Optimal Deterministic Debt Contracts

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Abstract:

This paper extends the costly enforcement model of optimal financing to the case of investment projects financed by several lenders. We consider the asymmetric situation when only one lender is a big strategic investor. All other lender are small passive investors. We first provide the sufficient and necessary condition for renegotiation proofness. Then we show that the optima verification is deterministic. We also discuss the conditions under which the optimal contract is a debt contract.

Keywords: Costly State Verification, Multiple Lenders, Debt

JEL: C72, D82, G33

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1   Introduction

In this paper we connect the literature dealing with absolute priority violations with the literature on optimal debt contracts with costly state verification. We show that the optimal financing contract with absolute priority violation is a deterministic simple debt contract. First we show this result in a general case and subsequently we provide an application to financial contracts with multiple lenders.

Absolute priority rule is one of the major principles of bankruptcy law. It says that creditors' claims take precedence over shareholders' claims in the event of a liquidation or reorganization. Shareholders are compensated only after creditors have been fully paid off. This principle is documented in both empirical, see Claessens and Klapper (2005), and theoretical studies, see Knot and Vychodil (2005).

Nevertheless absolute priority is quite often violated.For publicly traded firms Eberhart and Weiss (1998) and other authors cited by them show that absolute priority was violated in approximately 50-70% of out-of-court workouts and bankruptcies, depending on the particular sample used. Similarly Berkowitz and White (2004) document widespread absolute priority violation for small firms. Therefore we take absolute priority violation as given in this paper and we investigate its impact on the form of optimal credit contract.

Since the time Townsend (1979) introduced the costly state verification framework, it is known that contracts with stochastic verification Pareto dominate debt contracts. Recently Krasa and Villamil (2000, 2003) and Krasa, Sharma and Villamil (2004) proposed a related costly enforcement model in which they show that a deterministic simple debt contract is optimal when there is a limited commitment and enforcement is imperfect. Our paper uses the approach introduced by Krasa and Villamil (2000). We prove that allowing for non-commitment to the original contract and for absolute priority violation leads to simple

debt being the optimal financing contract. Then we show that this result may be naturally extended to the situation with multiple lenders with publicly observable monitoring results.

The rest of the paper is organized in the following way. Section 2 provides a general model of the financial contracting which we consider in this paper. In section 3 we solve for an equilibrium contract in this model. Section 4 provides application of our model to the situation with multiple lenders and section 5 concludes the paper.

2   The Model

We consider a model with one risk neutral firm and a large number of risk neutral possible investors. The firm has a project which requires one unit of financing to be provided by investors. The project leads to a random output $x \in X \equiv \{x_1, \ldots, x_n\}$, where $0 < x_1 < \cdots < x_n$. The output is privately observed by the firm. This private information is the only information asymmetry in the model. All other information is fully shared by all agents in the model. The value of output can by verified by a verification agency at a cost $c(x)$. The results of this verification are publicly revealed.

The model has 4 time periods. At the period $t = 0$ the firm and investor have a common prior $\mu(\cdot)$ over the possible realizations in the output space X. The firm specifies that if at time t=3 the verification agency is called upon to determine the state x the investor is entitled to an enforceable payment $G(x, v) \geq 0$. The payment $G(x, v)$ is a function of the true state $x \in X$ and the time $t = 1$ transfer v from firm to investor.

At the period $t = 1$ the firm privately observes the project realization and subsequently pays v to the investor. The firm is free to choose different values of v for different realizations of x. Since we do not allow for a commitment to the original contract we are not able to use the revelation principle which would otherwise imply that the firm truthfully announces the project realization x.

At the period $t = 2$ the investor may renegotiate the firm's offer v with the firm. He

may offer that the firm pays him additional money so that his payoff is now $v' > v$. We do not allow for the possibility that the investor returns some money to the firm. That is, we do not allow $v' < v$. If the firm pays the additional amount $(v' - v)$ it will not ask the verification agency for enforcement (in this paper we will use the terms verification and enforcement interchangeably). In the case this offer of additional money $(v' - v)$ is accepted, the game ends, otherwise the game proceeds to the next stage. In the case where the game ends, the payoffs are as follows. The payoff to the investor is $v' > v$. The payoff to the firm is $x - v'$.

At the period $t = 3$ the investor chooses whether to request enforcement. If no enforcement is requested, the investor's payoff remains v and the firm's payoff is $x - v$. If enforcement is requested, the verification agency determines the true state x and the investor pays the verification cost $c(x)$. We assume that $0 < c(x) < x$ and that $c(x)$ is a continuously differentiable nondecreasing function of x for all $x \in X$. The firm pays $G(x, v)$ to the investor.

We assume that the enforcement is imperfect in the sense that the absolute priority rule is violated. That is, firm is always able to keep $\delta(x)$ out of project outcome x. We assume that $0 < \delta(x) < x$ and that $\delta(x)$ is a continuously differentiable nondecreasing function of x for all $x \in X$. In order to avoid an unlimited liability problem for the investor, we assume $c(x_1) + \delta(x_1) < x_1$, $\frac{\partial c(x)}{\partial x} < 1$, and $\frac{\partial \delta(x)}{\partial x} < 1$ for all x.

The model presented above is very close to the standard costly state verification models of Townsend (1979), Gale and Hellwig (1985), and Williamson (1987). We use a generalized form of costly state verification model with a general verification cost function $c(x)$ and with stochastic reporting and verification strategies. In addition to the three time periods present in a standard costly state verification model we introduce the renegotiation period, which is a period $t = 2$. Crucial difference from standard costly state verification model is an assumption of absolute priority rule violation. This assumption together with possibility of renegotiation leads to the optimality of deterministic renegotiation proof simple debt

contract in our model.

3   The Equilibrium Contract

The investment problem presented in this model constitutes a dynamic game of incomplete information. We first define the behavioral strategies of the players in this game. We denote the set of all payments v as V and we define $\sigma_F(v|x), v \in V$ to be the firm's mixed strategy at $t = 1$. We denote the investor's action of asking for verification as $e = 1$ and that of not asking for verification as $e = 0$. By $\sigma_I(e|v)$ we denote the investor's behavioral strategy at $t = 3$. We define the firm's and investor's payoffs at time $t = 3$ as:

$$\pi_F(x, v, e) \;=\; x - v - eG(x, v) \tag{1}$$

$$\pi_I(x, v, e) \;=\; v + e[G(x, v) - c(x)] \tag{2}$$

We will solve the continuation game for its perfect Bayesian Nash equilibrium (PBNE). In this equilibrium $G(x, v)$ induces σ_F and σ_I which are foreseen by all players. That is, in the continuation game starting from $t = 1$, σ_F maximizes the firm's expected payoff given σ_I and the strategy σ_I maximizes the investor's expected payoff given $\mu(\cdot|v)$. The beliefs $\mu(\cdot|v)$ are derived by using Bayes rule whenever possible. From now on we will refer to $\{G, \sigma_I, \sigma_F\}$ as an equilibrium contract.

We assume that there is Bertrand competition among possible investors. Therefore in stage $t = 0$ we will assume that the investor chooses $\{G, \sigma_I, \sigma_F\}$ to maximize the firm's payoff subject to a set of restrictions. This means that the investor solves

Problem 1 *At $t = 0$ choose G, σ_I, σ_F to maximize*

$$E_0[u_F(x)] = \sum_{x \in X} \sum_{v \in V} \sum_{e=0}^{1} \pi_F(x, v, e)\sigma_I(e|v)\sigma_F(v|x)\mu(x) \tag{3}$$

subject to

$$E_0[u_I(x)] = \sum_{x \in X} \sum_{v \in V} \sum_{e=0}^{1} \pi_I(x, v, e)\sigma_I(e|v)\sigma_F(v|x)\mu(x) \geq \bar{u}_I \tag{4}$$

$$\sigma_F, \sigma_I, \mu, \mu(\cdot|v) \text{ is a PBNE at } t = 1 \tag{5}$$

$$v, G, \sigma_I \text{ is renegotiation proof } \forall x_i \in X \text{ and } \forall v | \sigma_F(v|x_i) > 0 \tag{6}$$

$$0 \le G(x, v) \le x - v - \delta(x) \forall x \in X. \tag{7}$$

We will characterize and simplify this Problem.

Note that (7) implies that $v \le x_i - \delta(x_i)$ for all $x_i \in X$ and for all v such that $\sigma_F(v|x_i) > 0$.

In order to provide the condition for renegotiation proofness we first formally define when the contract is renegotiation proof.

Definition 1 *A contract $\{G, \sigma_I, \sigma_F\}$ is renegotiation proof if and only if there does not exist v' that makes the investor strictly better off and the firm weakly better off in all states:*

$$v' > \sum_{x \in X} \sum_{e=0}^{1} \pi_I(x, v, e) \sigma_I(e|v) \mu(x|v) \tag{8}$$

$$x - v' \ge \sum_{e=0}^{1} \pi_F(x, v, e) \sigma_I(e|v), \forall x | \mu(x|v) > 0. \tag{9}$$

Given this definition we provide a sufficient and necessary condition for renegotiation proofness in Lemma 1. The intuition behind Lemma 1 is following. Assume that the expected continuation payoff from enforcement for the investor is smaller than the lowest payment which the firm has to investor in the case of enforcement. Then the firm is able to bribe the investor not to ask for verification. Condition (10) ensures that this possibility of bribing will not happen.

Lemma 1 *For given payments v and G, let σ_F, σ_I be PBNE strategies. Then v, G, σ_I, σ_F is renegotiation proof for all $v \in V$ with $\sigma_I(e = 1|v) > 0$ if and only if*

$$\sum_{x \in X} [G(x, v) - c(x)] \mu(x|v) \ge \min_{x \in X, \mu(x|v) > 0} G(x, v). \tag{10}$$

Proof. We first prove sufficiency. Assume that (10) holds. We show that G, σ_I, σ_F is renegotiation proof. Suppose by contradiction that G, σ_I, σ_F is not renegotiation proof.

Because we suppose that the contract is not renegotiation proof, we know that there exists v' which satisfies (9)

$$x - v' \geq x - v - 0G(x,v)\sigma_I(0|v) - 1[G(x,v)]\sigma_I(e=1|v), \forall x|\mu(x|v) > 0,$$

which simplifies as

$$v' \leq v + G(x,v)\sigma_I(e=1|v), \forall x|\mu(x|v) > 0.$$

Therefore

$$v' \leq v + \sigma_I(e=1|v) \min_{x \in X, \mu(x|v) > 0} G(x,v). \tag{11}$$

Because we suppose that the contract is not renegotiation proof, there also exists v' which satisfies (8)

$$v' > \sum_{x \in X} [v + 0G(x,v) - c(x)]\sigma_I(0|v) + 1[G(x,v) - c(x)]\sigma_I(e=1|v)]\mu(x|v)$$

$$v' > v + \sigma_I(e=1|v) \sum_{x \in X} [G(x,v) - c(x)]\mu(x|v). \tag{12}$$

Inequalities (11) and (12) imply

$$\sum_{x \in X} [G(x,v) - c(x)]\mu(x|v) < \min_{x \in X, \mu(x|v) > 0} G(x,v),$$

which contradicts (10). This completes the sufficiency part of the proof.

Now we prove that (10) is a necessary condition. Assume by contradiction that there exists contract $\{G, \sigma_I, \sigma_F\}$, which is renegotiation proof but violates (10) for some v. Let

$$v' = v + \sigma_I(1|v) \min_{x \in X, \mu(x|v) > 0} G(x,v). \tag{13}$$

Then (8) implies

$$v + \sigma_I(1|v) \min_{x \in X, \mu(x|v) > 0} G(x,v) > v + \sigma_I(1|v) \sum_{x \in X} [G(x,v) - c(x)]\mu(x|v),$$

which is a violation of (10) as assumed. After substituting (13) into the firm's renegotiation proofness condition (9) we obtain

$$x - v - \sigma_I(1|v) \min_{x \in X, \mu(x|v) > 0} G(x,v) \geq x - v - \sigma_I(1|v)G(x,v), \forall x|\mu(x|v) > 0,$$

which is true by the definition of minimum. Therefore contract $\{G, \sigma_I, \sigma_F\}$ is not renegotiation proof, which is a contradiction.

$$\text{Q.E.D.}$$

Note, that the necessity part of the proof is true for all possible values of $\min_{(x \in X, \mu(x|v) > 0)} G(x, v)$ including the zero value.

Lemma 1 straightforwardly implies the optimality of deterministic verification, which is formalized in the following Lemma.

Lemma 2 *If the contract $\{G, \sigma_I, \sigma_F\}$ solves Problem 1 with (10) holding as a strict inequality, then $\sigma_I(e = 1|v) \in \{0, 1\}$ for all v such that $\sigma_F(v|x) > 0$ for some $x \in X$.*

Proof. Assume that $\sigma_I(e = 1|v) > 0$. The right hand side of (10) is non-negative. Therefore the investor's continuation payoff from verification, which is given by left hand side of (10) holding as strict inequality, is strictly positive. Since the continuation payoff from not verifying is zero, the optimality of investor's decision requires $\sigma_I(e = 1|v) = 1$.

$$\text{Q.E.D.}$$

Now we show that the optimal contract solving Problem 1 has two characteristic features of the debt contract. The first feature is that firm either pays a fixed face value $\bar{v}$ or it does not pay anything and defaults. The second feature is that this default triggers the verification process.

Lemma 3 *If the contract $\{G, \sigma_I, \sigma_F\}$ solves Problem 1 with (10) holding as a strict inequality, then there exists contract $\{\tilde{G}, \tilde{\sigma}_I, \tilde{\sigma}_F\}$ which solves Problem 1 with (10) holding as a strict inequality. The contract $\{\tilde{G}, \tilde{\sigma}_I, \tilde{\sigma}_F\}$ has the following properties*

1. *At most the two payments 0 and $\bar{v}$ occur with positive probability. That is $\mu(v|x) = 0$ for all $x \in X, v \notin \{0, \bar{v}\}$.*

2. *Verification is requested if and only if $v < \bar{v}$. That is, $\sigma_I(e = 1|v) = 1 \Leftrightarrow v < \bar{v}$ and $\sigma_I(e = 1|v) = 0 \Leftrightarrow v \geq \bar{v}$.*

Proof. First we prove second part of this Lemma. Consider contract $\{G, \sigma_I, \sigma_F\}$ that solves Problem 1. Let v_j be some payment which takes place with positive probability from an ex ante perspective and which is followed by no verification. That is, $\sigma_F(v_j|x) > 0$ for some $x \in X$ and $\sigma_I(e = 1|v_j) = 0$. Optimality of σ_F then implies that $\sigma_F(v|x) = 0$ for all $v > v_j$ irrespective of the value of $\sigma_I(e = 1|v)$. Therefore without any consequence for the payoff of any agent we can set $\sigma_I(e = 1|v) = 0$ for all $v > v_j$. If $v_j = 0$, then the lemma is true. So let $v_j > 0$ and consider $v < v_j$. By Lemma 2, $\sigma(e = 1|v) \in \{0, 1\}$. If $\sigma_I(e = 1|v) = 0$, then optimality of σ_F implies that $\sigma_F(v_j|x) = 0$, which is a contradiction. So, $\sigma_I(e = 1|v) = 1$. Defining $\bar{v} \equiv v_j$ concludes the proof. Thus there is at most one payment $\bar{v}$ such that $\sigma_F(\bar{v}|x) > 0$ and $\sigma_I(e = 1|\bar{v}) = 0$.

Now we prove first part of this Lemma. If $\bar{v} = 0$ then by part two there is no verification for any v. This means that the firm optimally announces $\bar{v}$ for all $x \in X$. This means that the Lemma is true for $\bar{v} = 0$. So let $\bar{v} > 0$. We consider all payments $v < \bar{v}$ and we define for each x_i

$$\tilde{G}(x_i, 0) = v_k + G(x_i, v_k) \text{ for } v_k \text{ such that } \sigma_F(v_k|x_i) > 0 \tag{14}$$

$$\tilde{\sigma}_F(0|x_i) = \sum_{v_k < \bar{v}} \sigma_F(v_k|x_i). \tag{15}$$

Because we have shown that statement two of this Lemma holds and because of Lemma 2, $\tilde{\sigma}_I(e = 1|0) = 1$. Equation (15) means that in state x_i the firm chooses 0 instead of v_k, for all $v_k \in (0, \bar{v})$ when the $t = 3$ payment schedule following a $t = 1$ payment of 0 is given by $\tilde{G}(x_i, 0)$. By choosing 0 instead of v_k in state x_i the firm gets

$$x_i - \tilde{G}(x_i, 0) = x_i - v_k - G(x_i, v_k). \tag{16}$$

The firm is therefore for all $x \in X$ indifferent between 0 and any payment $v_k < \bar{v}$ such that $\sigma_F(v_k|x) > 0$. Because σ_F is optimal, $\tilde{\sigma}_F$ is also optimal. Now we show that $\tilde{G}(x_i, 0)$

is well defined and does not depend on the choice of v_k. For two distinct payments v_j and v_k such that $\sigma_F(v_j|x_i) > 0$ and $\sigma_F(v_k|x_i) > 0$ we have, due to the optimality of σ_F

$$x_i - v_j - G(x_i, v_j) = x_i - v_k - G(x_i, v_k),$$

which implies

$$\tilde{G}(x_i, 0) = v_j + G(x_i, v_j) = v_k + G(x_i, v_k) = \tilde{G}(x_i, 0).$$

Therefore $\tilde{G}(x_i, 0)$ does not depend on the choice of v_k. Next we show feasibility of the redefined enforcement payment. That is, we show $0 \leq \tilde{G}(x_i, 0) \leq x_i - \delta(x_i)$. The condition $\tilde{G}(x_i, 0) \geq 0$ holds by definition. The minimum payoff of the firm in state x_i is $\delta(x_i)$. Optimality of σ_F implies that the right hand side of (16), and hence the left hand side of (16) is not less than $\delta(x_i)$. This implies that $\tilde{G}(x_i, 0) \leq x_i - \delta(x_i)$.

We next show that contract $\{\tilde{G}, \tilde{\sigma}_I, \tilde{\sigma}_F\}$ is renegotiation proof. We denote by $\tilde{\mu}(x_i|0)$ the investor's updated belief given $\tilde{\sigma}_F$. The definition of $\tilde{\sigma}_F$ in (15) implies that $\tilde{\mu}(x_i|0) > 0$ if and only if $\mu(x_i|v_j) > 0$ for some $v_j < \bar{v}$. This and (16) imply

$$\min_{x_i \in X, \tilde{\mu}(x_i|0)>0} \tilde{G}(x_i, 0) = \min_{x_i \in X, \mu(x_i|v_j)>0} G(x_i, v_j) + v_j. \tag{17}$$

Since the contract $\{G, \sigma_I, \sigma_F\}$ is renegotiation proof we obtain

$$\sum_{x \in X} \tilde{G}(x, 0)\tilde{\sigma}_F(0|x)\mu(x) \tag{18}$$

$$= \sum_{x \in X} \sum_{v_j < \bar{v}} \tilde{G}(x, 0)\sigma_F(v_j|x)\mu(x) \tag{19}$$

$$= \sum_{x \in X} \sum_{v_j < \bar{v}} [G(x, v_j) + v_j]\sigma_F(v_j|x)\mu(x) \tag{20}$$

$$= \sum_{v_j < \bar{v}} \sum_{x \in X} [G(x, v_j) + v_j]\mu(x|v_j) \sum_{x \in X} \sigma_F(v_j|x)\mu(x) \tag{21}$$

$$\geq \sum_{v_j < \bar{v}} [\min_{x \in X, \mu(x|v_j)>0} G(x, v_j) + v_j + \sum_{x \in X} c(x)\mu(x|v_j)] \sum_{x \in X} \sigma_F(v_j|x)\mu(x) \tag{22}$$

$$= \sum_{v_j < \bar{v}} [\min_{x \in X, \tilde{\mu}(x|0)>0} \tilde{G}(x, 0) + \sum_{x \in X} c(x)\tilde{\mu}(x|0)] \sum_{x \in X} \sigma_F(v_j|x)\mu(x) \tag{23}$$

$$= [\min_{x \in X, \tilde{\mu}(x|0)>0} \tilde{G}(x, 0) + \sum_{x \in X} c(x)\tilde{\mu}(x|0)] \sum_{x \in X} \tilde{\sigma}_F(0|x)\mu(x). \tag{24}$$

The equalities (19) and (24) follow from (15). The equality (20) follows from (14). The equality (21) is given by Bayesian updating. The inequality (22) follows from (10). The equality (23) follows from (17). Dividing (18) and (24) by $\sum_{x \in X} \tilde{\sigma}_F(0|x)\mu(x)$ yields

$$\sum_{x \in X} [\tilde{G}(x,0) - c(x)\tilde{\mu}(x|0)] \geq \min_{x \in X, \tilde{\mu}(x|0)>0} \tilde{G}(x,0). \tag{25}$$

Because of Lemma 1, the contract $\{\tilde{G}, \tilde{\sigma}_I, \tilde{\sigma}_F\}$ is renegotiation proof. Also $\tilde{\sigma}_I(e=1|0) = 1$ is optimal because the right hand side of (25) is strictly positive.

Q.E.D.

As a consequence of Lemma 3 it is sufficient to consider strategies σ_F for which at most two payments occur in equilibrium. If payment $\bar{v}$ is made, then no verification occurs. We can therefore assume that $G(x, \bar{v}) = 0$. As a consequence, only payments $G(\cdot, 0)$ occur in equilibrium. We define $g(x) = G(x,0)$. We denote by $\sigma_d(x) = \sigma_F(0|x)$ the probability that the firm defaults. Similarly we define $\mu(x|d) = \mu(x|v=0)$.

Then we can rewrite Problem 1 as

Problem 2 *At $t = 0$ choose $\{\bar{v}, f, \sigma_d\}$ to maximize*

$$E_0[u_F(x)] = \sum_{x \in X} [x - g(x)\sigma_d(x) - \bar{v}(1 - \sigma_d(x))] \mu(x) \tag{26}$$

subject to

$$E_0[u_I(x)] = \sum_{x \in X} [(g(x) - c(x))\sigma_d(x) + \bar{v}(1 - \sigma_d(x))] \mu(x) \geq \bar{u}_I \tag{27}$$

$$\sigma_d(x) = \begin{cases} 1 & \text{if } \bar{v} > g(x) \\ 0 & \text{if } \bar{v} < g(x) \\ \alpha \in [0,1] & \text{if } \bar{v} = g(x). \end{cases} \tag{28}$$

If $\exists x \in X | \sigma_d(x) > 0$ then:

$$\sum_{x \in X} [g(x) - c(x)]\mu(x|d) \geq \min_{x \in X, \mu(x|d)>0} g(x) \tag{29}$$

$$0 \leq g(x) \leq x - \delta(x), \forall x \in X \tag{30}$$

In the following proposition we show that the solution of Problem 2 is a debt contract under which the firm pays the maximum enforceable payment to the lender in the case of default.

Proposition 1 *As long as renegotiation proofness condition (29) is satisfied as a strict inequality, the optimal payment schedule solving Problem 2 is $g(x) = x - \delta(x)$.*

Proof. The investor's participation constraint (27) binds in equilibrium. Suppose by contradiction that (27) does not bind. Then there exist $\epsilon > 0$ and $x' \in X$ with $\sigma_d(x') > 0$ such that (27) is still satisfied for $g'(x') = g(x') - \epsilon$ without violating conditions (29) – (30). Firm's payoff (26) is higher for $g'(x')$ than for $g(x')$ which leads to contradiction.

From binding (27) we obtain

$$\sum_{x \in X} \left[g(x)\sigma_d(x) + \bar{v}(1 - \sigma_d(x)) \right] \mu(x) = \bar{u}_I + \sum_{x \in X} c(x)\sigma_d(x)\mu(x).$$

After substituting for $\sum_{x \in X} \left[g(x)\sigma_d(x) + \bar{v}(1 - \sigma_d(x)) \right] \mu(x)$ in (26) we obtain

$$E_0[u_F(x)] = \sum_{x \in X} x\mu(x) - \bar{u}_I - \sum_{x \in X} c(x)\sigma_d(x)\mu(x).$$

Minimization of $\sum_{x \in X} c(x)\sigma_d(x)\mu(x)$ then requires $g(x) = x - \delta(x)$.

Q.E.D.

In the following section we will provide an application of the model of this section to the situation with multiple lenders.

4   Application to a Contract with Multiple Lenders

In this section we show that the optimality of simple debt contracts applies also to a situation with multiple investors. We consider a setting in which besides a small number of big strategic investors there is a huge number of small investors. These small investors take market conditions as given and do not strategically influence the market. These small

investors do not bargain with the borrowers about the conditions of the credit contract and leave the monitoring and other interactions with the borrowers to the big strategic investors. We adopt this structure of a financial market from the papers by Rajan and Winton (1995) and Menichini and Simmons (2002).

In this application we again consider an economy with a risk neutral firm and a large number of risk neutral investors. These investors are either quite big or very small. We denote the big investors as type I and the small investors as type T. The firm owns a technology which requires 1 unit of financing to be provided by the investors. We assume that β percent of financing is provided by one big strategic investor and the rest is equally provided by m small non-strategic investors. We assume that this proportion β is determined exogenously as a parameter of the model. Production and verification technologies and information asymmetry are the same as in the general model in Section 2.

The timing of the model corresponds to the timing in section 2 as closely as possible . In period $t = 0$ the firm and all investors have a common prior $\mu(\cdot)$ over the possible realizations in the output space X. The firm borrows β percent of the required finance from one strategic investor I and $\frac{(1-\beta)}{m}$ percent from each small investor. The firm specifies that if at time t=3 the verification agency is called upon to determine the state x the following happens. Each one of the small investors is entitled to $\frac{\eta_x x}{m}$. The strategic investor is entitled to an enforceable payment $\eta_f F(x, v) \geq 0$. This specification of enforceable payment is a special case of the general model in Section 2.

In period $t = 1$ the firm privately observes the project realization and announces what the value of transfer v will be. All investors immediately obtain their share of this payment v. Each investor obtains v units of money per unit of their financial investment provided at time $t = 0$.

In period $t = 2$ the strategic investor may renegotiate the firm's offer v with the firm. He may offer that the firm pay him additional money so that his payoff is now $v' > v$. If the firm pays this additional money he will not ask for enforcement. In the case this offer

is accepted, the game ends, otherwise the game proceeds to the next stage. In the case the game ends, the payoffs are as follows. The payoff of strategic investor is $\beta v' > \beta v$. The payoff of each of the small investors is $\frac{(1-\beta)v}{m}$. The payoff of the firm is $x - \beta v' - (1-\beta)v$.

In period $t = 3$ the strategic investor chooses whether to request verification. If no enforcement is requested, the investors' payoffs remain βv and $\frac{(1-\beta)v}{m}$ and the firm's payoff is $x - v$. If verification is requested, the strategic investor, who asked for it, pays cost c and the verification agency determines the true state x. Our assumption of constant verification cost c is a simplification of the more general form $c(x)$ which we used in Section 2. The firm pays $\eta_f F(x, v)$ to the strategic investor and $\frac{\eta_x x}{m}$ to each of the m small investors.

We assume that the cost c paid by the investor asking for enforcement is sufficiently high and the investment share $\frac{1-\beta}{m}$ of any of the small investors is sufficiently low so that it is never optimal for a small investor to initiate the costly state verification proceedings. We also assume that the enforcement is imperfect in the sense that the firm is always able to keep some part of project outcome. Formally we capture this by assuming:

$$0 < \eta_f < 1, 0 < \eta_x < 1, \eta_f + \eta_x < 1, 0 \leq F(x, v) \leq x - v - \frac{\eta_x}{\eta_f}v. \tag{31}$$

The upper bound on $F(x, v)$ is based on the following consideration. Given that v was paid in the period $t = 1$ and there was no renegotiation in the period $t = 2$, $x - v$ remains to be distributed at $t = 3$. We capture imperfect enforcement by assuming that the firm is able to keep at least $(1 - \eta_f - \eta_x)(x - v)$. The feasibility condition

$$x - v \geq \eta_f F(x, v) + \eta_x x + (1 - \eta_f - \eta_x)(x - v)$$

then leads to an upper bound on $F(x, v)$ in (31).

We define the firm's and investors' payoffs at $t = 3$ as:

$$\pi_F(x, v, e) = x - v - e[\eta_f F(x, v) + \eta_x(x - v)] \tag{32}$$

$$\pi_I(x, v, e) = \beta v + e[\eta_f F(x, v) - c] \tag{33}$$

$$\pi_i(x, v, e) = \frac{1 - \beta}{m}v + e\frac{\eta_x x}{m}, \forall i \in \{1, \ldots, m\}. \tag{34}$$

In the perfect Bayesian Nash equilibrium of the continuation game, $F(x, v)$ induces σ_F and σ_I which are foreseen by all players. Therefore we will from now refer to $\{F, \sigma_I, \sigma_F\}$ as an equilibrium contract.

We assume that there is Bertrand competition among possible strategic investors. Therefore in stage $t = 0$ we will assume that the strategic investor chooses contract $\{F, \sigma_I, \sigma_F\}$ to maximize the firm's payoff subject to a set of restrictions. This means that the strategic investor solves

Problem 3 *At $t = 0$ choose F, σ_I, σ_F to maximize*

$$E_0[u_F(x)] = \sum_{x \in X} \sum_{v \in V} \sum_{e=0}^{1} \pi_F(x, v, e) \sigma_I(e|v) \sigma_F(v|x) \mu(x) \tag{35}$$

subject to

$$E_0[u_I(x)] = \sum_{x \in X} \sum_{v \in V} \sum_{e=0}^{1} \pi_I(x, v, e) \sigma_I(e|v) \sigma_F(v|x) \mu(x) \geq \bar{u}_I \tag{36}$$

$$E_0[u_i(x)] = \sum_{x \in X} \sum_{v \in V} \sum_{e=0}^{1} \pi_i(x, v, e) \sigma_I(e|v) \sigma_F(v|x) \mu(x) \geq \bar{u}_i, \forall i \tag{37}$$

$$\sigma_F, \sigma_I, \mu, \mu(\cdot|v) \text{ is a PBNE at } t = 1 \tag{38}$$

$$v, F, \sigma_I \text{ is renegotiation proof } \forall x_i \in X \text{ and } \forall v | \sigma_F(v|x_i) > 0 \tag{39}$$

$$0 \leq F(x, v) \leq x - v - \frac{\eta_x}{\eta_f} v, \forall v \leq x, x \in X. \tag{40}$$

We will characterize and simplify this Problem.

First note that since all m small investors are identical and non-strategic, we can represent them by one aggregate non-strategic investor. We assume that the small lenders are not able to collude either because of some legal or institutional reasons or because of too high cost of coordination and collusion. Therefore instead of (34) we will use

$$\pi_T(x, v, e) = \sum_{i=1}^{m} \pi_i(x, v, e) \tag{41}$$

and instead of (37) we will use

$$E_0[u_T(x)] = \sum_{x \in X} \sum_{v \in V} \sum_{e=0}^{1} \pi_T(x, v, e) \sigma_I(e|v) \sigma_F(v|x) \mu(x) \geq \bar{u}_T = \sum_{i=1}^{m} \bar{u}_i. \tag{42}$$

Also note that (40) implies $v \leq \frac{\eta_f}{\eta_f + \eta_x} x$, which implies that $v < x$.

In order to provide the condition for renegotiation proofness we first formally define when the contract is renegotiation proof.

Definition 2 *A contract $\{F, \sigma_I, \sigma_F\}$ is renegotiation proof if and only if there does not exist v' that makes the strategic investor strictly better off and the firm weakly better off in all states:*

$$\beta v' > \sum_{x \in X} \sum_{e=0}^{1} \pi_I(x, v, e) \sigma_I(e|v) \mu(x|v) \tag{43}$$

$$x - (1 - \beta)v - \beta v' \geq \sum_{e=0}^{1} \pi_F(x, v, e) \sigma_I(e|v), \forall x | \mu(x|v) > 0. \tag{44}$$

Given this definition we provide a sufficient and necessary condition for renegotiation proofness in Lemma 4. The intuition behind Lemma 4 is following. Assume that the expected continuation payoff from enforcement for the strategic investor is smaller than the lowest payment which the firm has to pay both strategic and nonstrategic investors in the case of enforcement. Then the firm is able to bribe the strategic investor not to ask the verification agency for enforcement. Condition (45) ensures that this possibility of bribing will not happen.

Lemma 4 *For given v and F, let σ_F, σ_I be a PBNE strategies. Then v, F, σ_I, σ_F is renegotiation proof for all $v \in V$ with $\sigma_I(e = 1|v) > 0$ if and only if*

$$\sum_{x \in X} \eta_f F(x, v) \mu(x|v) - c \geq \min_{x \in X, \mu(x|v) > 0} \eta_f F(x, v) + \eta_x x. \tag{45}$$

Proof. In Appendix.

Lemma 4 implies that enforcement is deterministic.

Lemma 5 *If the contract $\{F, \sigma_I, \sigma_F\}$ solves Problem 3, then the verification is deterministic. That is, $\sigma_I(e = 1|v) \in \{0, 1\}$ for all v such that $\sigma_F(v|x) > 0$ for some $x \in X$.*

Proof. In Appendix.

Now we show that the optimal contract solving Problem 3 has two characteristic features of the debt contract. The first feature is that the firm either pays a fixed face value $\bar{v}$ or it does not pay anything and defaults. The second feature is that this default triggers the verification process.

Lemma 6 *Let $\{F, \sigma_I, \sigma_F\}$ solve Problem 3. Then there exists $\{\tilde{F}, \tilde{\sigma}_I, \tilde{\sigma}_F\}$ which solves Problem 3 with the following properties*

1. *At most the two payments 0 and $\bar{v}$ occur with positive probability, i.e., $\mu(v|x) = 0$ for all $x \in X, v \notin \{0, \bar{v}\}$.*

2. *Verification takes places if and only if $v < \bar{v}$, i.e., $\sigma_I(e = 1|v) = 1 \Leftrightarrow v < \bar{v}$ and $\sigma_I(e = 1|v) = 0 \Leftrightarrow v \geq \bar{v}$.*

Proof. In Appendix.

As a consequence of Lemma 6 it is sufficient to consider strategies σ_F for which at most two payments occur in equilibrium. If payment $\bar{v}$ is made, then no verification occurs. We can therefore assume that $F(x, \bar{v}) = 0$. As a consequence, only payments $F(\cdot, 0)$ occur in equilibrium. Define $f(x) = F(x, 0)$. Let $\sigma_d(x) = \sigma_F(0|x)$ be the probability of default. Let $\mu(x|d) = \mu(x|v = 0)$.

Then we can rewrite Problem 3 as

Problem 4 *At $t = 0$ choose $\{\bar{v}, f, \sigma_d\}$ to maximize*

$$E_0[u_F(x)] = \sum_{x \in X} \left[x - (\eta_f f(x) + \eta_x x)\sigma_d(x) - \bar{v}(1 - \sigma_d(x)) \right] \mu(x) \tag{46}$$

subject to

$$E_0[u_I(x)] = \sum_{x \in X} \left[(\eta_f f(x) - c)\sigma_d(x) + \beta\bar{v}(1 - \sigma_d(x)) \right] \mu(x) \geq \bar{u}_I \tag{47}$$

$$E_0[u_T(x)] = \sum_{x \in X} \left[\eta_x x \sigma_d(x) + (1 - \beta)\bar{v}(1 - \sigma_d(x)) \right] \mu(x) \geq \bar{u}_T, \tag{48}$$

$$
\sigma_d(x) = \begin{cases} 1 & \textit{if } \bar{v} > \eta_f f(x) + \eta_x x \\ 0 & \textit{if } \bar{v} < \eta_f f(x) + \eta_x x \\ \alpha \in [0,1] & \textit{if } \bar{v} = \eta_f f(x) + \eta_x x. \end{cases} \tag{49}
$$

If $\exists x \in X | \sigma_d(x) > 0$ *then:*

$$
\sum_{x \in X} \eta_f f(x) \mu(x|d) - c \geq \min_{x \in X, \mu(x|d) > 0} \eta_f f(x) + \eta_x x \tag{50}
$$

$$
0 \leq f(x) \leq x, \forall x \in X \tag{51}
$$

$$
\bar{v} \leq \frac{\eta_f}{\eta_f + \eta_x} x, \forall x \in X \, such \, that \, \sigma_d(x) = 0. \tag{52}
$$

In the following proposition we show that the solution of the Problem 4 is a debt contract under which the firm pays maximum enforceable payment to strategic lender in the case of default.

Proposition 2 *As long as renegotiation proofness condition (50) is satisfied as a strict inequality, the optimal payment schedule solving Problem 4 is* $f(x) = x$.

Proof. In Appendix.

5 Conclusion

In our paper we deal with the problem of deriving an optimal investment financing contract. Since we do not allow for the commitment to the original contract we are not able to use the standard revelation principle argument as introduced by Townsend (1979). Instead we use the Perfect Bayesian Equilibrium concept as used in related papers by Krasa and Villamil (2000) and Bester and Strausz (2001).

Since Townsend (1979) it is known that when the commitment to the original contract is possible then in the costly state verification model the contracts with stochastic enforcement dominate the contracts with deterministic enforcement. This result continues to hold in the costly state verification models with impossibility of commitment, as shown

by Choe (1998) and Khalil and Parigi (1998) in the models with one investor and only two possible outcomes of investment projects. Menichini and Simmons (2002) show that the optimality of stochastic enforcement survives an introduction of multiple investors into the costly state verification model with two possible outcomes and with impossibility of commitment.

In our model we show that allowing for more than two possible outcomes and allowing for absolute priority violation together with using costly state enforcement as introduced by Krasa and Villamil (2000) leads to different results. The optimal contract in our model is a renegotiation proof deterministic debt contract. We first show the optimality of simple debt contract for the situation with a single investor financing the project. Then we apply our model to the investment project with two classes of investors and we show that optimality of the deterministic debt contract continues to hold in this framework too.

6   Appendix

Proof of Lemma 4. We first prove sufficiency. Assume that (45) holds. We show that $\{F, \sigma_I, \sigma_F\}$ is renegotiation proof. Suppose by contradiction that $\{F, \sigma_I, \sigma_F\}$ is not renegotiation proof.

Because we suppose that the contract is not renegotiation proof, there exists v' which satisfies (44)

$$
\begin{aligned}
x - (1-\beta)v - \beta v' &\geq x - v - [\eta_f F(x,v) + \eta_x x]\sigma_I(e=1|v) \\
\beta(v'-v) &\leq [\eta_f F(x,v) + \eta_x x]\sigma_I(e=1|v) \\
\beta(v'-v) &\leq \sigma_I(e=1|v) \min_{x \in X, \mu(x|v)>0}[\eta_f F(x,v) + \eta_x x]. \quad (53)
\end{aligned}
$$

Because we suppose that the contract is not renegotiation proof, there also exists v' which satisfies (43)

$$
\beta v' > \sum_{x \in X}[\beta v + \eta_f F(x,v) - c]\sigma_I(e=1|v)\mu(x|v)
$$

$$\beta(v' - v) \;>\; \sigma_I(e = 1|v) \sum_{x \in X} \eta_f F(x, v)\mu(x|v) - c. \tag{54}$$

Inequalities (53) and (54) imply

$$\sum_{x \in X} \eta_f F(x, v)\mu(x|v) - c < \min_{x \in X, \mu(x|v)>0} \eta_f F(x, v) + \eta_x x,$$

which contradicts (45). This completes the sufficiency part of the proof.

Now we prove that (45) is necessary. Assume by contradiction that there exists contract $\{F, \sigma_I, \sigma_F\}$, which is renegotiation proof but violates (45) for some v. Let

$$v' = v + \sigma_I(1|v)[\min_{x \in X, \mu(x|v)>0} \eta_f F(x, v) + \eta_x x]. \tag{55}$$

Because we assume that (45) is violated, the strategic investor's expected payoff, which is given by the $v + [\sigma_I(1|v)$ times left hand side of (45)] is smaller than v' in (55). Therefore the strategic investor strictly prefers to renegotiate. After substituting (55) into the firm's renegotiation proofness condition (44) we obtain

$$x - (1 - \beta)v - \beta v - \beta\sigma_I(1|v)[\min_{x \in X, \mu(x|v)>0} \eta_f F(x, v) + \eta_x x] \geq$$
$$x - v - 0[\cdot]\sigma_I(0|v) - 1[\eta_f F(x, v) + \eta_x x]\sigma_I(1|v)$$

$$\eta_f F(x, v) + \eta_x x \geq \beta[\min_{x \in X, \mu(x|v)>0} \eta_f F(x, v) + \eta_x x].$$

Since $\beta < 1$ and $\eta_x x > 0$, we see that the condition (44) holds even as a strict inequality. This means that the entrepreneur is strictly better off. Therefore the contract $\{F, \sigma_I, \sigma_F\}$ is not renegotiation proof, which is a contradiction.

Q.E.D.

Proof of Lemma 5. Assume that $\sigma_I(e = 1|v) > 0$. The right hand side of (45) is strictly positive. Therefore the strategic investor's continuation payoff from verification, which is given by left hand side of (45), is strictly positive. But the continuation payoff from not enforcing is zero. Therefore, $\sigma_I(e = 1|v) = 1$.

Proof of Lemma 6. First we prove part two. Consider $\{F, \sigma_I, \sigma_F\}$ that solves Problem 3 and let v_j be some payment which takes place with positive probability from an ex ante perspective and which is followed by no verification, i.e., $\sigma_F(v_j|x) > 0$ for some $x \in X$ and $\sigma_I(e = 1|v_j) = 0$. Optimality of σ_F then implies that $\sigma_F(v|x) = 0$ for all $v > v_j$ irrespective of the value of $\sigma_I(e = 1|v)$. Hence without any consequence for the payoff of any agent we can set $\sigma_I(e = 1|v) = 0$ for all $v > v_j$. If $v_j = 0$, then the lemma is true. So let $v_j > 0$ and consider $v < v_j$. By Lemma 5, $\sigma(e = 1|v) \in \{0, 1\}$. If $\sigma_I(e = 1|v) = 0$, then optimality of σ_F implies that $\sigma_F(v_j|x) = 0$, which is a contradiction. So, $\sigma_I(e = 1|v) = 1$. Defining $\bar{v} \equiv v_j$ completes the proof. In this way we proved that there is at most one payment $\bar{v}$ for which $\sigma_F(\bar{v}|x) > 0$ and $\sigma_I(e = 1|\bar{v}) = 0$.

Now we prove the first part of this Lemma. If $\bar{v} = 0$ then according to second part of this Lemma the investor does not request verification for any v. This means that firm optimally announces $\bar{v}$ for all $x \in X$. This means that lemma is true for $\bar{v} = 0$. So let $\bar{v} > 0$. Consider all payments $v < \bar{v}$ and define for each x_i

$$\tilde{F}(x_i, 0) = \frac{v_k}{\eta_f} + F(x_i, v_k) \text{ for } v_k \text{ such that } \sigma_F(v_k|x_i) > 0 \tag{56}$$

$$\tilde{\sigma}_F(0|x_i) = \sum_{v_k < \bar{v}} \sigma_F(v_k|x_i). \tag{57}$$

Because we have shown that statement two of this Lemma holds and because of Lemma 5, $\tilde{\sigma}_I(e = 1|0) = 1$. Equation (57) means that in state x_i the firm chooses 0 instead of v_k, for all $v_k \in (0, \bar{v})$ when the $t = 3$ payment schedule following a $t = 1$ payment of 0 is given by $\tilde{F}(x_i, 0)$. By choosing 0 instead of v_k in state x_i the firm gets

$$x_i - 0 - \eta_f \tilde{F}(x_i, 0) - \eta_x x_i = x_i - v_k - \eta_f F(x_i, v_k) - \eta_x x_i. \tag{58}$$

The firm is therefore for all $x \in X$ indifferent between 0 and any payment $v_k < \bar{v}$ such that $\sigma_F(v_k|x) > 0$. Because σ_F is optimal, $\tilde{\sigma}_F$ is also optimal for the firm.

The payoff of strategic investor when the payoff schedule $\tilde{F}(x,0)$ is applied is

$$0 + \eta_f \tilde{F}(x,0) = v_k + \eta_f F(x,v_k).$$

This means that the payoff of the strategic investor is not changed by this transformation.

Now we will show that the transformed enforceable payment is feasible, that is $0 \leq \tilde{F}(x_i,0) \leq x_i$. By definition $\tilde{F}(x_i,0) \geq 0$. The minimum payoff of the firm in state x_i is $(1 - \eta_f - \eta_x)x_i$. Optimality of σ_F implies that the right hand side of (58), and hence the left hand side of (58) is not less than $(1 - \eta_f - \eta_x)x_i$. That is,

$$x_i - 0 - \eta_f \tilde{F}(x_i,0) - \eta_x x_i \quad \geq \quad (1 - \eta_f - \eta_x)x_i \tag{59}$$

$$\tilde{F}(x_i,0) \leq x_i. \tag{60}$$

We next show that $\{\tilde{F}, \tilde{\sigma}_I, \tilde{\sigma}_F\}$ is renegotiation proof. Let $\tilde{\mu}(x_i|0)$ be the strategic investor's updated prior given $\tilde{\sigma}_F$. The definition of $\tilde{\sigma}_F$ in (57) implies that $\tilde{\mu}(x_i|0) > 0$ if and only if $\mu(x_i|v_j) > 0$ for some $v_j < \bar{v}$. This and (58) imply

$$\min_{x_i \in X, \tilde{\mu}(x_i|0)>0} \eta_f \tilde{F}(x_i,0) = \min_{x_i \in X, \mu(x_i|v_j)>0} \eta_f F(x_i,v_j) + v_j. \tag{61}$$

It is also true that

$$\sum_{x \in X} \eta_f \tilde{F}(x,0)\tilde{\sigma}_F(0|x)\mu(x) \tag{62}$$

$$= \sum_{x \in X} \sum_{v_j < \bar{v}} \eta_f \tilde{F}(x,0)\sigma_F(v_j|x)\mu(x) \tag{63}$$

$$= \sum_{x \in X} \sum_{v_j < \bar{v}} [\eta_f F(x,v_j) + v_j]\sigma_F(v_j|x)\mu(x) \tag{64}$$

$$= \sum_{v_j < \bar{v}} \sum_{x \in X} [\eta_f F(x,v_j) + v_j]\mu(x|v_j) \sum_{x \in X} \sigma_F(v_j|x)\mu(x) \tag{65}$$

$$\geq \sum_{v_j < \bar{v}} [\min_{x \in X, \mu(x|v_j)>0} \eta_f F(x,v_j) + \eta_x x + v_j + c] \sum_{x \in X} \sigma_F(v_j|x)\mu(x) \tag{66}$$

$$= \sum_{v_j < \bar{v}} [\min_{x \in X, \tilde{\mu}(x|0)>0} \eta_f \tilde{F}(x,0) + \eta_x x + c] \sum_{x \in X} \sigma_F(v_j|x)\mu(x) \tag{67}$$

$$= [\min_{x \in X, \tilde{\mu}(x|0)>0} \eta_f \tilde{F}(x,0) + \eta_x x + c] \sum_{x \in X} \tilde{\sigma}_F(0|x)\mu(x). \tag{68}$$

The equalities (63) and (68) follow from (57). The equality (64) follows from (56). The equality (65) is given by Bayesian updating. The inequality (66) follows from (45). The equality (67) follows from (61). Dividing (62) and (68) by $\sum_{x \in X} \tilde{\sigma}_F(0|x)\mu(x)$ yields

$$\sum_{x \in X} \eta_f \tilde{F}(x, 0)\tilde{\mu}(x|0) - c \geq \min_{x \in X, \tilde{\mu}(x|0) > 0} \eta_f \tilde{F}(x, 0) + \eta_x x. \tag{69}$$

Because of Lemma 4, the contract $\{\tilde{F}, \tilde{\sigma}_I, \tilde{\sigma}_F\}$ is renegotiation proof. Also $\tilde{\sigma}_I(e = 1|0) = 1$ is optimal because the right hand side of (69) is strictly positive.

<div align="right">Q.E.D.</div>

Proof of Proposition 2. Strategic investor's participation constraint (47) binds in equilibrium. Suppose by contradiction that (47) does not bind. Then there exist $\epsilon > 0$ and $x' \in X$ with $\sigma_d(x') > 0$ such that (47) is still satisfied for $f'(x') = f(x') - \epsilon$ without violating conditions (48) – (52). Firm's payoff (46) is higher for $f'(x')$ than for $f(x')$ which leads to contradiction.

Case 1. Non-strategic investor's participation constraint (48) binds.

By adding (47) and (48) we obtain

$$\sum_{x \in X} \left[(\eta_f f(x) + \eta_x x)\sigma_d(x) + \bar{v}(1 - \sigma_d(x)) \right] \mu(x) = \bar{u}_I + \bar{u}_T + \sum_{x \in X} c\sigma_d(x)\mu(x).$$

After substituting for $\sum_{x \in X} \left[(\eta_f f(x) + \eta_x x)\sigma_d(x) + \bar{v}(1 - \sigma_d(x)) \right] \mu(x)$ in (46) we obtain

$$E_0[u_F(x)] = \sum_{x \in X} x\mu(x) - (\bar{u}_I + \bar{u}_T) - \sum_{x \in X} c\sigma_d(x)\mu(x).$$

Minimization of $\sum_{x \in X} c\sigma_d(x)\mu(x)$ then requires $f(x) = x$.

Case 2. Non-strategic investor's participation constraint (48) does not bind.

From binding (47) we obtain

$$\bar{v} \sum_{x \in X} [1 - \sigma_d(x)]\mu(x) = \frac{\bar{u}_I - \sum_{x \in X}[\eta_f f(x) - c]\sigma_d(x)\mu(x)}{\beta}. \tag{70}$$

After substituting (70) in (46) we get

$$E_0[u_F(x)] = \sum_{x \in X} x\mu(x) - \frac{\bar{u}_I}{\beta} + \frac{\sum_{x \in X}[(1-\beta)\eta_f f(x) - \beta\eta_x x - c]\sigma_d(x)\mu(x)}{\beta}. \quad (71)$$

Since right-hand-side of (71) is increasing in $f(x)$, setting $f(x) = x$ maximizes $E_0[u_F(x)]$.

<div align="right">Q.E.D.</div>

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