^{1-\pi_C}\beta_{YH} + p_{EH}^{1-\pi_C}(1-\beta_{YH})\right]^{\frac{1}{1-\pi_C}}$
Production functions	
$E = f_E(K_E, L_E)$	$E = \left[\alpha_E^{1/\sigma_E} K_E^{\theta_E} + (1-\alpha_E)^{1/\sigma_E} L_E^{\theta_E}\right]^{\frac{1}{\theta_E}}$
	Where $\theta_E = (\sigma_E - 1)/\sigma_E$
$G = f_G(K_G, L_G)$	$G = \left[\alpha_G^{1/\sigma_G} K_G^{\theta_G} + (1-\alpha_G)^{1/\sigma_G} L_G^{\theta_G}\right]^{\frac{1}{\theta_G}}$
	Where $\theta_G = (\sigma_G - 1)/\sigma_G$

$Y = f_Y\left(Q\left(K_Y, L_Y\right), E_Y\right)$	$Q = \left[\alpha_Q^{1/\sigma_Q} K_Q^{\theta_Q} + \left(1-\alpha_Q\right)^{1/\sigma_Q} L_G^{\theta_q}\right]^{\frac{1}{\theta_Q}}$ Where $\theta_Q = \left(\sigma_Q - 1\right)/\sigma_Q$ $Y = \left[\alpha_Y^{1/\sigma_Y} Q_Y^{\theta_Y} + \left(1-\alpha_Y\right)^{1/\sigma_Y} E_Y^{\theta_Y}\right]^{\frac{1}{\theta_Y}}$ Where $\theta_Y = \left(\sigma_Y - 1\right)/\sigma_Y$
Environmental damage	
$\nu(E)$	$\nu = A - \frac{\gamma}{2}E^2$
Exogenous variables	$\overline{K} = 16$ $\overline{G} = 8$ $\overline{T} = 47{,}5$

Source: Böhringer, Ch., Wiegard, W., Starkweather, C., Ruocco, A. (2003), p.88

4.1.6 Optimization

The optimization process is based on six equations of equilibrium with six solution variables (prices, quantities). According to Walras Law[56], it is possible to fix one price as a numeraire and omit one equation. I fix the interest rate r equal to one and omit the capital market equilibrium $\overline{K} = K_Y + K_E + K_G$. The solution variables are w, Y, E, G, u and the related five equilibrium conditions are following.

Labour market equilibrium requires the total labour supply being equal to demands for labour as inputs into production, i.e. $\overline{T} - l = L_Y + L_E + L_G$, where $\overline{T}$ is given exogenously, demand for leisure l is derived from household's utility maximization given its budget constraint and demands for labour are calculated from cost minimization conditions.

To reach energy market equilibrium, production of energy has to be equal to consumption of energy by household and producers, i.e. $E = E_H + E_Y$, where E is

[56] For further reading see Walras, L. (1954), Elements of Pure Economics, Homewood III: Richard Irwin, London: Allen and Unwin

calculated from the production function, E_H comes from household's utility maximization and E_Y from cost minimization condition.

Equilibrium on non-energy consumption market is conditioned by the validity of equation $Y = Y_H$, as no foreign trade is allowed. The amount of Y is determined by the production function and household's utility maximization yields demand for Y_H.

Public good equilibrium is set by the rule, that the value of public good provided depends on the tax revenues, specifically $q_G G = TR$, where TR denotes tax revenues calculated according to equation (4.21) and G is known from the production function.

Finally, the household budget constraint is necessary to complete the optimization, i.e. the equation $p_Y Y_H + p_E E_H + w(1 - t_w)l = r(1 - t_K)\overline{K} + w(1 - t_w)\overline{T}$ needs to hold.

4.1.7 Double Dividend

Moreover, this model allows for the double dividend hypothesis evaluation. The first dividend refers to the improvement of the environment. However, the concrete measurement is a bit problematic and thus two types of the first dividend are calculated. It is the percentage reduction in the overall energy consumption by households and producers marked as *D1(P)* and percentage change in welfare marked as *D1(W)*. The second dividend reflects changes in the costs of raising public revenues, also known as excess burden of the tax system. Under revenue neutrality, the excess burden can be measured as a change in household's income[57]. Hence, the second dividend is positive if financial situation of the representative household improves. Thus, the total change in household's welfare can be split into the first dividend and second dividend, as shown by the following equation:

$$(4.28)\quad \Delta U^j = \underbrace{\left(\frac{u^j - u^0}{u^0} INC^0 \frac{u^0}{U^0} \right)}_{D2} + \underbrace{\left(\frac{v^j - v^0}{v^0} INC^0 \frac{v^0}{U^0} \right)}_{D1}$$

Where *j* denotes the equilibrium after tax reform and *0* refers to the benchmark equilibrium. INC^0 is the benchmark income calculated from the right hand side of equation (4.2).

[57] The cost of a distortion is usually measured as the amount, expressed in monetary terms, which would have to be paid to households affected by it, in order to make them indifferent to the tax reform. This means, that under tax revenue neutrality assumption, the compensation is equal to income changes. See Myles G., Hindricks J.: "Intermediate Public Economics", p.444-447

4.2 Consistency Tests

Consistency tests have to be carried out before simulations so that one can rely on the right specification of the model. Two tax reform scenarios are checked, whose results are theoretically well supported. See Appendix C for exact outcomes.

First, labour income tax is partly offset by newly introduced payroll tax, assuming tax revenue neutrality. If the model is correctly specified, all quantitative variables remain unchanged as it is irrelevant who pays the taxes. Table 7 shows the initial equilibrium where only labour income tax rate 0.3 was assumed. Then, payroll tax rate 0.1 was introduced and labour income tax rate simultaneously decreased to 0.23, so that tax revenue neutrality is kept. All quantitative variables remained unchanged, as proposed by the theory.

Second, labour income tax is replaced by uniform consumption tax, which is levied on non-energy consumption and energy used in production. Therefore the burden of uniform commodity taxation falls on labour and capital simultaneously, and thus works as a labour income tax combined with lump-sum tax. Hence, labour market distortions decline, employment boosts and income increases, which gives rise to higher consumption and production. As a result, more energy is produced, making *D1(P)* positive and *D1(W)* negative. Initially, the labour income tax rate was 0.3 and was fully replaced by uniform consumption tax rate 0.17. Thanks to lower labour market distortions, employment really increases in all production sectors as well as energy production, as demonstrated by Table 8.

4.3 Simulations

Having constructed the CGE model, I carry out several simulations. The aim of these simulations is to find such conditions, under which the double dividend hypothesis holds, and assess the outcomes. I consider the tax revenue recycling through labour income tax t_W and through payroll tax t_L. For both cases, I define four initial systems, which differ in the elasticity of substitution. Moreover, for each system, I specify three initial tax schemes containing various tax rates. Finally, I increase the values of initial tax rates to investigate the dynamics. Table 9 in appendix C is to clarify the structure and states the share parameter values. The same tables apply to the recycling policy through t_L.

In the following parts, I describe the results of the simulations and explain them. Moreover, I compare the same systems under different elasticity assumptions.

4.3.1 Simulation Case "A"[58]

4.3.1.1 Case 1

The existence of the second dividend arises from household's behaviour. As the labour income tax falls, the income increases and households are better off; therefore, they supply more labour. However, the energy tax rate represents a significant mark-up in the price of energy, which motivates households to consume less and substitute energy for the non-energy good. Nonetheless, the elasticity of substitution between energy and non-energy goods is low, causing it more difficult for households to replace one good by another one. As a result, the consumption of the non-energy good grows slightly. Simultaneously, the elasticity of substitution between leisure and consumption is low, and therefore households cannot substitute higher consumption for significant amount of leisure. However,there is a break point, from which the real income declines, as well as consumption and utility. This causes the second dividend to shrink.

From the producer's point of view, larger labour supply reduces wage paid by producers to households, which decreases producer's costs and boosts production of all commodities.

Finally, the first dividend exists as a consequence of household's drop in energy consumption. Although the amount of energy put into production rises, it is offset by lower consumption by households. Hence, the higher energy tax rate, the better prospects for the first dividend.

Furthermore, these simulations have been carried out under three initial tax systems, which differ in the magnitude of the payroll tax. As the payroll tax increases, the second dividend curve becomes steeper, but reaches maximum at the same level of energy tax rate. The explanation lies in the pace of decline of the labour income tax. The higher initial payroll tax, the larger drop in the labour income tax rate and the higher net wage. On the other hand, the first dividend curve rises at lower pace. It is because the decrease in labour income tax is larger with higher payroll tax rate, and therefore household's income

[58] See Appendix C for Graph 7 and Graph 8

and consumption grow. This leads to higher demand for energy from the consumption as well as production side. As a result, the total downturn in the amount of energy becomes smaller.

4.3.1.2 Case 2

In the second case, all substitutions are very elastic. The results demonstrated in simulations are very similar to that ones in the first case. Thanks to higher substitution elasticities in consumption and between consumption and leisure, households can substantially react to all price changes, which gives better prospects for the magnitude of both dividends.

However, different characteristics of the dividend curves are obtained, if the payroll tax rises. The second dividend curve grows faster, but, in contrast to the first case, maximum is reached at higher energy tax rate and at better magnitude. Hence, the more labour put into production is taxed, the higher energy tax rate is optimal for the double dividend existence.

4.3.1.3 Case 3

The third case considers elastic substitution between energy and non-energy consumption, but very low elasticity between consumption and leisure. Even though the first dividend performs considerable values, the second dividend is negatives and further declines. Hence, the double dividend hypothesis does not hold.

The negative double dividend is a result of household's behaviour according to the substitution elasticities. The decrease in labour income tax rate gives rise to the net wage, which, in turn, motivates households to supply more labour. Consequently, the new energy tax stimulates households to purchase more non-energy good to compensate the lower consumption of energy induced by the energy tax. This is possible because of the high elasticity of substitution in consumption. However, as the prospects for substitution of consumption for leisure is not sufficient because of the low elasticity, households do not supply enough labour to fully compensate additional energy expenses (triggered due to the energy tax) by higher income (thanks to higher net wage). Hence, the real income declines with the introduction of the energy tax. As a result, the second dividend is negative and further deteriorates.

On the other hand, the first dividend flourishes. As energy and non-energy consumption can be replaced elastically, the decrease in energy consumption is large and more than offsets the increased usage of energy in production.

Finally, the first dividend curve becomes less steep as the payroll tax increases, while the first dividend curve shifts upwards, but stays negative. The relatively lower first dividend stems from lower reduction in energy in the economy. The improvement in the second dividend results from higher drop in labour income tax rate, which gives rise to net wage and through consumption and income to the second dividend.

4.3.1.4 Case 4

In the last case, households can elastically substitute consumption and leisure, however, replacement of energy by non-energy consumption is very inelastic. This yields positive second dividend and negative first dividend. Thus, the double dividend hypothesis is invalid for this system of substitution.

In fact, the explanations are opposite to that ones in the previous case. As the elasticity of substitution between consumption and leisure is high, households can compensate increased energy expenditures by improved income. Thus utility boosts and the double dividend occurs. However, the first dividend is negative and further declines. This development results from unexpected rise in energy consumption accompanied by higher usage of energy in production. As households supply a lot of labour, their soaring consumption elastically replaces the loss of leisure to keep the utility at least at the same level. Hence, the rise in energy consumption results from the domination of the elasticity effect, which prevents energy consumption to differ significantly from the non-energy consumption, above the price effect, which motivates household to purchase less energy because of its mark-up due to energy tax introduction. This surprising conclusion states, that the first dividend plummets with rising energy tax rate.

Moreover, the higher initial payroll tax, the better second dividend and the less negative first dividend. The reason lies in the pace of reduction in the labour income tax; the higher payroll tax, the higher reduction in the labour income tax, the better net wage and the larger second dividend. In contrast, the amount of total energy rises and the first dividend improves, even though it remains negative.

4.3.2 Simulation Case "B"[59]

4.3.2.1 Case 1

The explanation of the double dividend curves is the same as in the case A. However, if energy as an input into production is taxed instead of labour, the prospects for the validity of the double dividend hypothesis improve. Even though the decrease in labour income tax rate is lower, both dividends achieve better values. Put differently, it is more efficient from the double dividend point of view to introduce tax on energy consumption if the initial tax system consists of energy and labour income taxation, rather than payroll and labour income taxes. Moreover, as the energy tax rate levied on production rises, both dividends reach higher values.

4.3.2.2 Case 2

If all elasticities of substitution are high, the double dividend hypothesis also holds and performs better results than case one. Again, the reason lies in the elastic substitution in consumption and between consumption and leisure, which allows households to react to all price changes more flexibly. Moreover, it yields much better results for the first dividend, as a consequence of decrease in total amount of energy in the economy, and a bit worse results for the second dividend, due to relatively lower real income, compared to the case A. Furthermore, the higher initial energy tax rate imposed on production, the higher levels are reached by the second dividend and the higher optimal energy consumption tax rate becomes. Simultaneously, the first dividend curve increases, too.

4.3.2.3 Case 3

The third case gives considerably different outcomes, compared to the case A. The former negative second dividend now turns to be positive, as the real income rises up to a certain break point. The reason for local positivity of real income lies in inflation: while in the case B real labour income growth can more than compensate real capital income fall, it is not able to yield the same in the case A. Again, as the production energy tax rate increases, the first dividend curve becomes steeper and the second dividend curve reaches better maximum at higher energy tax rate.

[59] See Appendix C for Graph 9 and Graph 10.

4.3.2.4 Case 4

The last case performs different development than in the case A. Now, both dividend curves are positive, especially the first dividend, which is increasing in the energy tax rate. Compared to the case A, households are able to compensate higher use of energy in production by their declining energy consumption, co that the total amount falls. However, as the initial energy tax rate levied on producers increase, the slope of the first dividend curve lowers, but the second dividend curve becomes steeper and reaches higher maximum.

4.4 Theory Tests

Except for the simulations, I try to test some conclusions derived in the theoretical models in chapters 2 and 3. First, it is the model by Fuest and Huber, who suggest, that the environmental tax levied on the polluting consumption good increases with the degree of substitutability of this good with clean consumption good, provided the tax rate on clean consumption commodity is positive. Second, I test the validity of Metcalf's model. He shows, that in an economy with multiple distortions, the weak double dividend does not have to hold, so it might be more efficient to recycle additional tax revenues through transfers rather than through cuts in existing distortionary taxes.

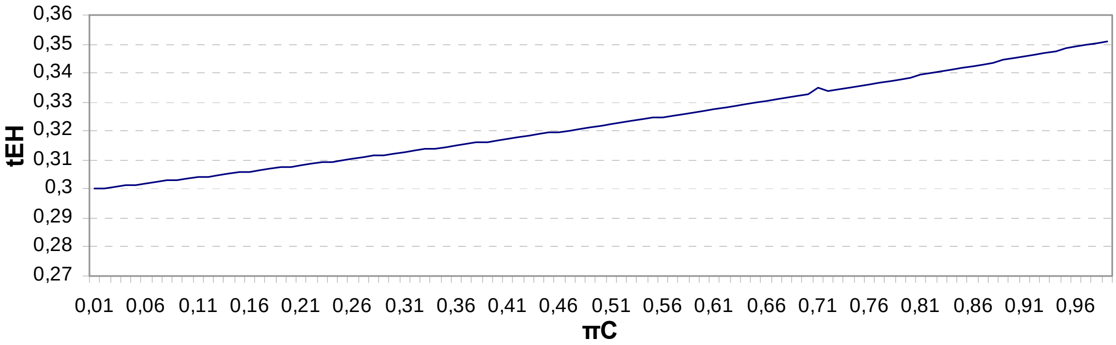
4.4.1 Fuest and Huber 1999

Fuest and Huber present a model, where households derive utility from consumption of leisure, clean commodity (Y_H), polluting commodity (E_H), public goods and environmental quality, and maximize it over optimal quantities of leisure and consumption. This description coincides with my CGE model. Moreover, Fuest and Huber consider a production function with labour as the only input. As the CGE model includes capital and energy as another factor of production, I select the parameter a_Q and a_Y equal to 0.01, to minimize the influence of additional inputs into production. Under the commodity tax system, there is a positive tax on clean commodity and no taxes on labour.

The tax rate 0.3 is introduced on energy consumption and the elasticity of substitution between consumption goods increases from 0.01 to 0.99. Then, I observe how the energy tax rate develops, keeping the tax revenues constant. According to the graph, the tax rate increases with the elasticity of substitution between consumption goods.

The intuition behind is following. If the two goods are substitutes, people tend to substitute the taxed good for the less taxed one, so in fact they avoid paying more taxes and therefore the tax rate on the polluting commodity must be higher to keep the same level of tax revenues. Alternatively, if the two goods are complements, people cannot replace one good by another and the tax rate on the dirty good is lower.

Graph 4: Tested model by Fuest and Huber



Source: Own calculations

4.4.2 Metcalf 2002

Another model I test is Metcalf's weak double dividend model, in which he shows that the weak double dividend does not have to hold in an economy with multiple distortions. In the model, there are two consumption goods and one of them creates pollution, which is considered as an input into production. This structure is compatible with my CGE model, if the two consumption goods are understood as energy and non-energy consumption and pollution as energy used in production. Furthermore, production factors are labour and pollution. Therefore I neglect the input of capital into production by selecting a_Q equal to 0.01. Energy as an input into production, (pollution, resp.), is taxed by 10%, while the two consumption goods are taxed at various rates, as indicated by the Table 6. Moreover, elasticity values are introduced as stated by Metcalf; elasticity of substitution between energy and non-energy consumption goods is 0.99, elasticity of substitution between inputs capital-labour composite and energy used in production of non-energy good is set 0.99. Other parameters remain unchanged.

However, some modifications of the CGE model are necessary to carry out this simulation. First, the consumer price of non-energy consumption good does not include the uniform commodity taxation and so becomes $p_E = (1+t_E^H)q_E$, instead of $p_E = (1+t_C)(1+t_E^H)q_E$. Second, the budget constraint of the public sector changes to

$$q_G G = t_C q_Y Y_H + t_w w(\overline{T} - l) + t_L w(L_Y + L_E + L_G) + t_K r\overline{K} + t_E^H q_E E_H + t_E^Y q_E E_Y$$

Metcalf's model is based on comparison of utility in two situations – when additional revenues are recycled through transfers and when through cuts in pre-existing distortionary tax on non-energy consumption good. The weak double dividend occurs if the welfare change is positive.

Now, I introduce taxes on energy consumed by households and energy used in production and calculate welfare changes for both types of substitution. The arising outcome is a subtraction of utility under transfer-recycling from utility under the recycling though tax rate, as depicted in Table 6.

Table 6: Weak double dividend tested under multiple distortions – welfare change (%)

		tEH			
		0.1	0.2	0.3	0.4
tC	0.1	0.07	-0.19	-0.33	-0.4
	0.2	0.24	-0.11	-0.29	-0.38
	0.3	0.41	-0.02	-0.24	-0.36
	0.4	0.6	0.08	-0.19	-0.33

Source: Own calculations

As indicated by Table 6, the weak double dividend exists only if the tax levied on non-energy consumption good is significantly higher than the tax levied on energy consumption. Contrary to this, the weak form does not hold and it is more efficient for the economy to return additional proceeds through transfer payments. To sum up, these results perfectly coincide with Metcalf's conclusions and claim the following: "If the environmental tax revenue is used to reduce the differential between two distorting taxes then a weak double dividend occurs. If the recycling increases the difference between existing distortions then recycling is welfare worsening compared with lump sum recycling even if it reduces an existing distortion."[60]

[60] Metcalf, G.E., Babiker, M.H., Reilly, J. (2002), p. 12

4.5 Conclusion

This chapter describes results of a CGE model that is used to assess outcomes of various tax reforms and points out such initial conditions, which give prospects for the validity of the double dividend hypothesis. Moreover, two theoretical models were tested.

If the initial tax system consists of a payroll tax and labour income tax, with revenues from environmental tax levied on household's consumption of energy recycled though cuts in the labour income tax, it is best to have all elasticities of substitution high, followed by the case, where the elasticity of substitution in the consumption bundle and between consumption and leisure are low, while the other elasticities remain high. This stems from the possibility to react to all changes sensitively, and therefore alleviates labour market distortions. In the second case, households are forced to keep their behaviour patterns due to low elasticities of substitution. As a result, both dividends achieve much lower magnitudes, but remain positive.

If the initial tax system consists of energy tax paid by producers and labour income tax, through which tax revenues are recycled, the double divided exists in all cases. However, the initial system with all elasticities of substitution high is still the best one. The reason lies in the taxation of inputs. If payroll taxes are substituted by energy taxes levied on production, the labour market distortions are reduced and the validity of the first dividend supported, as producers try to substitute relatively more expensive energy for other inputs.

Furthermore, I tested the theoretical results of the model by Fuest and Huber and by Metcalf, using the CGE model. The results obtained correspond to the theory. As Fuest and Huber, I come to the conclusion that the environmental tax levied on polluting commodity increases with the elasticity of substitution between polluting and non-polluting commodity. Moreover, as shown by Metcalf, I also prove that the weak form of the double dividend does not need to hold if it increases the difference between tax rates in the economy and therefore moves the system far from optimality.

5 Conclusion

"The art of taxation consists in so plucking the goose as to obtain the largest possible amount of feathers with the smallest possible amount of hissing."

- Jean Baptiste Colbert

To sum up, the purpose of this study was to examine the area of environmental tax reform. First, the optimal environmental tax was presented using the approach of Pigu and Ramsey and model by Fuest and Huber. The main conclusion is that the optimal environmental tax consists of the externality-correcting and revenue-raising term and the optimal level is determined by the degree of substitution or complementarity between polluting and non-polluting commodity and between polluting good and leisure, depending on the normalization chosen.

Second, the theory of the double dividend was outlined distinguishing weak and strong form. The results reveal that the kind of relationship between commodities themselves and between polluting commodity and leisure is crucial for the validity of the strong double dividend. If the polluting commodity is sufficiently weak substitute to leisure, the strong double dividend holds.

Furthermore, these results conform to the theory of optimal taxation. It is beneficial from the efficiency point of view to tax goods that are more complementary to leisure than others, as it moves the tax system closer to optimality. Hence, existence of the second dividend stems from the improved efficiency.

Third, a simple computable general equilibrium model was used to examine results of several tax reforms. There are two implications of this research. First, it is better to tax energy as an input into production instead as labour as in input, as it alleviates labour market distortions and gives rise to the first dividend. Second, the environmental tax reform achieves best outcomes, if all elasticities of substitution in the economy are high and therefore households and producers can react to all changes with a substantial sensitivity.

Lastly, results obtained in the CGE model simulations cannot be compared to those derived in the theoretical models, as both differ in an important assumption. While the theoretical double dividend model assumes that each consumption good has a different degree or substitution or complementarity to leisure, the CGE model assumes weak separability between leisure and consumption.

6 APPENDIX A

A.1 Sandmo 1975

Derivation of equation (4) from eq. (2), if independent demands $\partial C_i / \partial p_k = 0$ are assumed:

$$(2)\,\theta_k = (1-\eta)\left[-\frac{1}{p_k}\frac{\sum_{i=1}^{m-1} C_i J_{ik} + DJ_{mk}}{J}\right] \text{ for } k \neq m$$

$$(4)\,\theta_k = (1-\eta)\left(-\frac{1}{\varepsilon_{C_k C_k}}\right)$$

The initial Jacobian matrix is:

$$J^* = \begin{pmatrix} \frac{\partial C_1}{\partial P_{C1}} & \cdots & \frac{\partial D}{\partial P_{C1}} \\ \vdots & \ddots & \vdots \\ \frac{\partial C_1}{\partial p_D} & \cdots & \frac{\partial D}{\partial p_D} \end{pmatrix}$$

If independent demands are assumed, Jacobian matrix changes to:

$$J^* = \begin{pmatrix} \frac{\partial C_1}{\partial p_{C1}} & 0 & \cdots & 0 \\ 0 & \frac{\partial C_2}{\partial p_{C2}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{\partial D}{\partial p_D} \end{pmatrix}$$

Hence, the expression $\sum_{i=1}^{m-1} C_i J_{ik} + DJ_{mk}$ is calculated in the following way:

$$\sum_{i=1}^{m-1} C_i J_{ik} + DJ_{mk} = C_1 \begin{vmatrix} \frac{\partial C_2}{\partial p_{C2}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{\partial D}{\partial p_D} \end{vmatrix} + C_2 \begin{vmatrix} \frac{\partial C_1}{\partial p_{C1}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{\partial D}{\partial p_D} \end{vmatrix} + \cdots + D \begin{vmatrix} \frac{\partial C_1}{\partial p_{C1}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{\partial C_{m-1}}{\partial p_{Cm-1}} \end{vmatrix}$$

To proceed, I use the following definition and sentence[61]:

[61] Hájková, V., John, O., Kalenda, O.F.K., Zelený, M., Matematika, MATFYZPRESS, Praha 2006

Definition:

Let $A \in M(n \times n)$. *We say that A is upper triangular matrix, if* $a_{ij} = 0$ *for*

$i > j; i, j \in \{1, \cdots, n\}$.

Sentence:

Let $A \in M(n \times n)$ *be upper triangular matrix. Then* $\det A = a_{11} a_{22} \cdots a_{nn}$.

The matrix *J** fulfills the assumptions of the definition, thus using the sentence yields

$$J = \frac{\partial C_1}{\partial p_1} \cdot \frac{\partial C_2}{\partial p_2} \cdot \ldots \cdot \frac{\partial D}{\partial p_D}$$

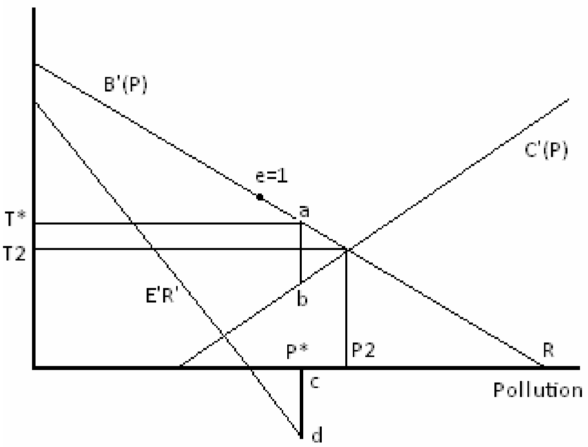
Now, consider only the commodity one. Then the optimal tax imposed on the commodity one is following:

$$\theta_1 = (1-\eta)\left[-\frac{1}{p_{C1}} \frac{C_1 J_{1k}}{J}\right] = (1-\eta)\left[-\frac{1}{p_{C1}} \frac{C_1 \cdot \frac{\partial C_2}{\partial p_{C2}} \cdot \frac{\partial C_3}{\partial p_{C3}} \cdot \ldots \cdot \frac{\partial D}{\partial p_D}}{\frac{\partial C_1}{\partial p_{C1}} \cdot \frac{\partial C_2}{\partial p_{C2}} \cdot \frac{\partial C_3}{\partial p_{C3}} \cdot \ldots \cdot \frac{\partial D}{\partial p_D}}\right] = (1-\eta)\left[-\frac{1}{p_{C1}} \cdot \frac{C_1}{\frac{\partial C_1}{\partial p_{C1}}}\right] = (1-\eta)\left[-\frac{1}{\varepsilon_{C_1 C_1}}\right]$$

The computation is identical for other commodities.

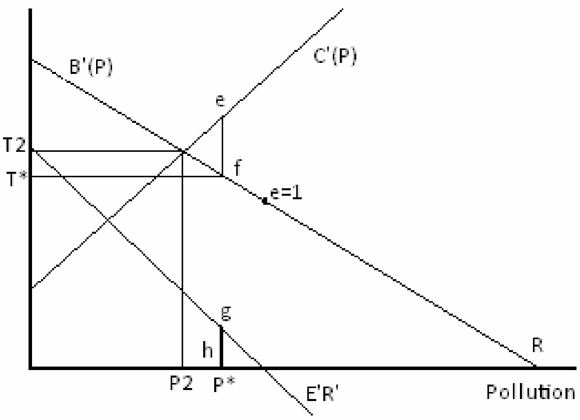
A.2 Terkla 1984, Lee and Misiolek 1986

Graph 5: Elasticity of pollution demand is less than one



Source: Lee, D.R., Misiolek, W.S., (1986), p.342

Graph 6: Elasticity of pollution demand is above one



Source: Lee, D.R., Misiolek, W.S., (1986), p.342

A.3 Bovenberg and van der Ploeg 1994

The material balance of the economy is given by:

(A.1) $\omega NL = \omega(1 - V) = \omega_C NC + \omega_D ND + \omega_X X + \omega_Y Y + \omega_A A$

Where ω denotes producer wage and subscript indicates the sector. Under the social planner's system, the aim is to maximize household's utility subject to the material balance condition, which yields:

(A.2) $u_C = \omega^{-1}u_V = Nu_X = u_D + Nu_E e_{ND} = N(u_Y + u_E e_Y) = Nu_E e_A$

Moreover, the household's budget constraint is

(A.3) $p_C C + p_D D = (1 - t_L)\omega L + T$

Where T denotes transfers received from the government. Maximizing household's utility over its budget constraint gives following optimality conditions:

(A.4) $\dfrac{u_C}{p_C} = \dfrac{u_D}{p_D} = \dfrac{u_V}{(1 - t_L)\omega} = \lambda$

To achieve the first best outcome characterized by (A.2), government levies zero tax rate on clean consumption and on labour and sets the dirty tax rate equal to:

(A.5) $t_{DP} = \dfrac{-Nu_E e_{ND}}{u_D + Nu_E e_{ND}} = \dfrac{-Nu_E e_{ND}}{u_C} > 0$

This is the Pigouvian tax.

Finally, the government determines the amount of transfers, so that the following condition holds:

(A.6) $NT = t_D ND - X - Y - A$

If a given amount of public expenditures has to be financed by distortionary taxes regardless of externalities, the following conditions have to hold:

(A.7) $\lambda D - \mu\left[D + t_D\left(\dfrac{\partial D}{\partial p_D}\right) + t_L\omega\left(\dfrac{\partial L}{\partial p_D}\right)\right] = 0$

(A.8) $\lambda L - \mu\left[L + t_D\left(\dfrac{\partial D}{\partial p_L}\right) + t_L\omega\left(\dfrac{\partial L}{\partial p_L}\right)\right] = 0$

Using Slutsky decomposition $(\partial I/\partial p_D) = S_{iD} - (\partial I/\partial T)D$ and $(\partial I/\partial p_L) = S_{iL} - (\partial I/\partial T)L$, where S represents Slutsky matrix, gives:

(A.9) $\begin{pmatrix} t_D \\ \omega t_L \end{pmatrix} = -\left(\dfrac{\mu - \lambda'}{\mu}\right)\begin{pmatrix} S_{DD} & S_{LD} \\ S_{DL} & S_{LL} \end{pmatrix}^{-1}\begin{pmatrix} D \\ -L \end{pmatrix}$

Where $\lambda' = \lambda + \mu\left(t_D\dfrac{\partial D}{\partial T} + \omega t_L\dfrac{\partial L}{\partial T}\right)$

Defining $\varepsilon_{ik} = p_k S_{ik} / I$ as compensated elasticity of demand for commodity i (I) with respect to the price of commodity k and using Slutsky symmetry $\varepsilon_{LD} = -\alpha_D \varepsilon_{DL}$, equation (A.9) can be rewritten as

$$\text{(A.10)}\quad \begin{pmatrix} \theta_D \\ \theta_L \end{pmatrix} = \begin{pmatrix} \dfrac{t_D}{p_D} \\ \dfrac{t_L}{\omega} \end{pmatrix} = \left(\frac{\mu - \lambda'}{\mu} \right) \begin{pmatrix} \varepsilon_{DD} & -\varepsilon_{DL} \\ -\varepsilon_{LD} & \varepsilon_{LL} \end{pmatrix}^{-1} \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

Which yields the following expressions for θ_D and θ_L:

$$\text{(A.11)}\quad \theta_L = \left(\frac{t_L}{1 + t_L} \right) = \left(\frac{\varepsilon_{LD} - \varepsilon_{DD}}{-\varepsilon_{DD}\varepsilon_{LL} + \varepsilon_{LD}\varepsilon_{DL}} \right) \left(\frac{\mu - \lambda'}{\mu} \right)$$

$$\text{(A.12)}\quad \theta_D = \left(\frac{t_D}{1 + t_D} \right) = \left(\frac{\varepsilon_{CL} - \varepsilon_{DL}}{\varepsilon_{CD} - \varepsilon_{DD}} \right) \theta_L$$

For the derivation of θ_D following expressions were used: $\varepsilon_{iL} = -(\varepsilon_{iC} + \varepsilon_{iD})$,

$\varepsilon_{LD} = -\alpha_D \varepsilon_{DL}$ and $\varepsilon_{LC} = -(1 - \alpha_D)\varepsilon_{CL}$.

Finally, if there is a need for tax revenues to finance public spending and if externalities are taken into account, equation (A.7) becomes

$$\text{(A.13)}\quad \lambda D - \mu \left[D + t_D \left(\frac{\partial D}{\partial p_D} \right) + t_L \omega \left(\frac{\partial L}{\partial p_D} \right) \right] - N u_E' e_{ND} \frac{\partial D}{\partial p_D} = 0$$

Where $u_E' = \left(\dfrac{u_E + \mu \left[t_D \dfrac{\partial D}{\partial E} + \omega t_L \dfrac{\partial L}{\partial E} \right]}{1 - N e_{ND} \dfrac{\partial D}{\partial E}} \right)$ denotes the marginal social utility of the environment. If the externality correcting term is defined as

$$\text{(A.14)}\quad t_{DP} = \left(\frac{-N e_{ND} u_E'}{u_C} \right) \left(\frac{1}{\eta} \right) = \left(\frac{-N e_{ND}}{1 - N e_{ND} \dfrac{\partial D}{\partial E}} \right) \left(\frac{u_E}{\mu} + t_D \frac{\partial D}{\partial E} + \omega t_L \frac{\partial L}{\partial E} \right)$$

Equations (A.7) and (A.8) become

$$\text{(A.15)}\quad \lambda D - \mu \left[D + (t_D - t_{DP}) \left(\frac{\partial D}{\partial p_D} \right) + t_L \omega \left(\frac{\partial L}{\partial p_D} \right) \right] = 0$$

$$\text{(A.16)}\quad \lambda L - \mu \left[L + (t_D - t_{DP}) \left(\frac{\partial D}{\partial p_L} \right) + t_L \omega \left(\frac{\partial L}{\partial p_L} \right) \right] = 0$$

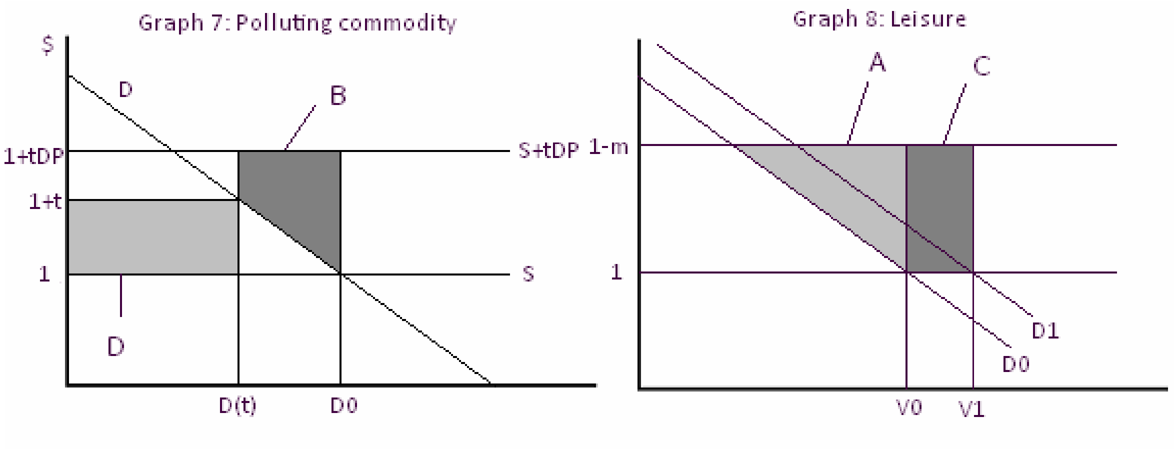
As the environmental tax rate is a composite of Ramsey and Pigouvian term, its expression, in general, is following:

(A.17) $\theta_D = \left(\frac{\varepsilon_{CL} - \varepsilon_{DL}}{\varepsilon_{CD} - \varepsilon_{DD}} \right) \theta_L + \theta_{DP}$, where $\theta_{DP} = \left(\frac{t_{DP}}{1 + t_D} \right)$

However, in this particular case, the expression of θ_D is based on eq. (A.17), (A.14) and (A.11)

(A.18) $\theta_D = \left(1 - \eta'^{-1}\right) \left(\frac{(\varepsilon_{CL} - \varepsilon_{DL})(1 - \alpha_D)}{-\varepsilon_{DD}\varepsilon_{LL} + \varepsilon_{DL}\varepsilon_{LD}} \right) + \eta^{-1} \left(\frac{-Ne_{ND}u'_E}{(1 + t_D)u_C} \right)$, where $\eta = \frac{\mu}{\lambda}; \eta' = \frac{\mu}{\lambda'}$

A.4 Parry 1995



Source: Parry, I. W.H., (1994), p.S-67

Source: Parry, I. W.H., (1994), p.S-67

The optimal environmental tax is a result of tax-interaction effect (IE) and revenue-recycling effect (RE). The IE stems from labour market distortions marked as area C calculated as:

(A.19) $C = t_L(V_1 - V_0) = t_L \frac{\partial V}{\partial p_D} t_D$

Moreover, there is another effect, which is defined as efficiency value per dollar of changes in aggregate tax revenues and is denoted Z. Using Slutsky symmetry $\partial V / \partial p_D = \partial D / \partial \omega$ and incorporating Z, the IE becomes:

(A.20) $IE = (1 + Z) \frac{t_L}{1 - t_L} t_D D^0 \varepsilon_{DL}$

Where $\varepsilon_{DL} = (\partial D/\partial \omega)(1-t_L)/D^0$ is compensated elasticity of demand for D with respect to the price of leisure. On the other hand, RE is a result of area D and effect of Z and is defined as:

(A.21) $RE = Zt_D D(t)$

If pollution tax revenues are used to cut labour tax rate, the proper definition of Z is marginal welfare cost of labour tax revenues. Hence, the formula for Z is following:

$$(A.21)\ Z = \frac{\frac{t_L}{1-t_L}\varepsilon_{LL}}{1-\frac{t_L}{1-t_L}\varepsilon_{LL}}$$

Therefore, expression (A.20) can be rewritten as

$$(A.22)\ IE = Zt_D D^0 \frac{\varepsilon_{DL}}{\varepsilon_{LL}}$$

However, marginal values of IE and RE are crucial for the optimal environmental tax rate. Differentiating eq. (A.22) and eq. (A.21) with respect to the reduction in D yields

$$(A.23)\ MC_{IE} = \frac{Z}{\varepsilon_{DD}}\frac{\varepsilon_{DL}}{\varepsilon_{LL}}$$

$$(A.24)\ MB_{RE} = Z\left\{\frac{1}{\varepsilon_{DD}}\frac{D(t)}{D^0} - t_D\right\}$$

Solving the optimal tax t^* from expression $t_{DP} = t^* + MC_{IE} - MB_{RE}$ using eq. (A.23) and (A.24) gives

$$(A.25)\ \bar{t}^* = \frac{1+\left(\frac{Z}{t_{DP}\varepsilon_{DD}}\right)\left(1-\frac{\varepsilon_{DL}}{\varepsilon_{LL}}\right)}{1+2Z}$$

Where $\bar{t}^* = t^*/t_{DP}$ denotes the optimal environmental tax a proportion of the Pigouvian tax. Moreover, if D is an average substitute for leisure, $\varepsilon_{DL} = \varepsilon_{LL}$ and eq. (A.25) simplifies to

$$(A.26)\ \bar{t}^* = \frac{1}{1+2Z}$$

7 APPENDIX B

Production sector E

The production function in the energy sector is:

$$E = \left[\alpha_E^{1/\sigma_E} K_E^{\theta_E} + (1-\alpha_E)^{1/\sigma_E} L_E^{\theta_E}\right]^{\frac{1}{\theta_E}}$$

$$\theta_E = (\sigma_E - 1)/\sigma_E$$

The marginal product of K_E is defined as a derivative of E according to K_E:

$$\frac{\partial E}{\partial K_E} = \frac{\partial \left[\alpha_E^{1/\sigma_E} K_E^{\theta_E} + (1-\alpha_E)^{1/\sigma_E} L_E^{\theta_E}\right]^{\frac{1}{\theta_E}}}{\partial K_E} = \frac{1}{\theta_E}\left[\alpha_E^{1/\sigma_E} K_E^{\theta_E} + (1-\alpha_E)^{1/\sigma_E} L_E^{\theta_E}\right]^{\frac{1-\theta_E}{\theta_E}} \alpha_E^{1/\sigma_E} \theta_E K_E^{\theta_E - 1} =$$

$$= \left[\alpha_E^{1/\sigma_E} K_E^{\theta_E} + (1-\alpha_E)^{1/\sigma_E} L_E^{\theta_E}\right]^{\frac{1}{\sigma_E - 1}} \alpha_E^{1/\sigma_E} K_E^{-1/\sigma_E} = E^{1/\sigma_E} \alpha_E^{1/\sigma_E} K_E^{-1/\sigma_E} = \left(\alpha_E \frac{E}{K_E}\right)^{\frac{1}{\sigma_E}}$$

The production function is concave and is considered within a perfect competition framework, therefore the value of the marginal product has to be equal to its price:

$$q_E \left(\alpha_E \frac{E}{K_E}\right)^{\frac{1}{\sigma_E}} = p_K$$

Hence, the demand for capital is given by:

$$K_E = \left(\frac{q_E}{p_K}\right)^{\sigma_E} \alpha_E E$$

The demand for labour is derived similarly:

$$L_E = \left(\frac{q_E}{w}\right)^{\sigma_E} (1-\alpha_E) E$$

The producer price of energy is given from the zero profit condition, given constant returns to scale production function and perfect competition framework:

$$q_E E - p_K K_E - w L_E = 0$$

Using the expressions of demand for K_E and L_E:

$$q_E E - p_K \left(\frac{q_E}{p_K}\right)^{\sigma_E} \alpha_E E - w \left(\frac{q_E}{w}\right)^{\sigma_E} (1-\alpha_E) E = 0$$

$$q_E - p_K \left(\frac{q_E}{p_K}\right)^{\sigma_E} \alpha_E - w \left(\frac{q_E}{w}\right)^{\sigma_E} (1-\alpha_E) = 0$$

$$q_E - q_E^{\sigma_E} p_K^{1-\sigma_E} \alpha_E - q_E^{\sigma_E} w^{1-\sigma_E} (1-\alpha_E) = 0$$

$$q_E^{1-\sigma_E} = p_K^{1-\sigma_E}\alpha_E + w^{1-\sigma_E}(1-\alpha_E)$$

$$q_E = \left[p_K^{1-\sigma_E}\alpha_E + w^{1-\sigma_E}(1-\alpha_E)\right]^{\frac{1}{1-\sigma_E}}$$

Production sector G

The production function in the public good sector is:

$$G = \left[\alpha_G^{1/\sigma_G} K_G^{\theta_G} + (1-\alpha_G)^{1/\sigma_G} L_G^{\theta_G}\right]^{\frac{1}{\theta_G}}$$

$$\theta_G = (\sigma_G - 1)/\sigma_G$$

Therefore, the derivation of the demand functions for inputs is the same as in the production sector E.

$$K_G = \left(\frac{q_G}{p_K}\right)^{\sigma_G} \alpha_G G$$

$$L_G = \left(\frac{q_G}{w}\right)^{\sigma_G} (1-\alpha_G)G$$

Similarly, the producer price of public good is

$$q_G = \left[p_K^{1-\sigma_G}\alpha_G + w^{1-\sigma_G}(1-\alpha_G)\right]^{\frac{1}{1-\sigma_G}}$$

Production sector Q

The production function in the sector Q is:

$$Q = \left[\alpha_Q^{1/\sigma_Q} K_Q^{\theta_Q} + (1-\alpha_Q)^{1/\sigma_Q} L_G^{\theta_q}\right]^{\frac{1}{\theta_Q}}$$

$$\theta_Q = (\sigma_Q - 1)/\sigma_Q$$

Therefore, the demand functions for inputs are:

$$K_Q = \left(\frac{q_Q}{p_K}\right)^{\sigma_Q} \alpha_Q Q$$

$$L_Q = \left(\frac{q_Q}{w}\right)^{\sigma_Q} (1-\alpha_Q)Q$$

And the producer price of the good Q is:

$$q_Q = \left[p_K^{1-\sigma_Q}\alpha_Q + w^{1-\sigma_Q}(1-\alpha_Q)\right]^{\frac{1}{1-\sigma_Q}}$$

Production sector Y

The production function the the sector Y is:

$$Y = \left[\alpha_Y^{1/\sigma_Y} Q_Y^{\theta_Y} + (1-\alpha_Y)^{1/\sigma_Y} E_Y^{\theta_Y}\right]^{\frac{1}{\theta_Y}}$$

$$\theta_Y = (\sigma_Y - 1)/\sigma_Y$$

With following demand functions for inputs:

$$Q_Y = \left(\frac{q_Y}{p_Q}\right)^{\sigma_Y} \alpha_Y Y$$

$$E_Y = \left(\frac{q_Y}{q_E}\right)^{\sigma_Y} (1-\alpha_Y)Y$$

Hence, the expression for the producer price of the non-energy consumption good Y is:

$$q_Y = \left[p_Q^{1-\sigma_Y}\alpha_Y + q_E^{1-\sigma_Y}(1-\alpha_Y)\right]^{\frac{1}{1-\sigma_Y}}$$

Composite consumption C

The equation for the composite consumption is following:

$$C = \left[\beta_{YH}^{1/\pi_C} Y_H^{\phi_C} + (1-\beta_{YH})^{1/\pi_C} E_H^{\phi_C}\right]^{1/\phi_C}$$

$$\phi_C = (\pi_C - 1)/\pi_C$$

Therefore, the demand functions are derived in the same way as in the case of production:

$$Y_H = \left(\frac{p_C}{p_{YH}}\right)^{\pi_C} \beta_{YH} C$$

$$E_H = \left(\frac{p_C}{p_{EH}}\right)^{\pi_C} (1-\beta_{YH})C$$

The price index of the consumption composite is expressed as

$$p_C = \left[p_{YH}^{1-\pi_C}\beta_{YH} + p_{EH}^{1-\pi_C}(1-\beta_{YH})\right]^{\frac{1}{1-\pi_C}}$$

Utility

The utility function is defined as:

$$u = \left[\beta_C^{1/\pi_u} C^{\phi_u} + (1-\beta_C)^{1/\pi_u} l^{\phi_u}\right]^{1/\phi_u}$$

$$\phi_u = (\pi_u - 1)/\pi_u$$

With the following demand functions for consumption composite and leisure:

$$C = \left(\frac{p_u}{p_C} \right)^{\pi_u} \beta_C u$$

$$l = \left(\frac{p_u}{w} \right)^{\pi_u} (1 - \beta_C) u$$

The price index of the utility is expressed as

$$p_u = \left[p_C^{1-\pi_u} \beta_C + w^{1-\pi_u} (1 - \beta_C) \right]^{\frac{1}{1-\pi_u}} .$$

8 APPENDIX C

Table 7: Consistency Test 1

t_L	0	0.1
t_W	0.3	0.23
Y	13.591	13.591
E	20.391	20.391
G	2.262	2.262
K_e	11.111	11.111
L_e	9.373	9.373
K_g	1.233	1.233
L_g	1.040	1.040
E_y	6.797	6.797
QQ	6.794	6.794
K_y	3.656	3.656
L_y	3.164	3.164
leisure l	33.924	33.924
cc	27.185	27.185
Y_H	13.591	13.591
E_H	13.594	13.594

Source: Own calculations

Table 8: Consistency Test 2

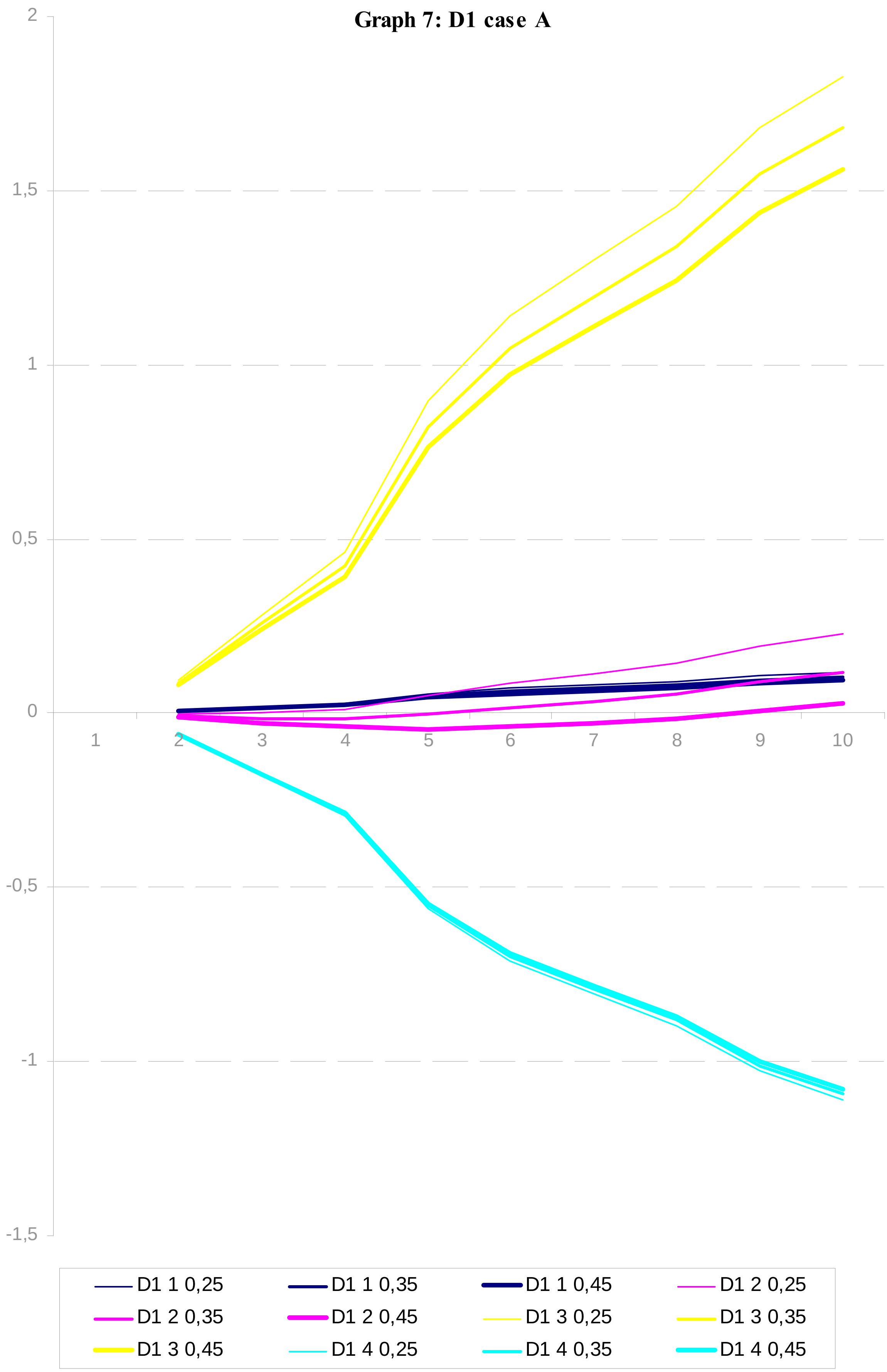
t_C	0	0.17
t_w	0.3	0
L_e	9.373	10.553
L_g	1.040	1.193
L_y	3.164	3.530
E	20.391	21.616
D1		-2.561

Source: Own calculations

Table 9: System of simulation

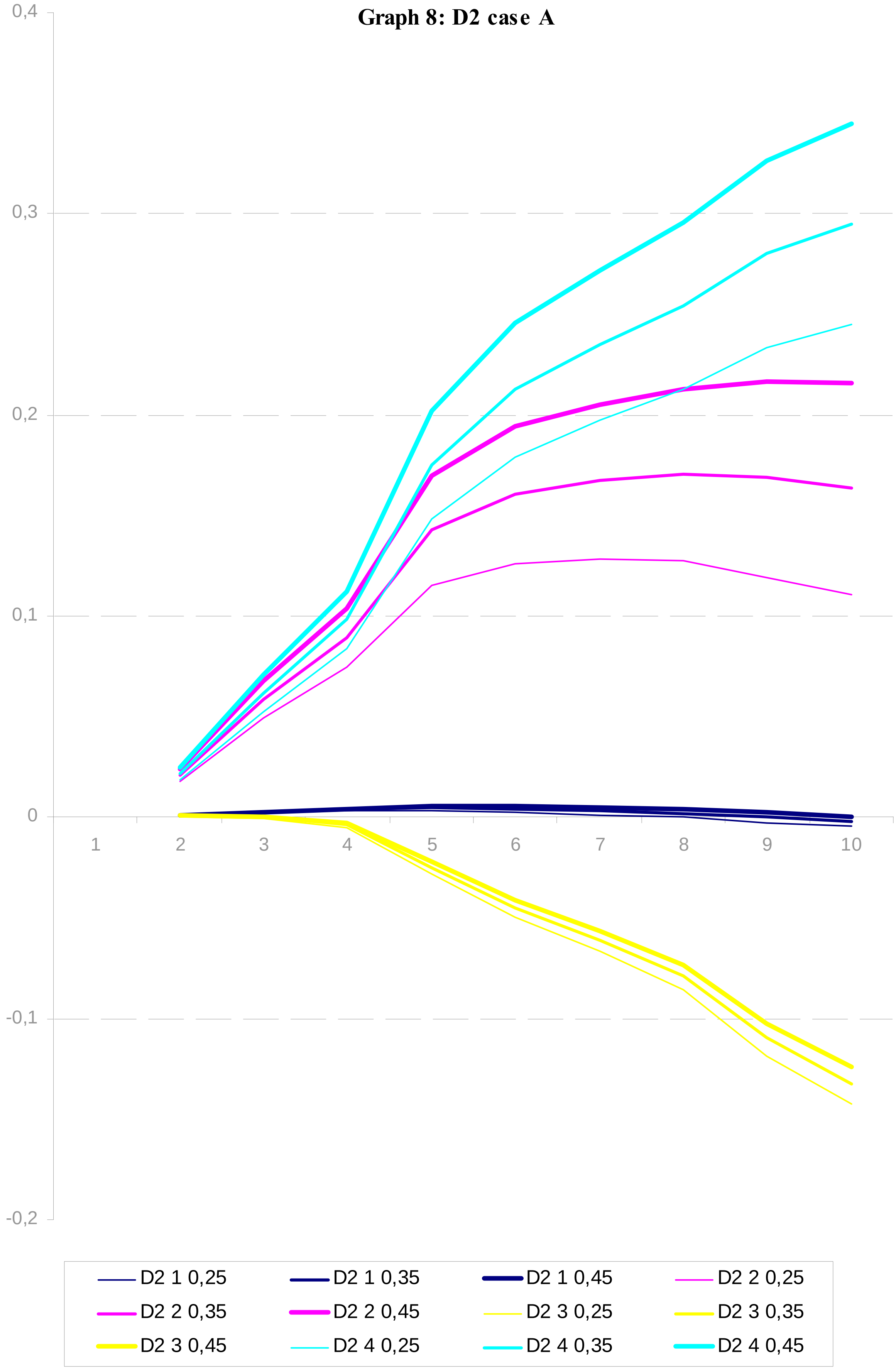
Case	π_C	π_U	σ	t_W	t_L	t_{EY}	t_{EH}
1A	0.1	0.05	0.8	0.3	0.25	0	0 to 0.2
	0.1	0.05	0.8	0.3	0.35	0	0 to 0.2
	0.1	0.05	0.8	0.3	0.45	0	0 to 0.2
1B	0.1	0.05	0.8	0.3	0	0.25	0 to 0.2
	0.1	0.05	0.8	0.3	0	0.35	0 to 0.2
	0.1	0.05	0.8	0.3	0	0.45	0 to 0.2
2A	0.95	0.95	0.95	0.3	0.25	0	0 to 0.2
	0.95	0.95	0.95	0.3	0.35	0	0 to 0.2
	0.95	0.95	0.95	0.3	0.45	0	0 to 0.2
2B	0.95	0.95	0.95	0.3	0	0.25	0 to 0.2
	0.95	0.95	0.95	0.3	0	0.35	0 to 0.2
	0.95	0.95	0.95	0.3	0	0.45	0 to 0.2
3A	0.95	0.05	0.8	0.3	0.25	0	0 to 0.2
	0.95	0.05	0.8	0.3	0.35	0	0 to 0.2
	0.95	0.05	0.8	0.3	0.45	0	0 to 0.2
3B	0.95	0.05	0.8	0.3	0	0.25	0 to 0.2
	0.95	0.05	0.8	0.3	0	0.35	0 to 0.2
	0.95	0.05	0.8	0.3	0	0.45	0 to 0.2
4A	0.1	0.95	0.8	0.3	0.25	0	0 to 0.2
	0.1	0.95	0.8	0.3	0.35	0	0 to 0.2
	0.1	0.95	0.8	0.3	0.45	0	0 to 0.2
4B	0.1	0.95	0.8	0.3	0	0.25	0 to 0.2
	0.1	0.95	0.8	0.3	0	0.35	0 to 0.2
	0.1	0.95	0.8	0.3	0	0.45	0 to 0.2

Source: Own calculations

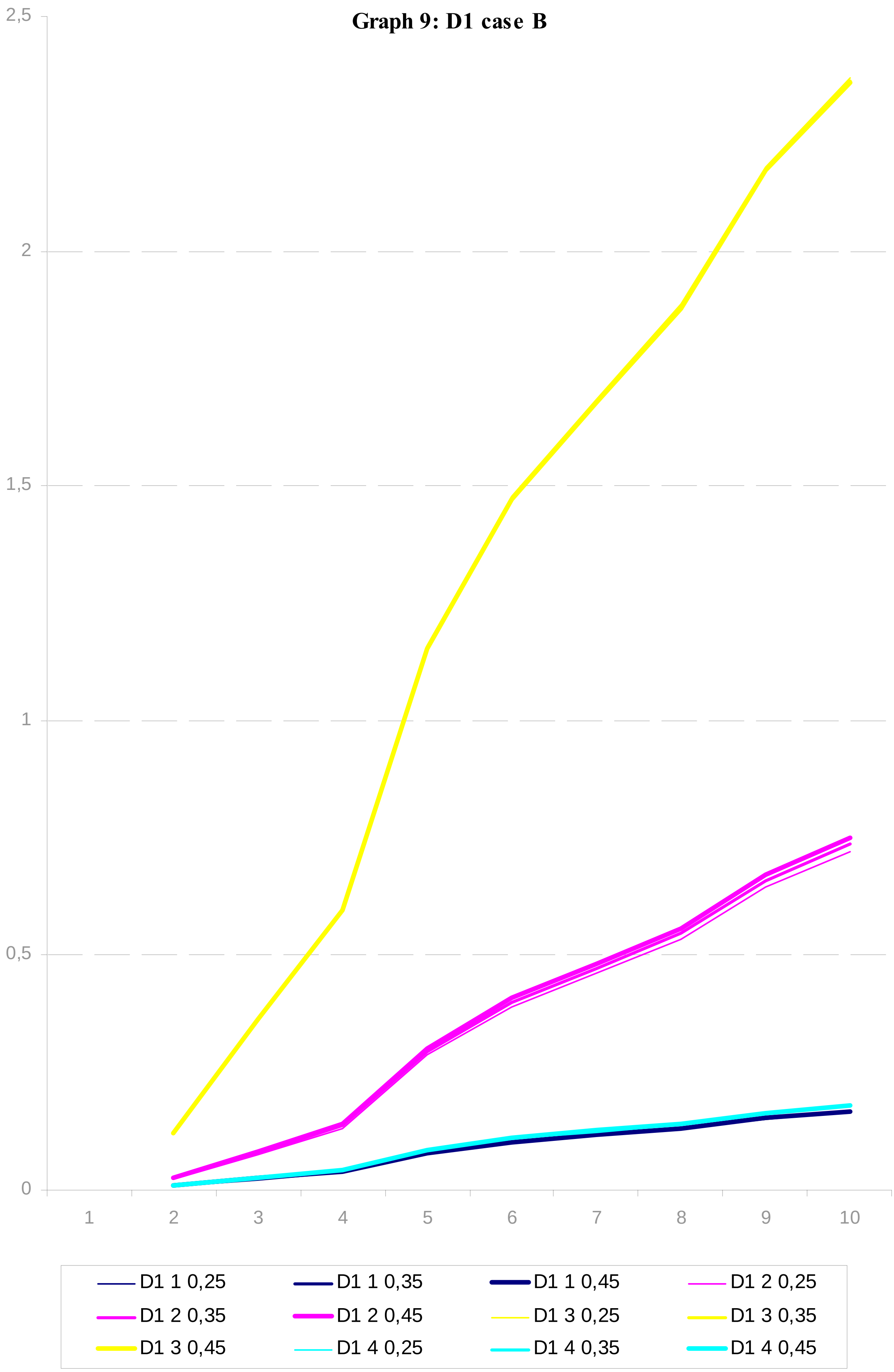


Source: Own Calculations

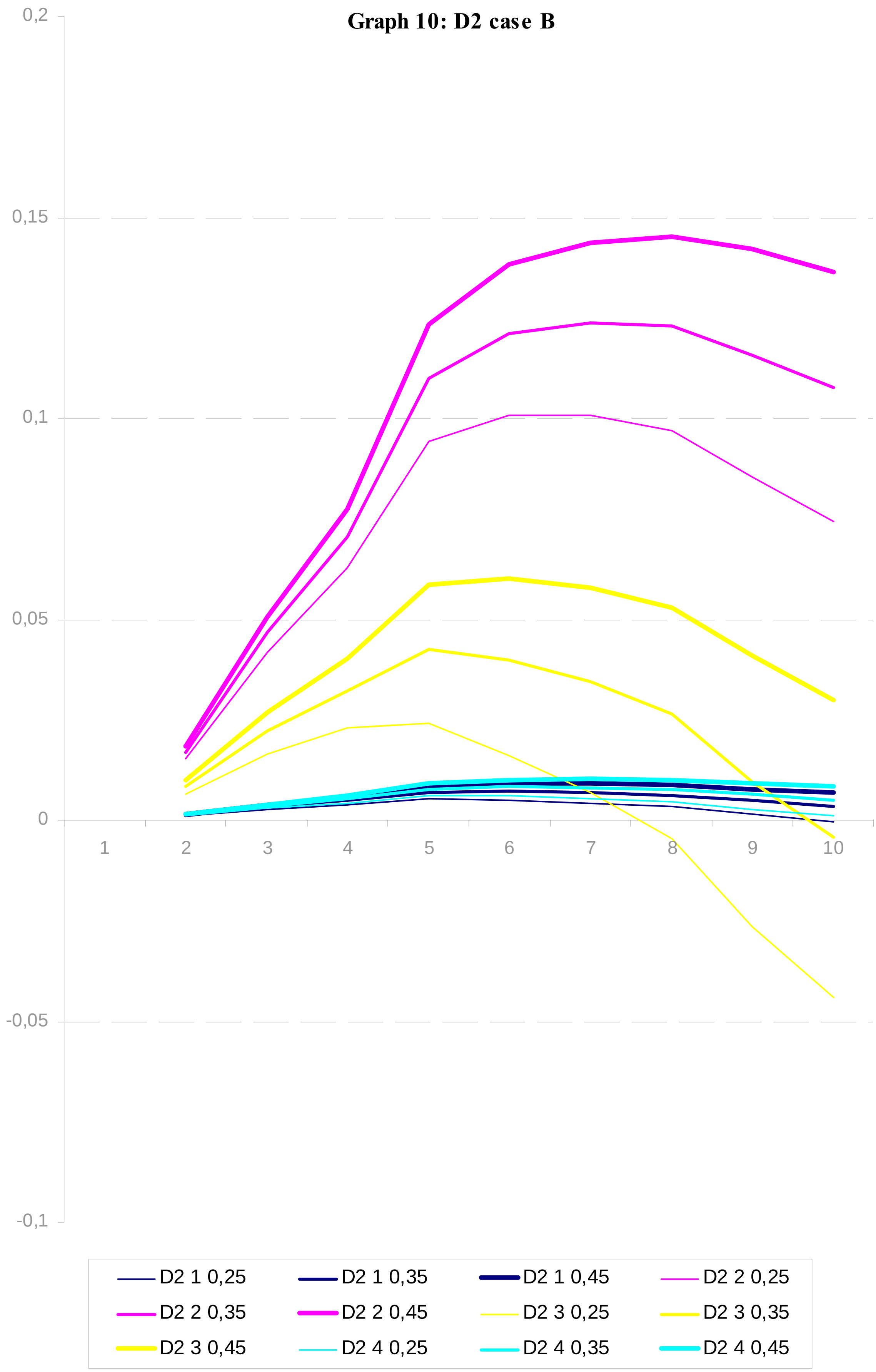
Graph 8: D2 case A



Source: Own calculations



Source: Own calculations



Source: Own calculations

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