Monetary and Fiscal Policy Interaction
With Various Degrees of Commitment[1]

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Abstract

Well before the global financial crisis, the long-term trend in fiscal policy had raised concerns about risks for the outcomes of monetary policy. Are fears of an unpleasant monetarist arithmetic justified? To provide some insight, this paper examines strategic fiscal-monetary interactions in a novel game theory framework with an asynchronous timing of moves. It generalizes the standard commitment concept of Stackelberg leadership by making it dynamic. By letting players move with a certain fixed frequency this framework allows policies to be committed or rigid for different periods of time. We find that the inferior non-Ricardian *(active fiscal, passive monetary)* regime can occur in equilibrium, and that this is more likely in a monetary union due to free-riding. The bad news is that, unlike under static commitment of Sargent and Wallace (1981), this may happen even if monetary policy acts as leader for longer periods of time than fiscal policy. The good news is that under some circumstances an appropriate institutional design of monetary policy may not only help the central bank resist fiscal pressure and avoid the unpleasant monetarist arithmetic, but also to discipline excessively spending governments. By acting as a credible threat of a costly policy tug-of-war, long-term monetary commitment may induce a reduction in the average size of the budget deficit and debt, and move the economy to a Ricardian *(passive fiscal, active monetary)* regime. More broadly, this paper demonstrates that our game theoretic framework with dynamic leadership can help uniquely select a Pareto-efficient outcome in situations with multiple equilibria where standard approaches do not provide any guidance.

Keywords: monetary vs fiscal policy interaction; Game of Chicken; commitment; dynamic leadership, asynchronous games; explicit inflation targeting

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1. INTRODUCTION

The recent financial crisis has shown that the borderline between fiscal (F) and monetary (M) policies is finer than most academics and policymakers had thought. This has highlighted the importance of understanding the *interaction* between the two policies.

The idea that M and F policies might interact goes back to Friedman (1948), Tinbergen (1954), Mundell (1962) and Cooper (1969). But until recently most models used for policy design treated each policy in isolation. The subsequent literature has concentrated on the government's direct institutional interventions into central banking.[5]

The focus of this paper is indirect interaction which is more subtle and less well understood. It works through spillovers of economic outcomes – variables such as inflation, output, or debt are affected by both policies, *and* their interactions in turn affect the optimal setting of both policies.[6]

We examine an increasingly important aspect of the indirect policy interaction that standard macro models do not capture, namely *strategic* policy behaviour. There exists an abundance of recent real world examples of strategic interactions between M and F policies. To name just one, despite the European Central Bank's initial distaste for any form of quantitative easing in the aftermath of the 2008 crisis, the Bank subsequently engaged in such actions, at least partly due to political pressures.

To fill the gap in the literature we will analyze the strategic interactions and possible policy spillovers through non-cooperative game theoretic techniques. Let us stress from the outset that our interest lies in *medium to long-term* outcomes of the policy interactions. We will *not* examine the optimal short-term mix of policy responses to a shock such as the global financial crisis and hence do not offer any assessment of the current F stimulus vs austerity debate. Our analysis describes the underlying long-term stance of M and F policies, and our policy implications therefore only apply once economies have recovered from the current global downturn.

Our perspective thus follows Sargent and Wallace (1981), Alesina and Tabellini (1987), Leeper (1991), Nordhaus (1994) and the subsequent literature, and links to current debates about F sustainability, eg Kotlikoff and Burns (2012) or Leeper (2010). We first motivate the analysis by outlining the large size of the F gap (unfunded liabilities) facing most advanced countries, primarily due to aging populations and ballooning health care costs. We then examine under what circumstances these F excesses may spill over to M policy.

In this long-term sense, we endogenize Leeper's (1991) policy regimes by deriving the circumstances under which we are likely to observe the Ricardian regime *(active M, passive F)*, and those under which the unpleasant monetarist arithmetic *(passive M, active F)* is likely to occur.

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[5]This includes the ability of the government (F policymaker) to affect M policy outcomes through the appointment of the central banker [Rogoff (1985)], optimal contract with the central banker [Walsh (1995)], or through overriding the central banker [Lohmann (1992)]

[6]Examples of this type of indirect interaction include Leeper (2010), Cochrane (2009), Adam and Billi (2008), Eusepi and Preston (2008), Chadha and Nolan (2007), Persson et al. (2006), Benhabib and Eusepi (2005), Gali and Monacelli (2005), Hughes Hallett and Weymark (2007, 2005), Eggertsson and Woodford (2004), Dixit and Lambertini (2003, 2001), van Aarle, et al. (2002), Leeper (1991) or Sargent and Wallace (1981).

Our analysis contributes to both macroeconomic policy and game theory. On the game theory front, we develop a novel framework with generalized timing. The policymakers no longer move simultaneously every period. Instead, each policy $i \in \{M, F\}$ can change its medium-run stance with a constant frequency - every $r^i \in \mathbb{N}$ periods (after a synchronized initial move). Figure 3 in Section 3 offers an illustration of such timing.

We interpret the number of periods r^F as the degree of F *rigidity* (which arguably depends on the size of the F gap). In contrast, we interpret r^M as the degree of *long-term M commitment*, as it describes the central bank's inability to change its target for average inflation.[7]

This framework is a generalization of the alternating move games of Maskin and Tirole (1988) and Lagunoff and Matsui (1997) for which the existing work provides a strong motivation. For example Cho and Matsui (2005) argue that: *'[a]lthough the alternating move games capture the essence of asynchronous decision making, we need to investigate a more general form of such processes. . . '.*

Allowing for $r^F \neq r^M$ leads to asynchronous policy moves with each player acting as a Stackelberg leader for at least some part of the stage game.[8] We show that conventional conclusions made under the standard commitment concept of Stackelberg leadership, which is static, may not be robust. This is because it cannot capture the cost of the policy conflict or mis-coordination - the Stackelberg follower moves 'immediately' after observing the leaders' move, so there are no such costs. This highlights the importance of incorporating the time dimension into the sequencing of policy actions: that is, the importance of modelling the commitment dynamics and leadership changes explicitly.

On the policy front, the paper's contribution is to show that concerns about long-term F excesses spilling over to monetary policy may be justified.[9] We are able to identify several variables and circumstances which can make it happen. We then show, using the asynchronous structure of the game, how institutional remedies can prevent such spillovers under some (but not all) circumstances.

Interestingly, our analysis implies that M policy commitment can sometimes indirectly discipline F policy, and achieve the socially optimal outcomes for *both* policies throughout the medium and long-run. Intuitively, if the inflation target is explicitly stated in the statutes or related legislation, the central banker is more willing to engage in a costly tug-of-war with the government, counter-acting (structurally) excessive F actions by a strong M tightening. As this would eliminate any political gains, the

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[7]This inability is likely to depend on how *explicitly* the long-term target is grounded in the central banking statues or legislation. It should be stressed that due to our medium-term focus r^M should *not* be interpreted as the frequency of the central bank's interest rate decisions. This is because r^M does not restrict the ability to make period by period policy stabilization decisions regarding zero mean shocks. It only restricts the frequency with which the average inflation level can be altered.

[8]This captures the observation of Tobin (1982) that *'Some decisions by economic agents are reconsidered daily or hourly, while others are reviewed at intervals of a year or longer...',* and follows Tobin's call: *...'It would be desirable in principle to allow for differences among variables in frequencies of change...'.*

[9]Let us state again that we are interested in structural imbalances; not cyclical ones caused by temporary F actions in response to cyclical downturns.

government's incentive to engage in excessive F actions or avoid tough F reforms fades away - leading to an improvement in the budget and debt.[10]

There is one important caveat to this conclusion, namely membership in a currency union. We show formally in Section 6 how accession to a currency union may introduce a free-riding problem. Intuitively, if a small member country engages in F profligacy, its impact on the inflation outcomes of the union as a whole is small. Because of that, the M punishment by the common central bank will be of a much smaller magnitude. Further, it will be spread across all member countries, see Masson and Patillo (2002). Therefore, if a small fiscally irresponsible country does not internalize the cost it imposes on others, it tends to spend excessively - more so than before joining the union when it would have borne the full weight of its own central bank's punishment. Recent developments in the Eurozone provide an example of that sort of phenomenon (although this is clearly *not* the only source of the current problems).

This constitutes a different type of moral hazard to the one commonly discussed with respect to union accession, where one country relies on a bailout by the rest of the members.

2. Strategic Policy Interactions in the Presence of a Fiscal Gap

This section lays out the simplest setup that can capture the main features of the long-term strategic policy interaction in the presence of intergenerational F imbalances. An advantage of such approach is the fact that our game theoretic insights are *not* model specific. They can be applied across a wide range of different monetary-fiscal interaction models in the literature.

2.1. **Budget Constraint.** Our exposition of inter-temporal F issues builds on Leeper and Walker (2011) but takes a long-term perspective and thus suppresses the dynamics (which have only second order effects). Consider two periods: period 0 representing the past and period 1 representing the future

$$(1) \qquad B_1 - \lambda \frac{Z_1 - T_1}{L_1} = B_0,$$

where B is the government debt (stock of bonds, where we normalize the interest rate to zero for parsimony) and L_1 is the future price level. $Z_1 - T_1$ is the level of *future net transfers* (transfers Z_1 minus taxes T_1) *promised* to the households by the existing legislation - in nominal terms ('dollars').

The fact that most advanced countries' F predictions suggest an excessive policy stance is uncontroversial, for a discussion see for example Kotlikoff and Burns (2012) or Leeper (2010). To provide just one piece of data, Figure 1 reports IMF (2009) estimates of the net present value of the F impact of aging-related spending and of the 2008 financial crisis (as a percentage of GDP). While the latter are non-negligible, they are

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[10]In relation to that, Section 8 presents a short case study written by Dr Don Brash, Governor of the Reserve Bank of New Zealand during 1988-2002. His contribution describes the developments in New Zealand shortly after the adoption of an explicit commitment to a low-inflation target, and highlights the disciplining effect this M arrangement has had on F policymakers. Empirical evidence on this effect using time-varying parameters VARs is presented in Franta, Libich, and Stehlík (2012) and discussed below.

dwarfed by the former - leading to a large predicted F gap.[11] The simplest way to incorporate such a F gap is to impose

(2) $$Z_1 - T_1 > 0.$$

The *actual* (delivered) level of future net transfers is however $\lambda(Z_1 - T_1)$. This implies that $(1 - \lambda) \in [0, 1]$ is a *reneging* parameter, and that $\frac{\lambda(Z_1 - T_1)}{L_1}$ expresses the delivered net transfers in real terms ('goods').

Intuitively, (1) states that existing debt including interest payments must be paid for by future primary surpluses or, up to a point, by issuing new bonds. Further, promised net transfers can be reneged upon by the government (passive F policy), or their real value inflated away by the central bank (passive M policy). The latter solution has been interpreted in a number of ways, most commonly as the unpleasant monetarist arithmetic of Sargent and Wallace (1981) or Leeper's (1991) fiscal theory of the price level.

Country	Crisis	Aging
Australia	30	482
Canada	21	726
France	31	276
Germany	29	280
Italy	35	169
Japan	35	158
Korea	20	683
Mexico	13	261
Spain	39	652
Turkey	22	204
United Kingdom	48	335
United States	37	495
Advanced G-20 Countries	35	409

FIGURE 1. Net present value of impact on fiscal deficit of the 2008 crisis and aging-related spending respectively, as % of GDP. Source IMF (2009).

2.2. **Policy Objectives.** Before we formally define the active and passive policy stance, we need to postulate the policymakers' utility functions. This is done in a way consistent with the standard intuition of the dynamic policy rules of Leeper (1991):

(3) $$U_i = -\phi_i(L_1 - L^T)^2 - \left(\frac{B_1}{L_1} - b^T\right)^2 - \rho_i(1 - \lambda)^2,$$

where $i \in \{M, F\}$ denotes the policymakers, b^T is the target for real debt, $\phi_i \geq 0$ represents the degree of their inflation conservatism and $\rho_i \geq 0$ is their aversion to reneging

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[11]For example for the United States, Batini et al (2011) provide a recent estimate of the fiscal gap, and conclude that: '... *under our baseline scenario, a full elimination of the fiscal and generational imbalances would require all taxes to go up and all transfers to be cut immediately and permanently by 35 percent. A delay in the adjustment makes it more costly.*'

upon promised net transfers, both relative to debt variability (the middle term).[12] This implies that while the policy targets are the same (for parsimony), the policy weights on these targets may differ between players. Let us define a *responsible* policymaker as one with $\phi_i > \rho_i$. In this paper we will restrict our attention to the relevant case of a responsible central bank

$$(4) \qquad \qquad \phi_M > \rho_M.$$

2.3. **Active/Passive Policies.** Next we adapt Leeper's (1991) terminology to our long-run environment.

Definition 1. *An **active policy stance,** A, provides no adjustment to balance the budget constraint (1). In contrast, a **passive policy stance,** P, is a level L_1^* for M and λ^* for F that provides the full adjustment necessary to balance the budget constraint and keep stable real debt - independently of the other policy (ie assuming the other policy is active).*

Using the A and P dichotomy we can analyze the strategic aspect of the policy interaction as a 2×2 game.

2.4. **Game Theoretic Representation.** The payoff matrix below summarizes the general game with $\{a, b, c, d, v, w, y, z\}$ denoting the policymaker's payoffs in the four possible policy regimes.[13]

(5)

		F	
		Passive PF	Active AF
M	Active AM	Ricardian $a, \quad v$	Explosive $b, \quad w$
	Passive PM	Mis-coordination $c, \quad y$	Non-Ricardian (unpleasant arithmetic) $d, \quad z$

The payoffs are obviously functions of the deep parameters of the underlying macroeconomic structure and policy preferences, in our case equations (1)-(4).[14] In truncating the setup to a 2×2 game we will follow Backus and Driffill (1985) and Cho and Matsui (2005) and offer a transparent example in which the number of free parameters is reduced by normalizing:

$$B_0 = L^T = b^T = T_1 = 1 < Z_1 = 3.$$

We focus on the future drivers of fiscal stress rather than past drivers by starting off with initial debt on target.[15] To derive the PF policy level from Definition 1 use these normalizations in the matrix (1) together with $L_1 = L^T$, which yields $B_1 = B_0$ and implies $\lambda^* = 0$. Similarly, the PM policy level L_1^* is obtained from (1) by imposing

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[12]Note that debt variability is closely positively related to output variability, which is a standard component of reduced-form policy preferences.

[13]As usual, the first payoff refers to the row player (M), and the second to the column player (F).

[14]Further examples of how the linkages between macro models and the game theoretic representation work can be found in the working paper version of this article, and in a different context in Libich and Stehlík (2010).

[15]Nevertheless, all our results apply to the case in which the exisiting debt is excessive and needs to be reduced, ie $\frac{B_0}{L_0} > b^T$.

$\lambda = 1$ and $\frac{B_1}{L_1} = b^T = 1$. This implies $L_1^* = 2$. Combining these with (1)-(2) yields the following debt outcomes in the four policy regimes:

(6)

		F	
		PF ($\lambda^* = 0$)	AF ($\lambda = 1$)
M	AM	Stable real debt $\frac{B_1}{L_1} = \frac{1}{1} = b^T$ Stable nominal debt	Rising real debt $\frac{B_1}{L_1} = \frac{3}{1} > b^T$ Rising nominal debt
	PM	Falling real debt $\frac{B_1}{L_1} = \frac{1}{2} < b^T$ Stable nominal debt	Stable real debt $\frac{B_1}{L_1} = \frac{2}{2} = b^T$ Rising nominal debt

In the Ricardian and non-Ricardian regimes [to use Woodford's (1994) terminology] the F gap is dealt with by F and M policy respectively, and the real debt burden is stable. By contrast, in the (AM, AF) regime neither policy deals with the problem, and hence real debt is on an explosive path. Finally, in the (PM, PF) regime both policies deal with the problem without coordination and hence real debt falls excessively.

2.5. **Possible Policy Scenarios.** Using (6) with (1)-(3) and Definition 1 we can derive the policymakers' payoffs:

(7)

		F		
		PF		AF
M	AM	$-\rho_M,$	$-\rho_F$	$-4, \quad -4$
	PM	$-\phi_M - \rho_M - \frac{1}{4},$	$-\phi_F - \rho_F - \frac{1}{4}$	$-\phi_M, \quad -\phi_F$

Several scenarios, reported in Figure 2, are possible depending on the policy weights.

Scenario	Pure Nash	Mixed Nash	Coordi- nation Problem	Policy Conflict	Parameter Range
Symbiosis	(AM, PF)	no	no	no	$\phi_M > 4 > \rho_F < \phi_F$
Discipline	(AM, PF)	no	no	no	$\phi_M > 4 > \rho_F > \phi_F$
Tug-of-war	(AM, AF)	no	no	yes	$\min\{\phi_M, \rho_F\} > 4$
Pure Coordination	(AM, PF) (PM, AF)	yes	yes	no	$\max\{\phi_M, \rho_F\} < 4$ $\phi_F > \rho_F$
Game of Chicken	(AM, PF) (PM, AF)	yes	yes	yes	$\max\{\phi_M, \rho_F\} < 4$ $\phi_F < \rho_F$
Neglect	(PM, AF)	no	no	no	$\rho_F > 4 > \phi_M$

FIGURE 2. Description of possible policy scenarios

The scenarios differ in the probability of F spillovers onto M policy (in ascending order).

(1) The *Symbiosis scenario* (using Dixit and Lambertini's terminology) whereby the (AM, PF) outcome is the unique Nash equilibrium and also both players' most preferred outcome. Thus F spillovers do not occur.

(2) The *Discipline scenario* in which the (AM, PF) outcome is still the unique Nash equilibrium. Hence the F spillovers are avoided, but the outcome is no longer F's most preferred outcome.

(3) The *Tug-of-war scenario* whereby the (AM, AF) outcome is the unique Nash equilibrium. While spillovers will not occur in the medium-run, this regime cannot obtain in the long-run as the government's budget constraint (1) is not satisfied.[16]

(4) The *Pure Coordination scenario* featuring two pure Nash equilibria, (AM, PF) and (PM, AF), and a mixed strategy Nash equilibrium that is Pareto inferior to both pure Nash equilibria. F spillovers onto M policy may or may not occur, but as both policymakers prefer the former pure Nash equilibrium they are unlikely.

(5) The *Game of Chicken scenario* similarly features two pure Nash equilibria, (AM, PF) and (PM, AF) and one in mixed strategies. But in this case F spillovers onto M policy are more likely as each policymaker prefers a different pure Nash equilibrium.

(6) The *Neglect scenario* in which (PM, AF) is the unique Nash equilibrium, so spillovers surely occur.

For three reasons, our attention will be primarily directed at the Game of Chicken summarized by[17]

$$(8) \qquad \phi_M \in (\rho_M, 4) \quad \text{and} \quad \rho_F \in (\phi_F, 4).$$

First, it is the most interesting scenario from the game theoretic point of view as there are equilibrium selection problems. Specifically, neither standard nor evolutionary game theoretic techniques can provide a clear choice between the pure Nash equilibria due to the symmetry of the game. It is also the only scenario of the six in which the timing of the actions determines the equilibrium. Under the standard commitment concept the Stackelberg leader will ensure his preferred outcome, whereas in the other five scenarios leadership does not alter the (likely) equilibrium play.

Second, the game features both a *conflict* (each player tries to secure his preferred pure Nash equilibrium) and a *coordination problem* (to avoid the inferior mixed Nash). These two characteristics seem to occur in many real world cases as well as in a wide range of policy interaction models: see Leeper (2010), Adam and Billi (2008), Branch, et al. (2008), Resende and Rebei (2008), Benhabib and Eusepi (2005), Dixit and Lambertini (2003, 2001), Barnett (2001), Bhattacharya and Haslag (1999), Artis and Winkler (1998), Blake and Weale (1998), Nordhaus (1994), Sims (1994), Woodford (1994), Leeper (1991), Wyplosz (1991), Petit (1989), Alesina and Tabellini (1987), or Sargent and Wallace (1981). The intuition of our findings will therefore apply to any of these diverse settings.[18]

Third, in this scenario neither policy's preferences are strongly skewed towards one objective. In particular, even a situation of a 'conservative' central bank (with ϕ_M well

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[16]In an important body of work, Davig and Leeper (2010) examine the combination of (AM, PF) - such as in Argentina in 2001 - and (PM, AF) - as many fear holds in the post Great Recession situation, that replaces the (AM, AF) outcome when the economy approaches/hits its F limit.

[17]The remaining scenarios of Figure 2, and alternative ones in which (4) does not hold, are discussed in Section 7.

[18]Switching between the two regimes is a possibility in many of these papers and analogous to our mixed Nash equilibrium.

above ϕ_F and 1) and a government putting more weight onto debt stabilization than buying votes (with ρ_F close to 0) will fall into this category. Put differently, in the non-Nash (AM, AF) and (PM, PF) regimes real debt is on an unstable path and thus these cannot be equilibria from a long-term perspective.

3. DYNAMIC COMMITMENT

This section postulates an asynchronous game framework with generalized timing of actions.[19] Our goal is to examine how the medium-run macroeconomic outcomes of the policy interaction depend on this timing and other variables.

3.1. **Timing.** After a synchronized initial move in period $t = 1$ each player i moves with a certain *constant frequency*, namely every $r^i \in \mathbb{N}$ periods. Figure 3 offers an example of such timing.

The variable r^i can be interpreted as the degree of commitment *or* rigidity of player i. These two concepts are formally identical in our framework, both referring to the players' inability to move. Nevertheless, in the real world such inability comes from different sources, which we will acknowledge by referring to r^M as long-term M commitment and to r^F as F rigidity. The degree of the latter is arguably closely linked to the size of the country's F gap, $Z_1 - T_1$, since many government outlays related to aging populations are mandatory. In terms of the former, let us stress again that r^M does not denote the frequency of the central bank's interest rate decisions in our medium-run environment. It should be interpreted as the frequency with which the central bank can alter its target for average inflation, which typically depends on whether/how its commitment to the target is legislated.

3.2. **Assumptions and Notation.** For maximum compatibility our framework adopts all the assumptions of a standard repeated game. First, commitment and rigidity r^i are exogenous and constant throughout each game. Second, they are common knowledge. Third, all past periods' moves can be observed. Fourth, the game starts with a simultaneous move. Fifth, players are rational, have common knowledge of rationality; and for expositional clarity they have complete information about the structure of the game and the opponent's payoffs. These assumptions can easily be relaxed (some alternatives, such as endogenous r^i, will be discussed below). They are introduced here so that the only difference from the standard repeated game is in allowing $r^i > 1$ values that differ across players.

This specification implies that the *stage game* of our asynchronous setup is itself an extensive-form game lasting T periods, where $T \in \mathbb{N}$ is the 'least common multiple' of r^M and r^F. For instance, the dynamic stage game in Figure 3 with $r^M = 5$ and $r^F = 3$ is $T = 15$ periods long.

Denoting n^i to be the i's player's n'th move (not period), and N^i the number of moves in the asynchronous stage game, it follows that $N^i = \frac{T(r^M, r^F)}{r^i}$. Also, M_n^l and F_n^l will denote a certain action $l \in \{A, P\}$ at a certain node n^i; eg F_2^A refers to AF in the government's second move.

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[19]For more details on the game theoretic aspects of the framework see Libich and Stehlík (2010).

For the rest of this section we assume some $r^i > r^j$ without loss of generality, where $i \in \{M, F\} \ni j$. We can then denote $\frac{r^i}{r^j} \geq 1$ to be the players' *relative* commitment/rigidity. Also, $\left\lfloor \frac{r^i}{r^j} \right\rfloor \in \mathbb{N}$ will be the integer value of relative commitment (the floor) and

$$(9) \qquad R = \frac{r^i}{r^j} - \left\lfloor \frac{r^i}{r^j} \right\rfloor = [0, 1),$$

denotes the fractional value of relative commitment (the remainder).[20] It will be evident that R plays an important role as it determines the exact type of asynchrony in the game. Note also that if $R > 0$ both players take leadership at some point during the stage game.

Further, we denote $B(.)$ to be the best response. For example, $F_1^P \in B(M_1^A)$ expresses that F_1^P is F's best response to M's initial A move, and $\{F_1^P\} = B(M_1^A)$ expresses that it is the unique best response. Finally, asterisk will denote an optimal play, eg $F_1^* \in B(M_1)$ expresses that F's optimal play in move 1 is the best response to M's first move.

3.3. **Recursive Scheme.** The fact that we will be able to present proofs for general values of r^i's is due to the existence of a recursive scheme in the moves. Formally, let k_n be the number of periods between the n^i-th move of player i and the immediately following move of player j (for a graphical demonstration see Figure 3). Using this notation we can summarize the recursive scheme of the game as follows:

$$(10) \qquad k_{n+1} = \begin{cases} k_n - R r^j & \text{if} \quad k_n \geq R r^j, \\ k_n + (1 - R) r^j & \text{if} \quad k_n < R r^j. \end{cases}$$

Generally, k_n is a non-monotonic sequence.

3.4. **History, Future, Strategies, and Equilibria.** By convention, history in period t, h_t, is the sequence of actions selected prior to period t. And the future in period t is the sequence of current and future actions. It follows from our perfect monitoring assumption that h_t is common knowledge at t. Let us refer, following Aumann and Sorin (1989), to moves in which a certain action $l \in \{A, P\}$ is selected for all possible histories as *history-independent*.

A strategy of player i is a vector that, $\forall h_t$, specifies the player's play $\forall n^i$. The asynchronous game will commonly have multiple Nash equilibria. To select among these we will use a standard equilibrium refinement, subgame perfection, that eliminates non-credible threats. Subgame perfect Nash equilibrium (SPNE) is a strategy vector (one strategy for each player) that forms a Nash equilibrium after any history h_t.

Given the large number of nodes in the game we focus on the equilibrium path of the stage game SPNE, ie the actions that would actually get played. In doing so we will use the following terminology.

Definition 2. *(i) Any SPNE of the asynchronous stage game that has, throughout its whole equilibrium path, the **Ricardian regime** (AM, PF) will be called a **Ricardian SPNE**. The case in which **all** SPNE of the game are Ricardian will be called a **Ricardian World**.*

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[20]For example, in Figure 3 we have $\left\lfloor \frac{r^M}{r^F} \right\rfloor = \lfloor \frac{5}{3} \rfloor = 1$, and $R = \frac{2}{3}$.
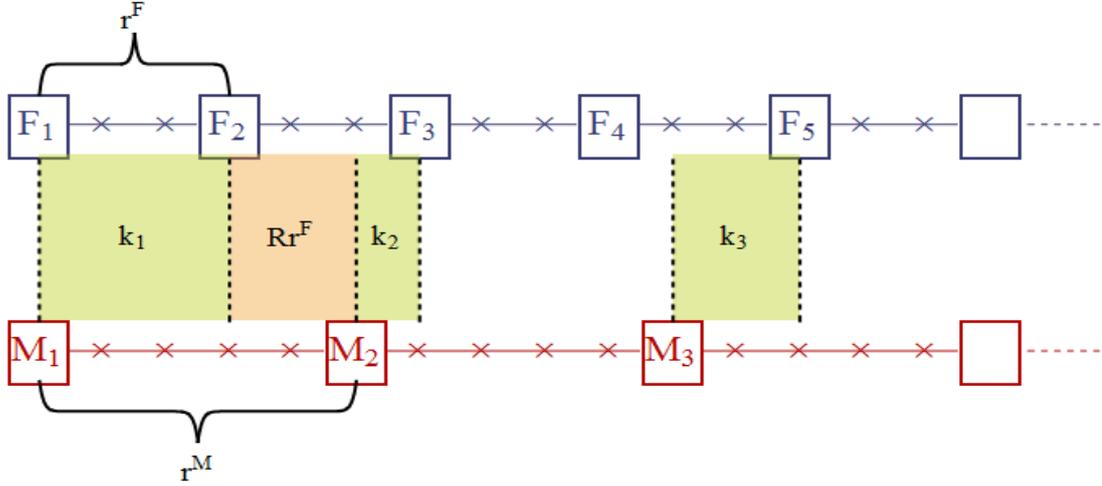
FIGURE 3. An asynchronous stage game with deterministic commitment/rigidity: illustration of the recursive scheme and the concepts of r^i, R, k and n^i under $r^F = 3, r^M = 5$.

*(ii) Any SPNE with the **non-Ricardian regime** (PM, AF) throughout its whole equilibrium path will be called a **non-Ricardian SPNE**. The case in which **all** SPNE of the game are Ricardian will be called a **non-Ricardian World**.*

*(iii) Any SPNE other than Ricardian and non-Ricardian will be called a **Regime-switching SPNE**. The case in which there exist multiple types of SPNE (any combination of Ricardian, non-Ricardian, and Regime-switching) will be called a **Regime-switching World**.*

3.5. **(Non)-Repetition.** As our focus is on conditions under which an efficient outcome uniquely obtains on the equilibrium path of the asynchronous stage game, its further repetition can be disregarded without loss of generality. Intuitively, if the effective minimax values of the players in the dynamic stage game [that are the infima of the players' subgame perfect equilibrium payoffs, see Wen (1994)] are unique and Pareto-efficient, then the effective minimax values of the repeated game (with any finite or infinite number of repetitions) will also be the same. Put differently, the set of Pareto-superior payoffs is empty as we are already on the Pareto-frontier. The uniqueness property also implies that we can focus on pure strategies only, without loss of generality.[21]

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[21]Repetition is commonly used to help alleviate inefficiency and enhance cooperation through reputational channels, see eg Mailath and Samuelson (2006), or in the monetary context Barro-Gordon (1983). The advantage of focusing on the dynamic stage game is that it provides the worst case scenario in which reputation cannot help secure cooperation.

4. Results in the Game of Chicken Without Discounting

It is important to recall Section 2 in which we showed that the Game of Chicken type interactions arise under *some* (but not all) parameter values depending on the model and policy preferences. To develop the intuition of the game theoretic framework, this section reports results for the specific payoffs in (7)-(8) derived from the macro setup, and fully patient players with discount factors $\delta_M = \delta_F = 1$. Section 5 will then allow for the policymakers' impatience, and solve the game for the general Game of Chicken payoffs in (5), namely

$$
(11) \qquad a > d > \max\{b, c\} \quad \text{and} \quad z > v > \max\{w, y\}.
$$

Proposition 1. *Consider the Game of Chicken (7)-(8) without discounting.*
*(i) The **Ricardian World** occurs if and only if M commitment is **sufficiently strong** relative to F rigidity*

$$
(12) \qquad r^M > \overline{r^M}\left(\underset{+}{r^F}, \underset{-}{\phi_M}, \underset{+}{\rho_M}, \underset{+}{\rho_F} \right) > r^F.
$$

*(ii) The **non-Ricardian World** occurs if and only if M commitment is **sufficiently weak** relative to F rigidity*

$$
(13) \qquad r^M < \underline{r^M}\left(\underset{+}{r^F}, \underset{+}{\rho_F}, \underset{-}{\phi_F}, \underset{-}{\phi_M} \right) < r^F.
$$

*(ii) The **Regime-switching World** occurs if and only if M commitment is neither sufficiently strong nor sufficiently weak relative to F rigidity*

$$
(14) \qquad r^M \in \left[\underline{r^M}, \overline{r^M} \right].
$$

Proof. Appendix A derives the exact form of the necessary and sufficient thresholds $\overline{r^M}$ and $\underline{r^M}$. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ □

Figure 4 summarizes these results graphically. The $\frac{r^M}{r^F}$ space is expanded compared to a one shot (or simultaneously repeated) game where $\frac{r^M}{r^F} = 1$, and the static concept of commitment (Stackelberg leadership) where $\frac{r^M}{r^F} \in \{0, \infty\}$.

The space can now be broken into three main equilibrium regions as defined in Definition 2. Note that, in contrast to Stackelberg leadership, our framework gives us additional valuable information. Among other things, it tells us the exact degree of commitment/rigidity required if a player's preferred outcomes is to be achieved. The threshold is a function of several variables: see (16) below for an example. Furthermore, it shows that once richer dynamics/leadership are allowed there may still be regime switches. This is unlike the static Stackelberg leadership where the intermediate region does not exist.

Let us sketch the intuition behind this fairly complex proof using the simple case $r^M = 2$, $r^F = 1$ (which is a special case of $R = 0$). For the central bank to ensure the Ricardian World obtains, it must be willing to engage in a costly tug-of-war. That is, the bank's compatibility relation between the costs and benefits of alternative courses
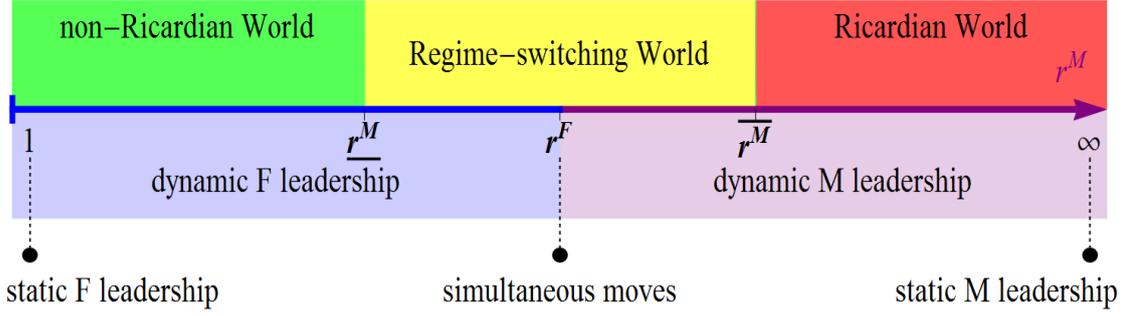
FIGURE 4. The r^M space featuring the thresholds and regions of SPNE.

of action, $B(F_1^A) = \{M_1^A\}$, has to hold

(15)
$$\underbrace{br^F}_{\substack{(AM, AF) \\ \text{policy conflict}}} + \underbrace{a(r^M - r^F)}_{\substack{(AM, PF) \\ \text{M victory}}} > \underbrace{dr^M}_{\substack{(PM, AF) \\ \text{M surrendering}}}.$$

Rearranging, and using the payoffs in (7)-(8) yields the following threshold value

(16)
$$r^M(R=0) > \overline{r^M}(R=0) = \frac{\overbrace{(4 - \rho_M)}^{\text{M conflict cost}}}{\underbrace{\phi_M - \rho_M}_{\text{M victory gain}}} r^F.$$

If this condition is satisfied then the central bank's 'victory reward' more than offsets its initial 'conflict cost', and hence the bank is not willing to accommodate excessive F policy. Such M determination to fight if necessary eliminates the incentive of F to run structural deficits and accumulate debt. In other words since the M threat of a tug-of-war is credible, there is in fact no such policy conflict in equilibrium as F 'surrenders' from its initial move.

The implication is that in a Game of Chicken sufficiently strong M commitment has two consequences. Is not only capable of shielding the central bank from F pressure and spillovers, but it can also discipline F policy by improving the government's incentives and equilibrium play. Section 7.3 presents a short case study by Dr Don Brash documenting how this actually happened in New Zealand. He explains that adoption of a stronger M commitment gave him, as Governor, more ammunition to stand-up to excessive F policy; and that this in turn has had a disciplining effect on F policymakers.

In contrast, if $r^M \in \left[\underline{r^M}(r^F), \overline{r^M}(r^F)\right]$ then the victory reward is insufficient to compensate M for the initial conflict cost with F. The threat of not accommodating M policies is no longer fully credible, and thus we move from the Ricardian World to the Regime-switching World in which F spillovers may occur. If the degree of M commitment (relative to the size of the F gap) is even lower, $r^M < \underline{r^M}(r^F)$, then we enter the non-Ricardian World in which F spillovers surely occur. It is now F who is willing to engage in a costly tug-of-war with M.

The proof of Proposition 1 (Appendix A) also shows that the nature of this special case $R = 0$ carries over to more general asynchronous cases with $R > 0$. This may appear surprising because in the latter case both players take the role of the leader during the dynamic stage game. For example in Figure 3 there are four changes in leadership (in F_2, M_2, F_3, and M_3); and because of that there are multiple periods of potential policy conflict with the decisions about them intertwined. The intuition for this result is twofold. First, for any R, the player with lower r^i makes the last revision which can be exploited by the opponent. Second, the most 'important' action happens in the initial simultaneous move since the conflict cost is at its maximum and would last the longest relative to the victory reward. This move therefore yields the strongest incentive compatibility condition. Formally proving this result - which is not obvious by any means - is one of the methodological contributions of this paper. This results also highlights the usability of our asynchronous game: users can focus on the start of the game and 'ignore' the rest of the complicated dynamics.

The findings are in contrast to those under standard Stackelberg commitment, whereby the leader (committed player) wins the game independently of any structural or policy parameters. The results of Proposition 1 can be viewed as a refined version of the conventional result. They offer a possible explanation for the observed institutional differences across countries, and a richer basis for policy recommendations.

Corollary 1. *(i) Greater aversion of the government to reneging on promised net transfers, ρ_F, increases the parameter region of the non-Ricardian World, and (together with the central bank's reneging aversion ρ_M) decreases the region representing a Ricardian World.*
(ii) A more explicitly committed central banker (higher r_M) can be less conservative (have lower ϕ_M) while still ensuring a Ricardian World.

Claim (i) is intuitive: greater reneging aversion increases the perceived (political) cost of F reform, and thus makes the non-Ricardian solution to the F gap problem (and the unpleasant monetarist arithmetic) more likely.

Claim (ii) is perhaps surprising as it implies partial substitutability of strict and explicit inflation targeting, but it can be seen in (12) where $\overline{r^M}$ is decreasing in ϕ_M. The more explicitly committed the M regime is, the less strict on inflation it has to be. Claim (ii) is at odds with concerns by inflation targeting sceptics such as Greenspan (2003) or Kohn (2005) who believed that an explicit inflation target reduces the period by period M policy flexibility needed to stabilize the real economy. But it is in line with Woodford who called such concerns the '*traditional prejudice of central bankers*' and Svensson (2008) who argued that: '*it is desirable to do flexible inflation targeting more explicitly*'. Similarly, recent empirical evidence of Kuttner and Posen (2011) or Creel and Hubert (2010) shows that explicit inflation targets have *not* led to stricter M policy during the global financial crisis.

It is apparent that, in the Regime-switching World, the *variability* of (trend) inflation and debt is generally higher than in a Ricardian World due to changes in the policy stance. Cycles in average inflation and debt can be generated under some circumstances. We leave a more detailed investigation of this intermediate region for future research, and just report one finding that qualifies the intuition of the standard Stackelberg commitment in an important way.

Proposition 2. *Consider the Game of Chicken (7)-(8) and some $r^M \in \left(r^F, \overline{r^M}\right)$. Despite the central bank acting as the leader for longer periods of time than the government, there are parameter values under which the bank's preferred Ricardian SPNE does not exist, but the government's preferred non-Ricardian SPNE does.*

Proof. Appendix B shows that this happens if $\phi_M < \overline{\phi_M}$ and $\rho_F > \overline{\rho_F}$, ie when the cost of surrendering (passive policy) is sufficiently low for the central bank, but sufficiently high for the government. $\square$

The fact that the player with a longer leadership period (higher r^i) cannot achieve its preferred outcome, while the opponent can, is in stark contrast with the standard Stackelberg leadership conclusion.

It is useful, as a matter of methodology, to recognize how this game with dynamic commitments has been solved. In a conventional dynamic game, an equilibrium solution can be found by backwards recursions to each decision point, taking account of any subsequent decisions to be made and conditional on the last known state of the system before that decision. But such an approach is only valid if each player's problem can be written as one of optimizing an additively recursively separable constrained objective function at each t, so that Bellman's (1961) principle of optimality applies. As an approach it breaks down if the constraints contain forward looking behaviour: for example when rational expectations of future outcomes are present.

Our solution also has that feature since, at each decision point, each player has to know if a given strategy is currently superior to the others, *and* will continue to remain superior long enough for that player to enjoy those gains long enough to dominate the payoffs available under the other strategies. The latter component of the calculation is the forward looking element; and will depend on the relative discount factors, and the relative lengths of commitment/rigidity (which determine when the opponent has the next chance to wrest control of the gains back to his own preferred strategy).

The methodology point here is that forward and backward dependencies arise in dynamic models with rational expectations, of course. However, our analysis shows that asynchronous games, even without explicit rational expectations in the constraints, are a second case in point.

5. EXTENSION I: DISCOUNTING AND GENERAL PAYOFFS

This section introduces discounting for both players, $\delta_M < 1, \delta_F < 1$, and solves the Game of Chicken for general payoffs (11). It will become apparent that while the nature of the above game theoretic analysis is robust to discounting, the players' impatience may change the outcomes in an important way. We will focus on deriving conditions of the Ricardian World. But as Proposition 1 demonstrates, the results apply analogously for the non-Ricardian one.

Proposition 3. *Consider the general Game of Chicken (11) with discounting. The Ricardian World occurs iff the M policymaker is both sufficiently patient*

$$(17) \qquad \delta_M > \overline{\delta_M} = \sqrt[r^F]{\frac{d-b}{a-b}},$$

and sufficiently strongly committed

$$(18) \qquad r^M > \overline{r^M} \left(r^F, \delta_M, \delta_F, a, b, d, v, w, y, z \right) \geq r^F.$$

If M is insufficiently patient, $\delta_M < \overline{\delta_M}$, then $\overline{r^M}$ does not exist, and even infinite commitment $\frac{r^M}{r^F} \to \infty$ cannot ensure the Ricardian World.

Proof. See Appendix C that derives the exact form of the necessary and sufficient threshold in (18), namely equation (39).[22] □

This implies that $\overline{r^M}$ is a step function of F's payoffs, specifically increasing in v and y, and decreasing in w and z. In terms of the other variables while we cannot formally prove the relationships for all R, valuable insights can nevertheless be obtained from the special case of $R = 0$ which was shown above to be representative of the more asynchronous cases $R > 0$.

The threshold $\overline{r^M}(0)$ is increasing in r^F, δ_F and d, and decreasing in δ_M, a and b: see Figure 5 for a graphical demonstration.[23] Intuitively, M's impatience strengthens the necessary and sufficient condition; it makes it more difficult (and eventually impossible) for M to dominate and ensure a Ricardian World. The intuition is similar to a standard repeated game in which it is harder to deter an impatient player from defecting since the future reward for not defecting has a smaller present value. The policy implication is therefore the following: a less patient central banker needs to commit more explicitly to guarantee his preferred medium-term outcomes. This implies partial substitutability between an explicit inflation target and longer mandates for central banks.

Proposition 3 not only refines the standard result obtained under Stackelberg leadership, it also qualifies its intuition substantially. Under static commitment, the (more) committed player *always* ensures its preferred regime. Under dynamic commitment there are parameter regions in which he *may* do so, and parameter regions in which he *never* does so. If the more committed player is highly impatient, then even an infinitely strong commitment cannot ensure his preferred regime regardless of the opponent's discount factor. Hence the insights obtained under the standard commitment concept are not robust.

Relating this result back to Figure 4, if $\delta_M < \overline{\delta_M}$ there are only non-Ricardian and Regime-switching Worlds; and only the latter if $\delta_M < \overline{\delta_M}$ and $\delta_F < \overline{\delta_F}$ since in such case neither $\underline{r^M}$ nor $\overline{r^M}$ exist.

6. EXTENSION II: FISCAL HETEROGENEITY IN A MONETARY UNION

The debt crisis in the Eurozone has recently received a lot of attention. Our dynamic commitment framework can offer some insights as it can easily incorporate any number of players. Let us examine the case in which F policy is heterogeneous focusing on two types of heterogeneity: in economic size and in F rigidity. This describes the situation in the European Monetary Union, and the United States to some extent, with a common currency and hence common M policy, but somewhat independent F policies.

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[22]The patience and commitment thresholds for the non-Ricardian World are again mirror images of $\overline{\rho_M}$ and $\overline{r^M}$.

[23]Formal proofs of these relationships appear in the working paper version of this article.
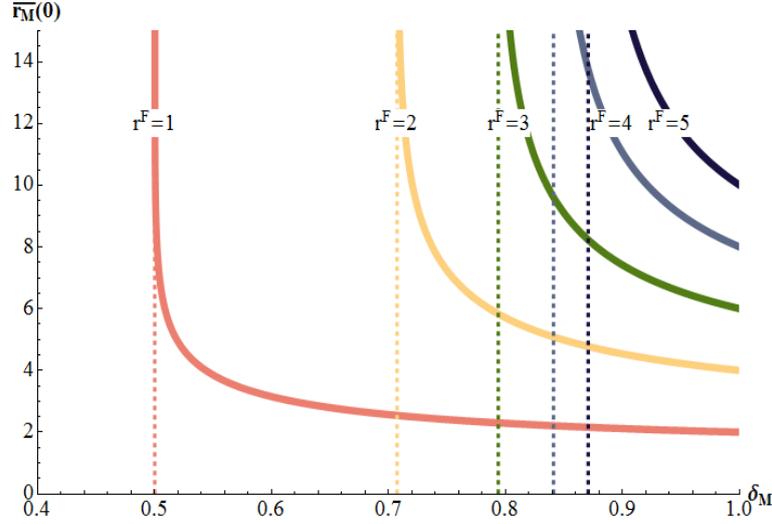
FIGURE 5. Dependence of $\overline{r^M}(0)$ on δ_M for various r^F from the necessary and sufficient condition (31) for the game in (7) (for illustration using $\phi_M = 2$ and $\rho_M = 0$). The Ricardian World occurs in the area to the *right* of the curves. The dotted asymptotes correspond to the bounds $\overline{\delta_M}(0)$ for each particular r^F.

Formally, the set of players is now $I = \{M, F^j\}$ where $j \in [1, J]$ denotes a certain member country, r_j^F its degree of F rigidity, and s_j its relative economic size such that $\sum_{j=1}^J s_j = 1$. We assume that the overall payoff of M is a weighted average of the bank's payoffs gained from the interaction with each F^j - with weights s_n. The payoff of each independent government is, however, directly determined by its own actions and those of the common central bank as shown in (5).[24]

In extending our analysis to this case we will first assume the absence of free-riding by union members. In that case, the nature of the above results remains unchanged.

Remark 1. *In a monetary union, the necessary and sufficient threshold $\overline{r^M}$ in the Game of Chicken is increasing in the weighted average of the F rigidities of member countries with the weights being the country sizes s_j. Formally, r^F in (12)-(13) and (17)-(18) is replaced by $\sum_{j=1}^J s_j r_j^F$.*[25]

However, a moral hazard problem may occur on the part of individual governments. This is because the political benefits of F spending accrue primarily to the fiscally irresponsible country, whereas the economic costs in terms of tighter M policy are spread across all countries [Masson and Patillo (2002)].

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[24]Indirectly, the actions of other governments also have an impact since they determine the action of the central bank, and hence the equilibrium outcomes in each country.

[25]To offer a numerical example, assume $\phi_M = 1 > \rho_M = 0$; and a union of two countries with one double the size and double the F rigidity of the other (assumed to be $r^F = 2$). For the Ricardian World in the whole union in the $R_j = 0, \forall j$ case it is required that $r^M > \overline{r^M} = \frac{4}{\phi_M} \sum_{j=1}^J s_j r_j^F = \frac{2}{3}4 + \frac{1}{3}2 = \frac{10}{3}$.

In particular, the smaller a country is relative to the union, the less impact its F policy has on average inflation and output forecasts in the union - and hence on the interest rate response of the common central bank. Furthermore, the punishment in the form of an ensuing M contraction is also spread across the union as a whole. Hence even disciplined governments are penalized. The incentives for free-riding, whether deliberate or out of myopia or neglect, can therefore rise rapidly especially for the smaller countries.

To formalize this, denote by $m_j \in [0, 1]$ the *degree of free-riding*: ie the extent to which a member country j does *not* internalize the negative impact of its F excesses onto the rest of the union members. The value $m_j = 0$ denotes no free-riding, whereas $m_j = 1$ denotes extreme free-riding in which country j totally ignores its impact on others.

The effect of this free-riding can be can be incorporated in our analysis through the payoff v_j. That parameter represents the government's surrender payoff relative to the conflict cost. It seems natural to assume the j government's conflict cost to be increasing in the country's weight s_j, and decreasing in its degree of free-riding m_j:

$$(19) \qquad \frac{\partial v_j (m_j, s_j)}{\partial m_j} < 0, \qquad \text{and} \qquad \frac{\partial v_j (m_j, s_j)}{\partial s_j} > 0 \text{ for all } m_j > 0.$$

That is, if the government of member country j decides to free-ride, it will face (and internalize) a smaller punishment from the common central bank than it would have encountered from the country's independent central bank prior to joining the monetary union.

Let us further assume that v_j is a monotone function of m_j, and that $v_j (m_j = 1) < w_j$. We can now contrast the outcomes in some country j before ($\mathcal{B}$) and after ($\mathcal{A}$) joining the M union. The case before joining the union is naturally $m_j^{\mathcal{B}} = 0$ and $s_j^{\mathcal{B}} = 1$. After joining, we have some $m_j^{\mathcal{A}} \in [0, 1]$ and $s_j^{\mathcal{A}} \in (0, 1)$.

Proposition 4. *(i) Consider country j described by the Game of Chicken (11) and $r^M > \overline{r^M}$ ensuring a Ricardian World. After joining a monetary union, if the degree of j's free-riding is above a certain (country specific) threshold, $m_j > \overline{m_j}\left(s_j, v_j^{\mathcal{B}}\right)$, then j's accession leads to a deterioration of its stance from PF to AF.*
(ii) The common central bank's commitment threshold $\overline{r^M}$ that ensures AM is increasing in the combined size of such countries with $m_j > \overline{m_j}$. If this size is sufficiently high then the Ricardian World is not achieved even if the bank is both fully patient and infinitely strongly committed, $\delta_M = 1, r^M \to \infty$.

Proof. See Appendix D. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $\square$

Intuitively, the threat of punishment by the common central bank is no longer enough to discipline the government of a sufficiently small union member country with a sufficiently high degree of free-riding. It is no longer within the bank's powers to discourage such countries from F excesses through its M actions.[26] This is because AF becomes a strictly dominant strategy for the free-riding government in the underlying game, and the scenario from its perspective changes from the Game of Chicken to Neglect.

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[26]Greece comes to mind as an example of this type of behaviour.

In terms of claim (ii), the central bank is worried about the cost of the policy conflict and therefore once the combined active stance of F policies reaches a certain level the bank will no longer play AM as the resulting conflict would be too widespread and costly. The bank therefore starts accommodating such F policy, which leads to sustained overshooting of its optimal inflation level - even if it is highly explicit. In such case the central bank's instrument independence has been seriously compromised along the lines of the unpleasant monetarist arithmetic.[27] It remains to be seen whether such narrative applies to the situation in the Euro area. But it demonstrates the overexpansion bias remains even if the impact on interest rates is small.

7. Alternative Scenarios and The Real World

7.1. **Alternative Scenarios.** It should by now be apparent that in the *Symbiosis, Discipline, Tug-of-war* and *Neglect scenarios* dynamic commitment - like static commitment - will not alter the outcomes of the game. This is because the underlying normal-form game has a unique (and efficient) Nash equilibrium with at least one player having a strictly dominant strategy.

In the *Pure coordination scenario*, the condition for selecting a Ricardian equilibrium is the same, both qualitatively and quantitatively, as in the Game of Chicken. Even if it is not satisfied, the probability of arriving in a non-Ricardian regime is arguably lower due to the focal point argument. Moreover, if there exists uncertainty about the type of government (it may change over time with elections over the political cycle), implementing a high M commitment can act as 'credibility insurance' against M policy being undermined by future irresponsible governments.

If both policymakers are irresponsible - a situation more likely to be observed in countries without a formally independent central bank - under some circumstances we can observe two additional scenarios of interest. First, in the *Battle of the Sexes scenario* there are two pure Nash equilibria, (AM, AF) and (PM, PF), each preferred by a different player. The intuition is analogous to the Game of Chicken: a sufficiently patient player that is sufficiently strongly committed (relative to the opponent) will ensure his preferred outcome. While Pareto-efficiency is ensured in such case, we never reach a Ricardian World.

Second, in the *Prisoner's dilemma* the non-Ricardian regime is the unique Nash despite being Pareto dominated by the Ricardian outcome. In such a case, static commitment does not alter the outcomes, and thus dynamic commitment cannot help escape the inefficient equilibrium either. One would need to allow for repetition of the dynamic stage game to enable various tit-for-tat strategies to emerge and help the policies cooperate.

7.2. **Real World Interpretation.** It is not possible to unambiguously connect real world countries with the above scenarios. This is not only because policy preferences and payoffs change over time. It is also because we observe the actual outcomes rather

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[27]As Bernanke (2005) argued: '*No monetary-policy regime, including inflation targeting, will succeed in reducing inflation permanently in the face of unsustainable fiscal policies - large and growing deficits.*'

than the underlying preferences, and these may already be influenced by legislated commitment devices. Moreover, observed outcomes are not necessarily the equilibrium ones; they may reflect a transitory (off-equilibrium) phase.

To give an example, countries with observed medium-run (AM, PF) such as Australia, New Zealand, and most Nordic countries, could in principle be described by the Symbiosis, Discipline, or Pure coordination scenarios under any $\frac{r^M}{r^F}$, or by the Game of Chicken under $r^M > \overline{r^M}\left(r^F\right)$. Similarly, countries in which we observe AF - most industrial ones including the United States and many Eurozone members - could fall into the Tug-of-war scenario, or the initial 'conflict phase' of the Game of Chicken.

Despite these caveats, it is important to note that such uncertainty does not alter the main prescription of our analysis: M policy should be made strongly/explicitly committed in the long-term (but not necessarily more conservative in its short-term responses). This will increase the range of circumstances under which the socially optimal outcomes obtain. Nevertheless, we have seen that this is not sufficient in all scenarios. Therefore, in order to 'cover all bases' and guarantee the Ricardian regime regardless of the type of government, transparent and accountable commitments should apply directly to F policy as well, with oversight by an independent F policy council. This has been argued forcefully by Leeper (2010) and others before him,[28] but only implemented in a minority of countries.

7.3. **Case Study on the Effect of Monetary Commitment on Fiscal Policy.** Dr Don Brash, Governor of the Reserve Bank of New Zealand during 1988-2002 in which period the Bank pioneered its explicit inflation targeting framework, wrote in private correspondence the following in response to our analysis (quoted with permission):

'*New Zealand provides an interesting case study illustrating the arguments in the article. We adopted a very strong commitment by the monetary authority, the Reserve Bank of New Zealand, when the Minister of Finance signed the first Policy Targets Agreement (PTA) with me as Governor under the new Reserve Bank of New Zealand Act 1989 early in 1990. The PTA required me to get inflation as measured by the CPI to between 0 and 2% per annum by the end of 1992, with the Act making it explicit that I could be dismissed for failing to achieve that goal unless I could show extenuating circumstances in the form, for example, of a sharp increase in international oil prices. At the time, inflation was running in excess of 5%.*

In the middle of 1990, the Government, faced with the prospect of losing an election later in the year, brought down an expansionary budget. I immediately made it clear that this expansionary fiscal policy required firmer monetary conditions if the agreed inflation target was to be achieved, and monetary conditions duly tightened.

Some days later, an editorial in the "New Zealand Herald", New Zealand's largest daily newspaper, noted that New Zealand political parties could no longer buy elections because, when they tried to do so, the newly instrument-independent central bank would be forced to send voters the bill in the form of higher mortgage rates.

I was later told by senior members of the Opposition National Party that the Bank's action in tightening conditions in response to the easier fiscal stance had had a profound effect on thinking about fiscal policy in both major parties in Parliament.

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[28]For example, in Hughes Hallett and Weymark and Hughes Hallett (2008).

Some years later, in 1996, the Minister of Finance of the then National Party Government announced that he proposed to reduce personal income tax rates subject to this being consistent with the Government's debt to GDP target being achieved, to the fiscal position remaining in surplus, and to the fiscal easing not requiring a monetary policy tightening. The Minister formally wrote to me asking whether tax reductions of the kind proposed would under the economic circumstances then projected, require me to tighten monetary conditions. Given how the Bank saw the economy evolving at that time, I was able to tell the Minister that tax reductions of the nature he proposed would not require the Bank to tighten monetary conditions in order to stay within the inflation target.'

8. Summary and Conclusions

The stance of F policy in a number of countries has raised concerns about the degree of discipline, and about the risks for the credibility and outcomes of M policy. While the global financial crisis contributed to the problem, the underlying causes of these concerns had existed long before the crisis.[29] This debate can be summarized as: Under what circumstances will we observe the Ricardian regime (AM, PF) and the non-Ricardian regime (PM, AF) respectively?

To contribute to this debate and derive the policy regimes endogenously, we use a novel asynchronous game theory framework that generalizes the standard Stackelberg leadership commitment concept from static to dynamic in an intuitive way.[30] We show that the conventional wisdom derived under standard static commitment is not robust, and that the risks of F spillovers undermining M policy may be greater than a conventional analysis would suggest.

Our investigation shows that the effect of M commitment on economic outcomes of the policy interaction crucially depends on its explicitness *relative* to the degree of F rigidity, the size of the F gap, as well as other policy parameters. The problem is that under a range of circumstances inferior M policy outcomes (higher inflation and lower credibility) can occur due to spillovers from an excessive F policy - even if the central bank is independent, responsible, patient, and strongly committed. As Davig et al (2010) note: *'Without significant and meaningful fiscal policy adjustment, the task of meeting inflation targets will become increasingly difficult.'*

To offer some constructive conclusions, we have identified the scenarios and circumstances under which M policy outcomes will not be compromised by long-term F excesses; ie the active M passive F policy equilibrium prevails. They require the central bank to be sufficiently patient as well as sufficiently strongly committed. Interestingly,

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[29]For example the IMF (2009) estimates the contribution of the crisis to the observed fiscal stress to only be 10.8% of that of the aging population related spending in G20 countries.

[30]The existing literature on asynchronous games has not fully investigated the commitment properties associated with infrequent timing that have important policy implications. This is because it focused on either the alternating move case in which the commitment periods are the same across the players [eg Lagunoff and Matsui (1997)], or are fixed multiples of each other [Wen (2002)]. These cases could not detect our key result that the ratio of commitments must be beyond a certain threshold to obtain discipline. The literature has also considered cases where the periods of commitment and hence timing of moves are without any consistent time pattern [eg Takahashi and Wen (2003) or Yoon (2001)] which means we are no longer dealing with a world of commitment games.

under those conditions M policy may not only resist F pressure coming from an ambitious F setting, its commitment may also discipline the government by reducing its payoff from excessive spending through a credible threat of a costly tug-of-war. We formally examine how the explicitness of long-run M commitment r^M can tip the balance between the two policies.

Our proposed channel is different from Rogoff (1985) and Walsh (1995). It highlights the (desirable) constraints associated with a legislated long-term objective, and may explain why many inflation targeting countries achieved sound outcomes without becoming excessively strict on inflation or legislating a formal incentive contract/dismissal procedure for the central bank.

What are the policy implications of these results? As we stressed at the outset our analysis does not examine or offer any recommendations for short-term stabilization. In particular, it does not assess the case for F stimulus vs austerity in the current situation of a weak post financial crisis recovery. But it does offer *medium to long-term* recommendations, ie what to do once economies have fully recovered from the financial crisis.

The long-term lesson for M policymakers is that, to discourage and/or counter-act (structurally) excessive F policies, they should if possible commit to low average inflation more explicitly. The Federal Open Market Committee's January 25, 2012 press release 'subscribing' to the 2% long-term inflation objective more openly than ever before is consistent with the prescription of our analysis.[31]

This is desirable primarily in non-inflation targeting countries facing long-term F sustainability issues such as the United States, Switzerland, and Japan, and the January 2012 announcements of the Fed go in this direction. The implication for F policymakers is that imposing such M commitment onto their central banks may provide a way to indirectly tie their hands, and gain political support for reforms towards F sustainability.[32]

We identify two important caveats to this finding. First, we show that if the government is too myopic then it will not be disciplined even by a fully patient and infinitely strongly committed central bank. Second, we show that the disciplining channel is unlikely to be effective in a currency union where a moral hazard problem due to free-riding of small member countries occurs naturally. If countries ignore the negative externality they impose on others, the M punishment they face from the common central bank is not strong enough. In such cases direct F commitment arrangements, ie legislated *and* enforceable F rules are necessary to discipline F policy over the long term. Such rules, if correctly formulated, seem to be desirable - as an 'insurance' - in all countries given that political preferences and realities often change.

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[31]And so is the Committee's explanation that: 'Communicating this inflation goal clearly to the public helps keep longer-term inflation expectations firmly anchored.' Similarly are the European Central Bank's president's statements such as (from end-May 2012): 'It's not our duty, it's not in our mandate ... (to) ... fill the vacuum left by the lack of action by national governments on the fiscal front.'

[32]Let us however reiterate that our focus has been on long-term issues. This paper therefore does not provide any guidance in terms of whether the required F adjustment should start immediately or only after recovery from the recent financial crisis is fully under way. The analysis thus cannot be interpreted as advocating or rejecting austerity measures recently undertaken by some countries.

The paper has several implications that can be taken to the data. Specifically, our analysis implies that for some but not all parameter values, a more explicit long-term M commitment can have three effects. First, it can reduce the average level and the variability of inflation, and increase M policy credibility. This is consistent with results due to Fang and Miller (2010), Neyapti (2009), Corbo, Landerretche and Schmidt-Hebbel (2001) or Debelle (1997) among others.

Second, M commitment can act as a partial substitute for central bank goal independence (patience δ_M and/or conservatism ϕ_M) in achieving credibility. This is in line with the negative correlation between central bank (goal) independence and accountability reported by Briault, Haldane and King (1997), de Haan, Amtenbrink and Eijffinger (1999), and Sousa (2002).

Third, M commitment may be able to discipline F policy and induce reductions in the average level and the variability of budget deficits and debt (except for small free-riding members of a M union). In addition to Don Brash's account in Section 7.3 and other narrative evidence, in our companion paper Franta, Libich and Stehlík (2012) we find some support this hypothesis formally using a novel empirical methodology that combines time varying parameters Vector Autoregressions with sign, magnitude, and contemporaneous restrictions. The paper shows that countries that have adopted an explicit inflation target (such as Australia, Canada, and the United Kingdom) have improved their F outcomes markedly post-adoption, which contrasts the developments in comparable non-targeters (such as the United States, Japan, and Switzerland) and most small EMU members.

There are two issues regarding robustness and extensions worth noting. First, our long-run M commitment is flexible in the sense that the central bank is still able to choose the desired short-run stabilization actions every period without any restrictions on *how* these choices need to be made. Put differently, since shocks have a zero-mean over the business cycle, our M commitment is compatible with a discretionary solution, an instrument rule such as Taylor (1993), as well as the timeless perspective type of commitment of Woodford (1999), or the quasi commitment of Schaumburg and Tambalotti (2007).

Second, commitment and rigidity can easily be endogenized in our framework. We could incorporate into the payoffs some cost of increasing M policy commitment (such as implementation cost of inflation targeting), $\frac{\triangle C^M}{\triangle r^M} > 0$, and some political cost of reducing F rigidity (such as loss of votes from an unpopular welfare or pension reform), $\frac{\triangle C^F}{\triangle r^F} < 0$. This would enable us to derive the equilibrium values of r^M and r^F that are optimally selected by the policymakers.[33]

9. References

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[33]We do not follow this route here as it would merely alter the key variable from r^i to C^i without obtaining additional theoretic insights: M's sure-win would obtain if $C^M < \underline{C^M}\left(C^F, .\right)$. In addition, r^i seems easier to identify and interpret than C^i. Nevertheless, this is done in a different context in Libich and Stehlík (2011).

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APPENDIX A. PROOF OF PROPOSITION 1

Proof. We solve the game backwards and prove the claims by mathematical induction, initially focusing on $r^M > r^F$. First, we derive conditions under which AM will be played in M's last move on the equilibrium path, $n^M = N^M$ (the inductive basis). Specifically, part A) of the proof will examine the case $R = 0$, and part B) the case $R > 0$. Second, supposing that this holds for some $n^M \leq N^M$, we show in part C) the conditions under which the same is true for $n^M - 1$ as well.[34]

A) $\mathbf{n^M = N^M}$ **under** $\mathbf{R = 0}$. Here we have $T(r^M, r^F) = r^M$, and therefore $N^M = 1$ and $N^F = \frac{r^M}{r^F}$. Solving backwards, we know F would like to play the best response to M's initial action, $F_n^* \in B(M_1), \forall n^F$. From her second move till the end of the dynamic stage game F can observe M_1, and will hence rationally respond with PF to M_1^A, and AF to M_1^P.

Moving backwards, M uses this information and hence knows that if he opens with AM he will from period r^F onwards be rewarded by payoff a. But M also knows that such inducement play may be costly, payoff b, if F plays F_1^A. Therefore, to achieve the Ricardian World M's victory reward must more than offset his conflict cost, in which case M's optimal play in period 1 will be AM even if he knows with certainty that F_1^A will be played. Formally, the incentive compatibility condition (15) in the main text needs to hold. Using (7)-(8) and rearranging yields equation (16). The $\overline{r^M}(0)$ threshold is therefore the necessary and sufficient degree of M commitment that delivers the Ricardian World under $R = 0$.

B) $\mathbf{n^M = N^M}$ **under** $\mathbf{R > 0}$: We know that the number of M's moves is $N^M = \frac{T(r^M, r^F)}{r^M} > 1$. A condition analogous to (15) is $br^F R + a(r^M - r^F R) > dr^M$ which implies, using (7)-(8) and rearranging,

$$(20) \qquad\qquad r^M > \frac{4 - \rho_M}{\phi_M - \rho_M} R r^F.$$

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[34]It will become evident that for most parameter values satisfying (12) there will be a *unique* Ricardian SPNE. Nevertheless, since our attention is on the equilibrium path we will not examine the exact number of SPNE (off-equilibrium behaviour).

C) $\mathbf{n^M + 1 \to n^M}$ (if applicable, ie if $1 \leq n^M < N^M$): The proof proceeds by induction. We first assume that M's unique best play in the (n^M+1)-th step is AM regardless of F's preceding play (ie that M_{n+1} is history-independent), and we attempt to prove that this implies the same assertion for the n^M-th step. Intuitively, this means that if M inflates he finds it optimal to immediately disinflate. Two scenarios are possible in terms of the underlying F behaviour that determines the costs of the disinflation. If F runs a deficit, AF, the conflict costs b and w will occur for at least one period. In contrast, if F switches to PF pre-emptively (in anticipation of the disinflation, it will only be accompanied by the payoffs a and v and hence costless. This implies that one of the following two conditions analogous to (15) will apply at any move n^M

$$(21) \qquad bk_n + a(r^M - k_n) + a[r^F - (r^F - k_{n+1})] > dr^M + \underbrace{b[r^F - (r^F - k_{n+1})]}_{(AM,AF): \text{ costly disinflation}} ,$$

$$(22) \qquad bk_n + a(r^M - k_n) > d[r^M - (r^F - k_{n+1})] + \underbrace{a(r^F - k_{n+1})}_{(AM,PF): \text{ costless disinflation}}.$$

Which of these two conditions is relevant for a certain n^M depends on F's payoffs $\{v, w, y, z\}$, and importantly on k_{n+1}. In particular, if

$$(23) \qquad \underbrace{z(r^F - k_{n+1})}_{(PM,AF)} + \underbrace{wk_{n+1}}_{(AM,AF)} \geq \underbrace{y(r^F - k_{n+1})}_{(PM,PF)} + \underbrace{vk_{n+1}}_{(AM,PF)} ,$$

then (21) obtains, otherwise (22) is the relevant condition. Now, we will show that if the conditions (21) and (22) are satisfied at $n^M = 1$, then they hold in all other n^M as well. This interesting feature notably simplifies the solution of the game.

Lemma 1. *Consider the Game of Chicken in which (7)-(8) hold and $\delta_F = \delta_M = 1$. For any given R, out of the necessary and sufficient conditions for the Ricardian World, $\{M_n^A\} = B(F^A)$, the one regarding the initial move $n^M = 1$ yields as least as high $\overline{r^M}(R)$ as any other n^M. Therefore, $\{M_1^A\} = B(F_1^A)$ is the sufficient condition.*

Proof. Equations (21) and (22) can be, respectively, rearranged into

$$(24) \qquad r^M > \frac{(k_n - k_{n+1})(a-b)}{a-d} \quad \text{and} \quad r^M > r^F + \frac{k_n(a-b)}{a-d} - k_{n+1}.$$

Using the specific payoffs in (7)-(8) yields

$$(25) \qquad r^M > \frac{(k_n - k_{n+1})(4 - \rho_M)}{\phi_M - \rho_M} \quad \text{and} \quad r^M > r^F + \frac{k_n(4 - \rho_M)}{\phi_M - \rho_M} - k_{n+1}.$$

The strength of both conditions is increasing in k_n and decreasing in k_{n+1}. Thus the strongest condition is guaranteed by the maximum of $(k_n - k_{n+1})$. From (10) it follows that $k_n - k_{n+1} \leq Rr^F$. The fact that $k_1 - k_2 = Rr^F$ then proves the claim for $R > 0$. Realizing that for $R = 0$ we have $N^M = 1$ finishes the proof. $\qquad \square$

Continuing the proof of Proposition 1, Lemma 1 means that regardless of the exact dynamics/asynchrony R, it suffices to focus on the initial simultaneous move (similarly to a one-shot game) *assuming* that all further relevant conditions hold. If the strongest

condition for $n^M = 1$ is satisfied we then know that a unique (type of) equilibrium outcome obtains throughout. Lemma 1 therefore implies, in combination with the recursive scheme, that throughout the proof we can use the following:

$$(26) \qquad k_n = k_1 = r^F \ \text{ and } k_{n+1} = k_2 = (1-R)r^F.$$

Substituting (26) into (25) we obtain, together with (16)

$$(27) \qquad r^M > \overline{r^M}(R) = \begin{cases} \frac{4-\rho_M}{\phi_M-\rho_M}r^F & \text{if } R = 0, \\ \left(\frac{4-\rho_M}{\phi_M-\rho_M} + R\right)r^F & \text{if } R \le \bar{R} = \frac{4(4-\rho_F)}{17}, \\ \frac{4-\rho_M}{\phi_M-\rho_M}Rr^F & \text{if } R > \bar{R} = \frac{4(4-\rho_F)}{17}, \end{cases}$$

where the threshold $\bar{R} \in (0,1)$ is derived from (23). The $\overline{r^M}(R)$ variable is the necessary and sufficient threshold for the Ricardian World (note that all three are at least as strong as the condition for N^M in (20)). By inspection, $\overline{r^M}(R)$ is, for all R, monotonically increasing in r^F and ρ_M, and decreasing in ϕ_M.[35] It is also increasing in ρ_F which follows from the fact that the condition for $R \le \bar{R}$ is stronger than the one for $R > \bar{R}$, and hence a higher ρ_F increases $\bar{R}$ and leads to strengthening of (27). This completes the proof of claim (i).

In terms of claim (ii), by symmetry the necessary and sufficient condition for the non-Ricardian World is

$$(28) \qquad r^F > \overline{r^F}(R) = \begin{cases} \frac{4-\phi_F}{\rho_F-\phi_F}r^M & \text{if } R = 0, \\ \left(\frac{4-\phi_F}{\rho_F-\phi_F} + R\right)r^M & \text{if } R \le \bar{R} = \frac{4(4-\phi_M)}{17}, \\ \frac{4-\phi_F}{\rho_F-\phi_F}Rr^M & \text{if } R > \bar{R} = \frac{4(4-\phi_M)}{17}. \end{cases}$$

Notice that the threshold $\overline{r^F}(R)$ is just a 'mirror-image' of the threshold $\overline{r^M}(R)$. Furthermore, the former threshold can be expressed in terms of r^M rather than r^F to obtain the threshold $\underline{r^M}(R)$ in the main text. Specifically, switching sides of r^M and r^F (28) can be re-written as

$$(29) \qquad r^M < \underline{r^M}(R) = \begin{cases} \frac{\rho_F-\phi_F}{4-\phi_F}r^F & \text{if } R = 0, \\ \left(\frac{\rho_F-\phi_F}{4-\phi_F+R(\rho_F-\phi_F)}\right)r^F & \text{if } R \le \bar{R} = \frac{4(4-\phi_M)}{17}, \\ \frac{\rho_F-\phi_F}{(4-\phi_F)R}r^F & \text{if } R > \bar{R} = \frac{4(4-\phi_M)}{17}. \end{cases}$$

By inspection, $\underline{r^M}(R)$ is, for all R, increasing in r^F and ρ_F, and decreasing in ϕ_F. In addition, given that the condition for $R \le \bar{R}$ is now weaker than the one for $R > \bar{R}$, the threshold $\underline{r^M}(R)$ is also decreasing in ϕ_M (in a step manner). This completes the proof of Proposition 1. □

APPENDIX B. PROOF OF PROPOSITION 2

Proof. To prove this existence claim it suffices to provide a specific example. Let us consider the simplest case of $R > 0$, namely $r^M = 3, r^F = 2$ (implying $R = \frac{1}{2}$) and the payoffs in (7)-(8) with $\rho_M = 0$. To prove that there exists no Ricardian SPNE it suffices

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[35]Equation (27) implies that the conditions for the $R > 0$ cases only differ quantitatively from the $R = 0$ case, not qualitatively.

to show that F will play AF in one of her moves regardless of the preceding move of M. To prove that there exists at least one non-Ricardian SPNE it suffices to show that in neither of his moves M will play AM regardless of F's preceding move.

Focus on the condition for M's last move to be uniquely AM in equation (20), $\frac{r^M}{r^F} > \frac{4R}{\phi_M}$. Notice that since $R = \frac{1}{2}$, under $\phi_M < \overline{\phi_M} < \frac{4}{3}$ the condition is *not* satisfied. Therefore, M_2 is not history-independent and it will be the best response to F's preceding move, F_2. Moving backwards, player F takes this into account in comparing the continuation payoffs from F_2^P and F_2^A. Under M_1^A the continuation payoff from playing F_2^P is $-4\rho_F$, whereas from playing F_2^A it is $-4-3\phi_F$. Therefore, if $\rho_F > \overline{\rho_F} > 1 + \frac{3\phi_F}{4}$ then F_2 is history-independent - regardless of M's preceding move, M_1, F will uniquely play F_2^A in order to ensure the non-Ricardian regime levels for the remaining four periods of the stage game. This proves that in this case there exists no Ricardian SPNE as there will never be F_2^P on the equilibrium path.

In order to prove that there exists a non-Ricardian SPNE it suffices to note that, similarly to M_2, the M_1^A level is not a unique play regardless of the level played in F_1. Put differently, we have $M_1^P \in B(F^A)$ since M knows that F_2^A is always played and there would be no victory reward from M_1^A. This implies that the non-Ricardian SPNE with $\left(F_1^A, M_1^P, F_2^A, M_2^P, F_3^A\right)$ on the equilibrium path belongs to the set of SPNE. $\square$

Appendix C. Proof of Proposition 3 (can be removed)

Proof: The derivation of the generalized necessary and sufficient threshold is analogous in all its aspects to that of Proposition 1. In part A) the condition corresponding to (15) under M's impatience, $\delta_M < 1$, is

$$(30) \qquad b\sum_{t=1}^{r^F} \delta_M^{t-1} + a \sum_{t=r^F+1}^{r^M} \delta_M^{t-1} > d\sum_{t=1}^{r^M} \delta_M^{t-1}.$$

This can, using the formula for a sum of a finite series and rearranging, be written as

$$\delta_M^{r^M} < \frac{(a-b)\,\delta_M^{r^F} + b - d}{a - d}.$$

Taking the logarithms yields

$$(31) \qquad r^M > \overline{r^M}\,(0) = \log_{\delta_M} \frac{(a-b)\,\delta_M^{r^F} + b - d}{a - d}.$$

The condition of part B) is again weaker than that. To prove part C) let us extend the result of Lemma 1 under the general payoffs and players' impatience.

Lemma 2. *Lemma 1 holds $\forall \delta_M \leq 1, \forall \delta_F \leq 1$, and any general payoffs satisfying (11).*

Proof. Lemma 1 shows this claim to hold under $\delta_M = \delta_F = 1$. The proof of Proposition 1 showed that the payoffs of the less committed player, F in our case, only affect the necessary and sufficient condition through the threshold $\bar{R}$. The same will thus be true for the value of δ_F. Let us therefore consider the effect of M's impatience. Under

$\delta_M < 1$, the inequality in (21) that applies to the case of $R > \bar{R}$ becomes

$$(32) \qquad b \sum_{t=1}^{k_n} \delta_M^{t-1} + a \sum_{t=k_n+1}^{r^M} \delta_M^{t-1} + a \sum_{t=r^M+1}^{r^M+k_{n+1}} \delta_M^{t-1} > d \sum_{t=1}^{r^M} \delta_M^{t-1} + b \sum_{t=r^M+1}^{r^M+k_{n+1}} \delta_M^{t-1}.$$

This can be, after some manipulation, rearranged into

$$(33) \qquad (a-b) \sum_{t=1}^{r^M} \delta_M^{t-1} - (a-d) \sum_{t=1}^{r^M} \delta_M^{t-1} < (a-b) \delta_M^{k_n} \frac{1 - \delta_M^{r^M+k_{n+1}-k_n}}{1 - \delta_M}.$$

Since $\delta_M < 1$ we see that, analogously to Lemma 1, the strength of the condition is increasing in k_n and decreasing in k_{n+1}. Hence the same argument applies. We can readily check, using (22) under $\delta_M < 1$, that the same is true for $R \leq \bar{R}$. $\qquad \square$

We will now complete the proof of Proposition 3 using this result. Lemma 2 implies that we need to substitute (26) into (33) for the costly disinflation case. Using formulas for finite sums, rearranging, and taking the logarithms yields

$$(34) \qquad r^M > \log_{\delta_M} \frac{(a-d) - (a-b)\left(1 - \delta_M^{r^F}\right)}{(a-d) - (a-b)\left(1 - \delta_M^{r^F(1-R)}\right)}.$$

For the costless disinflation case, the analog of (22) under $\delta_M < 1$ is, using Lemma 2

$$(35) \qquad b \sum_{t=1}^{r^F} \delta_M^{t-1} + a \sum_{t=r^F+1}^{r^M} \delta_M^{t-1} > d \sum_{t=1}^{r^M-r^F R} \delta_M^{t-1} + a \sum_{t=r^M-r^F R+1}^{r^M} \delta_M^{t-1},$$

and after rearranging

$$(36) \qquad r^M > \log_{\delta_M} \frac{b\left(1 - \delta_M^{r^F}\right) + a\delta_M^{r^F} - d}{a\left(1 + \delta_M^{-r^F} - \delta_M^{-r^F(1-R)}\right) - d\delta_M^{r^F R}}.$$

The threshold $\bar{R}$ determining whether the costly disinflation case of (34) or the costless disinflation case of (36) applies is derived from the generalization of (23) under F's impatience. Specifically, under $\delta_F < 1$ if

$$(37) \qquad z \sum_{t=1}^{r^F R} \delta_M^{t-1} + w \sum_{t=r^F R+1}^{r^F} > y \sum_{t=1}^{r^F R} \delta_M^{t-1} + v \sum_{t=r^F R+1}^{r^F},$$

then (34) obtains, otherwise (36) is the relevant condition. After rearranging this implies the following threshold

$$(38) \qquad \bar{R} = \frac{1}{r^F} \log_{\delta_F} \frac{z - y + (v - w)\delta_F^{r^F}}{z - y + v - w}.$$

Combining (31), (34), (36), and $\bar{R}$ from (38) yields the following generalized necessary and sufficient condition for the Ricardian World

$$
(39) \qquad r^M > \overline{r^M}(R) = \begin{cases} \log_{\delta_M} \frac{(a-b)\delta_M^{r^F}+b-d}{a-d} & \text{if } R = 0, \\[3mm] \log_{\delta_M} \frac{(a-d)-(a-b)\left(1-\delta_M^{r^F}\right)}{(a-d)-(a-b)\left(1-\delta_M^{r^F(1-R)}\right)} & \text{if } R \leq \bar{R}, \\[3mm] \log_{\delta_M} \frac{b\left(1-\delta_M^{r^F}\right)+a\delta_M^{r^F}-d}{a\left(1+\delta_M^{r^F}-\delta_M^{r^F(R-1)}\right)-d\delta_M^{Rr^F}} & \text{if } R > \bar{R}. \end{cases}
$$

We can now use this condition to prove the claims of Proposition 3. Examining (38) and (39) reveals that $\overline{r^M}(R)$ is a function of r^F, both players' discount factors δ_M and δ_F, and all the payoffs except c.

In terms of the patience threshold, consider the logarithm's numerator of (31), (34), and (36). For the threshold $\overline{r^M}(R)$ to exist for all R it must hold that $(a - b)\delta_M^{r^F} + b - d > 0$. Rearranging this inequality yields the necessary patience threshold $\overline{\delta_M}(.)$ in (17).

Finally, note that if $\delta_F < \overline{\delta_F}$ then there are cases in which $\overline{r^M} = r^F$ as claimed in (18) where $\overline{r^M} \geq r^F$. In such case *any* $r^M > r^F$ uniquely ensures discipline of both policies. The easiest way to see this is to consider $\delta_F = 0$. Such an impatient F will never reduce spending before the start of disinflation as she fully ignores the future. Therefore, disinflation will always be costly for both players, ie (36) no longer applies and (34) becomes the relevant condition $\forall n^M, R \in (0,1)$, and for all $\{a, b, d, v, w, y, z\}$ satisfying (11). This completes the proof of Proposition 3. $\qquad\square$

Appendix D. Proof of Proposition 4

Proof. After joining the union, there is a decrease in s_j and a possible increase in m_j. If in country j the degree of free-riding is above a certain threshold

$$
(40) \qquad m > \overline{m_j}\left(s_j, v_j^{\mathcal{B}}\right) \quad \text{where} \quad \frac{\partial \overline{m_j}\left(s_j, v_j^{\mathcal{B}}\right)}{\partial s_j} > 0 \quad \text{and} \quad \frac{\partial \overline{m_j}\left(s_j, v_j^{\mathcal{B}}\right)}{\partial v_j^{\mathcal{B}}} > 0,
$$

then the value of $v_j^{\mathcal{A}}$ will fall below $w_j = 0$. This follows, using a continuity argument, from the monotonicity of $v_j(m_j)$ and the assumed $v_j(m_j = 1) < w_j = 0$. In such case the underlying game after accession for country j is no longer the Game of Chicken but the Neglect scenario since AF becomes a strictly dominant strategy. Therefore, we will observe AF for any level of $\delta_{M,j}$ and r_j^F, even if the common central bank has $\delta_M = 1$ and $r^M \to \infty$.

In terms of claim (ii), denote the number of countries in which $m_j > \overline{m_j}\left(s_j, v_j^{\mathcal{B}}\right)$ by $\gamma \in \mathbb{N}$, and order the member countries such that those $\{1, ..., \gamma\}$ feature $m_j > \overline{m_j}$, and those $\{\gamma + 1, ..., J\}$ feature $m_j \leq \overline{m_j}$. Let us report the conditions only for the special case $R = 0$ and $\delta_M = 1$ as it was shown to be representative of the other cases as well.

The condition analogous to (15) becomes

$$\underbrace{b\sum_{j=1}^{J}s_j r_j^F}_{(AM,AF)} + \underbrace{b\sum_{j=1}^{\gamma}s_j(r^M - r_j^F)}_{(AM,AF):\;\forall j\leq\gamma} + \underbrace{a\sum_{j=\gamma+1}^{J}s_j(r^M - r_j^F)}_{(AM,PF):\;\forall j>\gamma} > \underbrace{dr^M}_{(PM,AF):\forall j}.$$

Note that the condition only differs from the no free-riding case in the second element on the left hand side, which is now the payoff b rather than a since the γ countries with $m_j > \overline{m_j}$ will not switch to PF. Intuitively, the cost of conflict is higher and the victory reward is lower. Rearranging yields

$$(41) \qquad\qquad r^M > \overline{r^M} = \frac{(a-b)\sum_{j=\gamma+1}^{J}s_j r_j^F}{b\sum_{j=1}^{\gamma}s_j + a\sum_{j=\gamma+1}^{J}s_j - d}.$$

By inspection, $\overline{r^M}$ is increasing in the total size of the γ free-riders, $\sum_{j=1}^{\gamma}s_j$. Since the numerator is positive, if the denominator is negative then the threshold $\overline{r^M}$ does not exist. This means that even $\delta_M = 1$ and $r^M \to \infty$ do not guarantee the AM outcome. By inspection this happens if $\sum_{j=1}^{\gamma}s_j$ is above a certain threshold that is an increasing function of b, and a decreasing function of a and d.[36]   □

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[36]Alternatively, this condition can be expressed as $d > \bar{d}\left(\underset{+}{a}, \underset{-}{b}, \underset{-}{\gamma}, s_j\right)$, where $\bar{d}$ is decreasing in s_j for all $j \leq \gamma$, and increasing in s_j for all $j > \gamma$.